Comparing methods of image reconstruction in electrical impedance tomography

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The problem of the image reconstruction in Electrical Impedance Tomography (EIT) is a highly ill-posed inverse problem. There are mainly two categories of image reconstruction algorithms, the direct algorithm and the iterative algorithm which was used in this publication. This paper presents the applications of the level set function for identification the unknown shape of an interface motivated by Electrical Impedance Tomography (EIT) by using a several level set algorithms. The conductivity values in different regions are determined by the finite element method. The applications were based on the level set method, the variational level set algorithm and the Mumford-Shah algorithm to solve the inverse problem.

1. Introduction

There are many algorithm of the image reconstruction in the electrical impedance tomography (Fig. 1). In this paper was proposed a method based on the combination level set idea and a few numerical methods to solve the inverse problem [2, 8]. The representation of the boundary shape and its evolution during an iterative reconstruction process is achieved by the level set method [5, 6, 7]. The shape derivatives of this problem involve the normal derivative of the potential along the unknown boundary. Numerical algorithm is a combination of the level set method. Mumford-Shah model [4] and variational level set method [3] to solve the inverse problem in the electrical impedance tomography (Fg.2). The conductivity values in different regions are determined by the finite element method. Numerical iterative algorithm is a combination of the level set methods for following the evolving step edges and the finite element method for computing the velocity. The objective function is defined as the difference between the potential due to the applied current and the measured potential. This function is minimized.

Methods of the image reconstruction in the electrical impedance tomography are following [1]:

A. Deterministic methods
- back-projection method,
- perturbation method,
- layer stripping,
- Kaczmarza method,
- Wexlera method,

- gradients mehod:
  - Newtona-Rapsona method,
  - steepest descent method,
  - Levenberg–Marquardt algorithm,
  - conjugate gradient method,
  - bell functions,
- level set methods.

B. Stochastic methods
- genetic algorithm (GA),
- simulated annealing (SA),
- Monte-Carlo method (MC).

C. Neural Networks.

D. Hybrid model of the optimization.

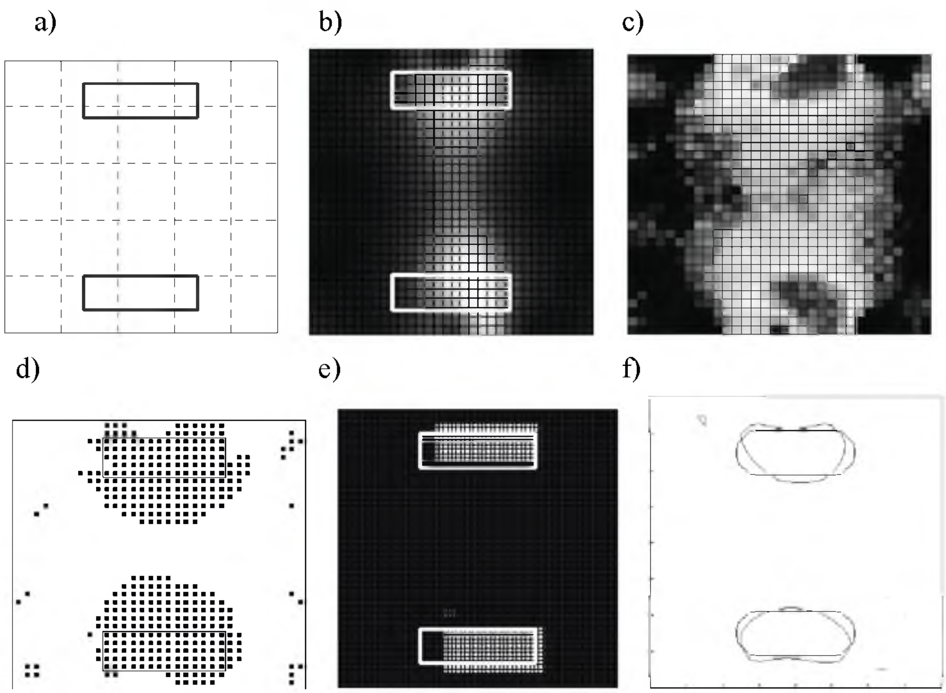

Fig. 1. The image reconstruction: a) model, b) conjugate gradient method, c) simulated annealing, d) genetic algorithm, e) bell functions, f) level set method

