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THERMAL MODELLING FOR PRODUCTION OF HOT STRIP OF MAGNESIUM ALLOY

The paper deals with a thermal modelling of a magnesium coil in the magnesium strip production. The magnesium coil heating is the preparation step for the magnesium hot strip rolling and occurs in the special air circulating furnace. The cooling of the coil takes place on the several steps such as a transport of coil after heating from the furnace to a mill coiler. The aim of the work is to develop a tool to accurately simulate the temperature distribution within coil during the magnesium coil heating in the furnace. The major thermal properties for material and furnace were determined indirectly based on experimental data. Simulation results are compared to data obtained from experimental trials.

1. INTRODUCTION

Magnesium alloys are used nowadays for a wide variety of application. It is due to its low density and high strength-to-weight ratio. One of the most efficient and applicable technology to produce the magnesium strip as semi-finished product is the developed technology at the Institute of Metal Forming, TU Bergakademie Freiberg [5]. This technology is based on three main technological steps:

- (i.) Twin-Roll Casting (TRC),
- (ii.) Heating and
- (iii.) Reverse strip rolling.

The intermediate product of these steps is a coiled strip. The state of the coiled strip depends on the technological step. It could be a rough strip, annealed/heated and finish rolled coiled strip. All of these steps are occurring at the temperature above 225°C due to magnesium's hexagonal lattice system.

The presented paper deals with the thermal modelling of magnesium coil after TRC during heating in the furnace. In addition to heating, the air cooling of the coil is also taken

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into consideration. The paper is organised in the following order. Firstly, the features of the furnace are presented with respect to the modelling of heating process. Subsequently, the modelling of the heat transfer according to boundary conditions in the coil is described. The numerical results are discussed in conclusion.

2. DESCRIPTION AND ANALYSIS OF COIL HEATING

Furnace for magnesium coil heating Based on the described technology of magnesium strip production in [5] the coil heating of rough coiled strip is the preparation stage before a hot rolling process. The coil heating occurs in the special engineered air circulation furnace for magnesium. The principle of the heating process in the furnace is based on the circulation of the heated air within a chamber by means of furnace's fans. Thereby, the temperature distribution in the furnace chamber is set during heating process very homogeneous. The air circulation furnace for magnesium is suitable not only for heating but also for cooling. However, a separated furnace model has not been developed. It is assumed therefore the uniform changing of the furnace temperature in the whole chamber during the heating. This assumption enables us to simulate the temperature within the furnace chamber during the heating process without modelling the furnace. The temperature distribution within furnace can be calculated and compared with the experimental data. The obtained experimental data yields information about the temperature in the furnace and in the coil. Furthermore, the essential thermal coefficients such as heat transfer coefficient and thermal conductivity during the heating are based on this information. The determination of these coefficients is discussed later.

Air cooling of coil After the heating the coil is transported from the furnace to the mill coiler. During this transport the coil is subjected to the air cooling due to temperature difference between the coil surface and the environment. To determine the thermal coefficients such as heat transfer coefficient, thermal conductivity during cooling we used again experimental data. The modelling of this stage is caused by the pre-planned modelling purposes.

Heat transfer mechanisms Since the temperature in this process ranges from 350°C up to 450°C, no radiation occurs and therefore radiation is not included in the modelling. The outer and inner surfaces of the coil mantels are being heated in the furnace by air-forced convection due to a circulating of hot air. The inner cross section of the coil is heated through the conduction process, which is retarded due to the air gaps between the strips and the contact zone properties. Several researchers concerning this topic for a steel coil have made studies. The main issue of these works was the modelling of thermal conductivity in radial direction throughout the coils. They consider and model the cross section of the coil as a layered structure. This layered structure consists of a material layer, an oxide or/and dirt layer and a contact layer. This contact layer is composed of actual contact zones and air gaps between them. Based on this structure, they calculated equivalent the radial thermal conductivity. Park et al. [3] took it into account via defining equivalent unit contact conductivity. Similar approach was used by Saboonchi et al. [4] for multi-stacking coils.

The description of contact conditions of both groups is based on a preliminary work performed by Mikic [2]. The value of the equivalent radial thermal conductivity obtained by these researchers depends on the contact pressure or radial stress across the interfacial zone, surface quality and material properties.

However, with our equipment it is not possible to measure the contact pressure and there are no results for magnesium. Therefore the solution of this problem was found by means of inverse method, which is based on the experimental measurements. As a result the coefficients for heat transfer within the furnace, see Eqn. (1), and radial thermal conductivity, see Eqn. (2