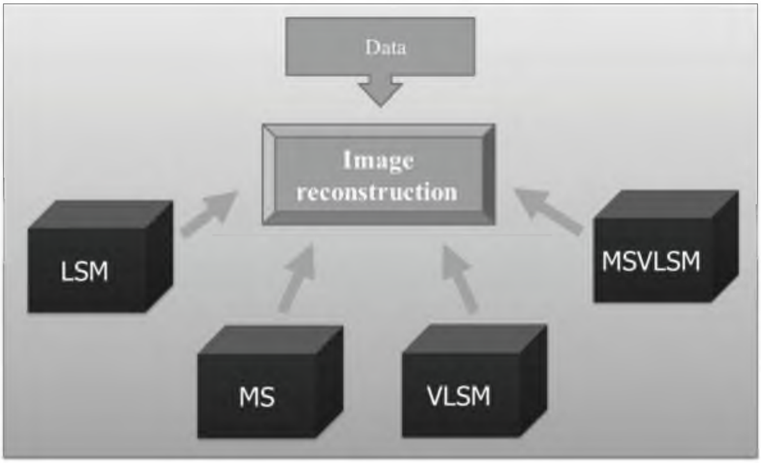

Fig. 2. The reconstruction methods

2. Level set method

The level set method tracks the motion of an interface by embedding the interface as the zero level set of the signed distance function. The motion of the interface is matched with the zero level set, and the resulting initial value partial differential equation for the evolution of the level set function. The idea is merely to define a smooth function $\phi(x,t)$, that represents the interface as the set where $\phi(x,t)=0$. The motion is analyzed by the convection the ϕ values (levels) with the velocity field. The Hamilton-Jacobi equation of the form [6]:

$$\frac{\partial\phi}{\partial t}+\mathbf{v}\,\nabla\phi=0 \tag{1}$$

where **v** is the velocity on the interface.

When flat or steep regions complicate the determination of the contour, the reinitialization is necessary. This reinitialization procedure is based by replacing by another function that has the same zero level set but behaves better. This is based on following partial differential equation:

$$\frac{\partial}{\partial t}\phi+\mathrm{S}(\phi)(\nabla\phi-1)=0 \tag{2}$$

where $\mathrm{S}(\phi)$ is defined as:

$$\mathrm{S}(\phi)=\begin{cases}-1 \text{ for } \phi<0\\ 0 \text{ for } \phi=0\\ 1 \text{ for } \phi>0\end{cases}$$

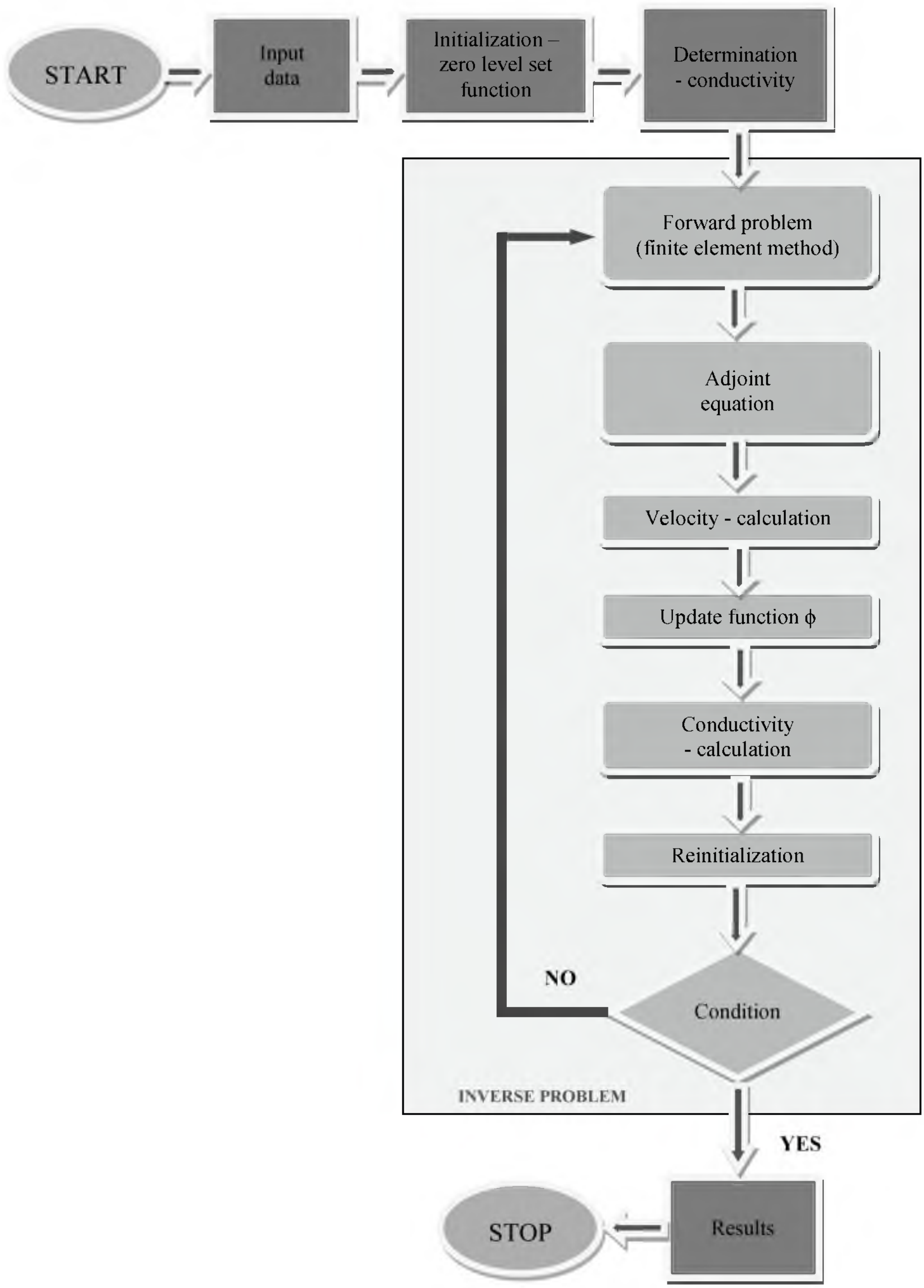

Fig. 3. The iterative algorithm

The iterative algorithm was shown in Figure 3. The Figure 4 presents the images of reconstruction by using the level set methods and the finite element method.

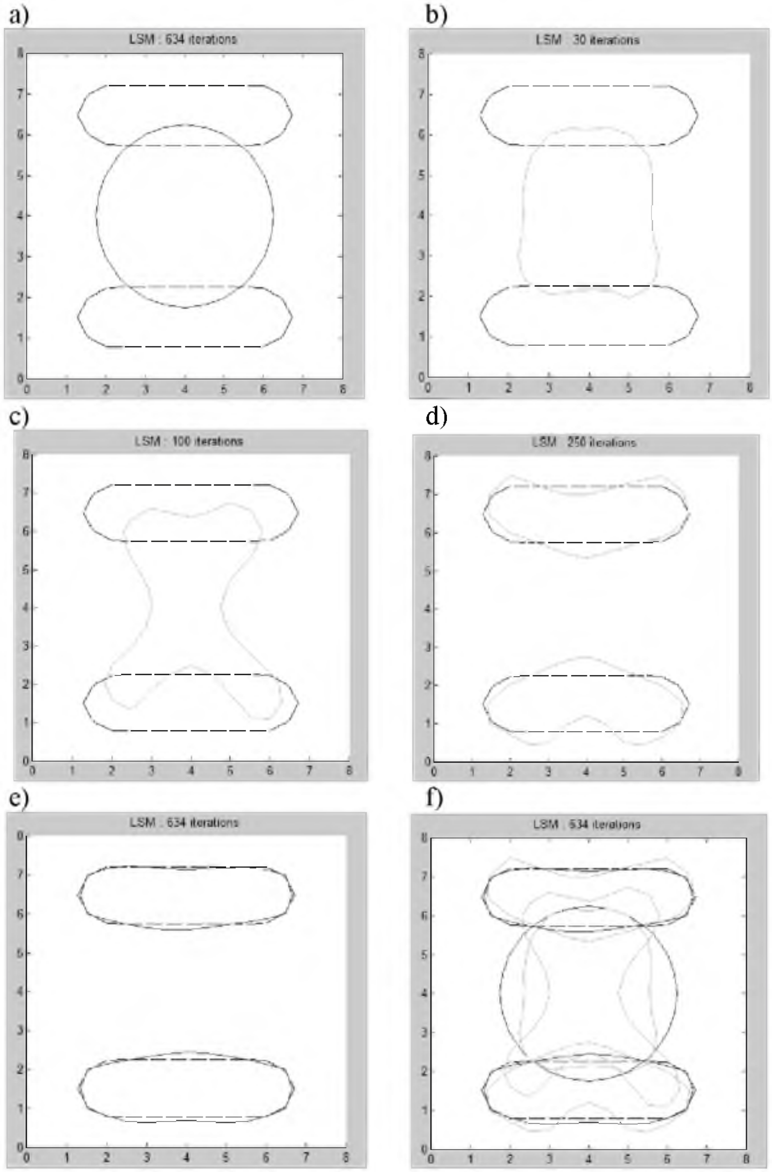

Fig. 4. The image reconstruction (mesh size 16×16) – the level set method: a) the original object and the zero level set function, b) after 30 iterations, c) after 100 iterations, d) after 250 iterations, e) the final reconstruction, f) the process reconstruction

3. Mumford Shah model

The Mumford-Shah algorithm sets formulation and minimization problem in image processing, to compute piecewise-smooth optimal approximations of a given image. The proposed model follows and fully generalizes work [4], where there was proposed an active contour model without edges based on a 2-phase segmentation and level sets γ_1 and γ_2.

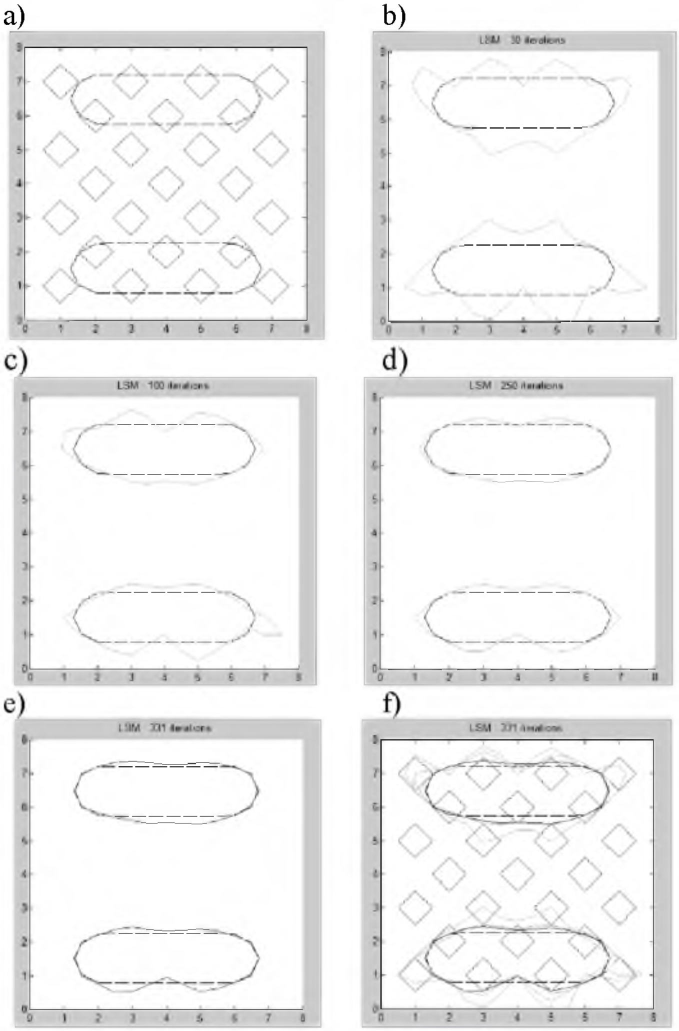

Fig. 5. The image reconstruction – Mumford-Shah model: a) the original object and the zero level set function, b) after 30 iterations, c) after 100 iterations, d) after 250 iterations, e) the final reconstruction, f) the process reconstruction

Conductivity γ is represented as:

$$\boldsymbol{\gamma} = \boldsymbol{\gamma}_{\mathrm{I}}\mathbf{H}(\boldsymbol{\phi}) + \boldsymbol{\gamma}_{\mathrm{II}}(1-\mathbf{H}(\boldsymbol{\phi})) \tag{3}$$

where **H** is the Heaviside function.
The derivative of F with respect to γ is given by

$$\left[\frac{\partial \mathrm{F}}{\partial \boldsymbol{\gamma}}\right] = -\sum_{j=1}^{\mathrm{p}} \nabla \mathbf{u}_j \nabla \mathbf{p}_j \tag{4}$$

Level set function is updated the following iterative scheme:

$$\boldsymbol{\phi}^{k+1} = \boldsymbol{\phi}^{k} - \mu\left[\frac{\partial \mathrm{F}}{\partial \boldsymbol{\phi}}\right] \tag{5}$$

where coefficient $\mu>0$ and

$$\left[\frac{\partial \mathrm{F}}{\partial \boldsymbol{\phi}}\right] = \left[\frac{\partial \mathrm{F}}{\partial \boldsymbol{\gamma}}\right]\frac{\partial \boldsymbol{\gamma}}{\partial \boldsymbol{\phi}} = \left[\frac{\partial \mathrm{F}}{\partial \boldsymbol{\gamma}}\right](\boldsymbol{\gamma}_{\mathrm{I}} - \boldsymbol{\gamma}_{\mathrm{II}})\,\delta(\boldsymbol{\phi}) \tag{6}$$

where δ is the Dirac delta function.

Figure 5 presents model of computer simulation an image reconstruction by using the Mumford-Shah model.

4. Variational level set method

The formulation of the variational level set method consists of an internal energy term that penalizes the deviation of the level set function and an external energy term that drives the motion of the zero level set toward the desired image features. Variational formulation for geometric active contours that forces the level set function to be close to a signed distance function, and therefore completely eliminates the need of the costly reinitialization procedure. The resulting evolution of the level set function is the gradient flow that minimizes the overall energy functional [3]:

$$\boldsymbol{P}(\boldsymbol{\phi}) = \int_{\boldsymbol{\Omega}} \frac{1}{2}(|\nabla\boldsymbol{\phi}| - 1)^2\,dxdy \tag{7}$$

An external energy for a function $\phi(x, y)$ is defined as below:

$$\boldsymbol{E}(\boldsymbol{\phi}) = \mu\,\boldsymbol{P}(\boldsymbol{\phi}) + \boldsymbol{E}_m(\boldsymbol{\phi}) \tag{8}$$

$\boldsymbol{P}(\boldsymbol{\phi})$ – internal energy,

$\boldsymbol{E}_m(\boldsymbol{\phi})$ – external energy.

Denoting by $\dfrac{\partial \mathbf{E}}{\partial \boldsymbol{\phi}}$ the Gateaux derivative of the functional **E** receiving the following evolution equation:

$$\frac{\partial \phi}{\partial t} = -\frac{\partial E}{\partial \phi} \tag{9}$$

The process for minimization of the functional E is the following:

$$\frac{\partial \phi}{\partial t} = \mu \left[\Delta \phi - \operatorname{div}\left(\frac{\nabla \phi}{|\nabla \phi|}\right) \right] + \nu \delta(\phi) \tag{10}$$

Figure 6 presents model of computer simulation an image reconstruction by using the variational level set methods.

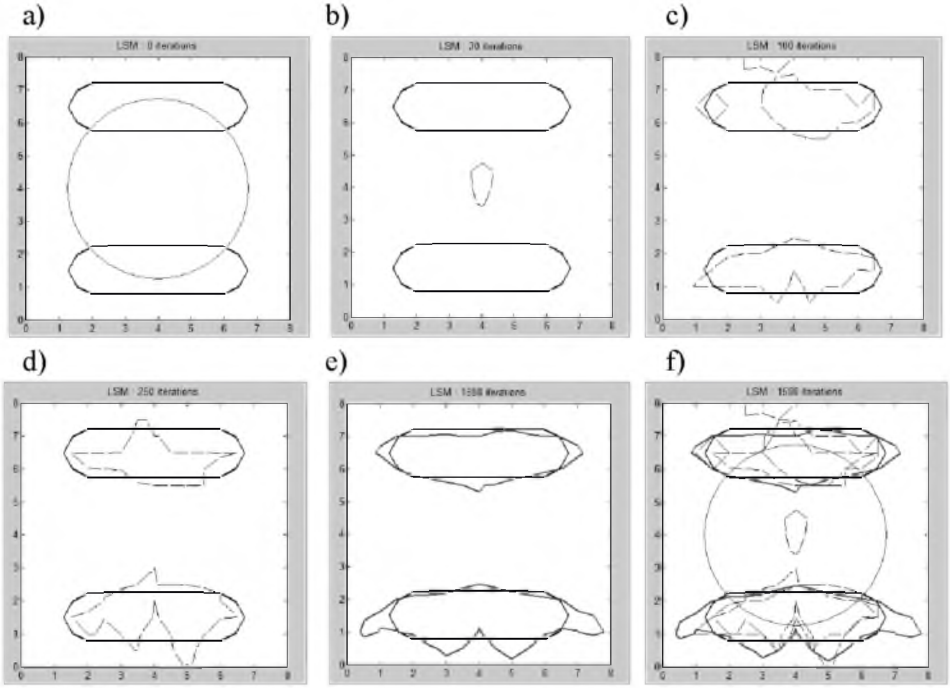

Fig. 6. The image reconstruction – the variational level set method: a) the original object and the zero level set function, b) after 30 iterations, c) after 100 iterations, d) after 250 iterations, e) the final reconstruction, f) the process reconstruction

5. Variational level set method with Mumford-Shah model

The method is based on the Mumford-Shah algorithm the variational level set method.

Figure 7 presents model of computer simulation an image reconstruction by using the variational level set methods and Mumford-Shah model.

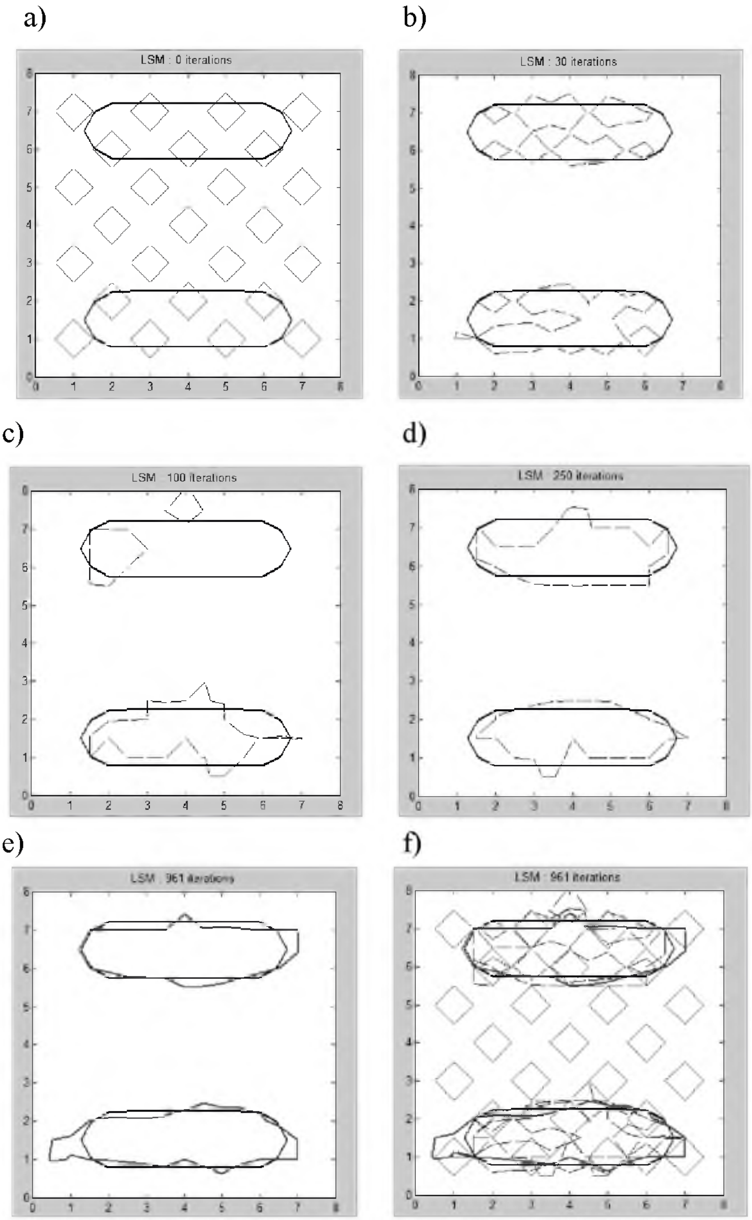

Fig. 7. The image reconstruction – the variational level set method with Mumford-Shah model: a) the original object and the zero level set function, b) after 30 iterations, c) after 100 iterations, d) after 250 iterations, e) the final reconstruction, f) the process reconstruction

6. Comparing methods

Table 1 shows values of the objective function for one and two objects by using the various algorithms. The Figure 8 presents an image reconstruction by using the level set methods, the variational level set method and Mumford-Shah model. The Figures 9 shows minimum values of the objective function.

Table 1. Values of the objective function

ALGORITHMS	Objective function (min)	
	1 object	2 objects
Level Set Method	145	611
Mumford-Shah model	194	332
Variational Level Set Method	90	1598
Variational Level Set Method and Mumford-Shah Model	1455	961

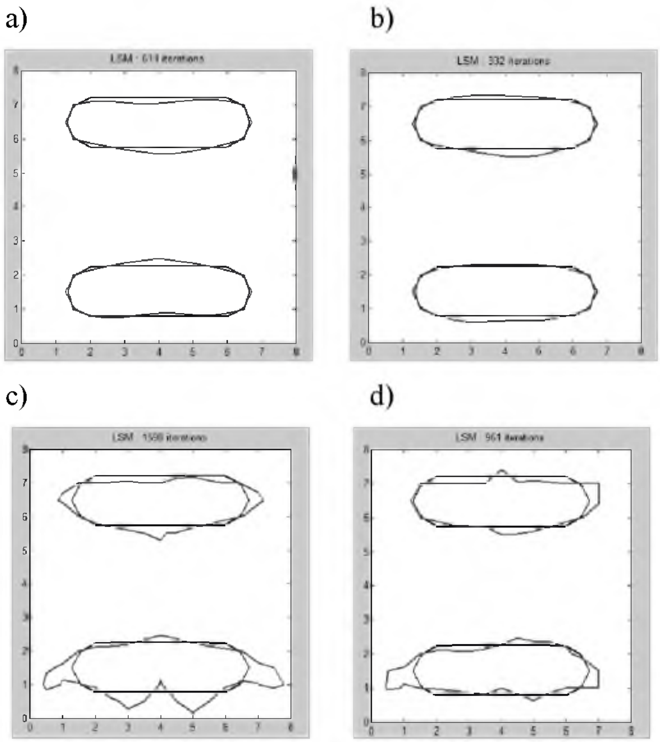

Fig. 8. Comparing methods – 2 objects: a) the level set method, b) Mumford-Shah model, c) the variational level set method, d) the variational level set method with Mumford-Shah model

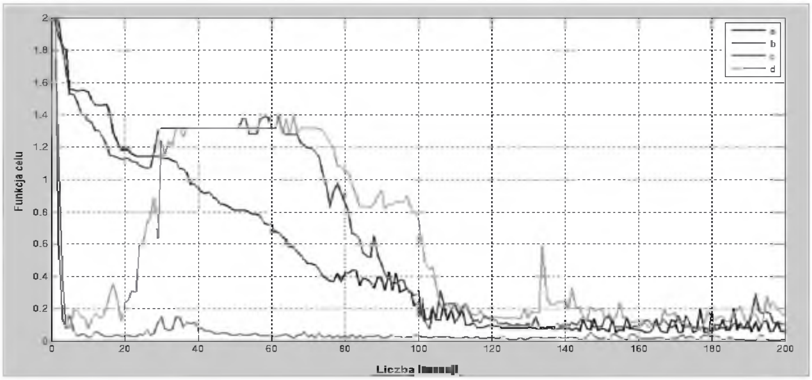

Fig. 9. The objective function - 2 objects

7. Conclusion

This paper has introduced the level set algorithms of approximation of material coefficient. The applications of the level set function, the variational level set method, Mumford-Shah model and the finite element method for the electrical impedance tomography were presented. The level set idea is known to be a powerful and versatile tool to model evolution of interfaces. Variational formulation is the faster the traditional level set because completely eliminates the need of the costly reinitialization procedure. The Mumford-Shah model gives the best quality reconstruction unknown areas with many objects.

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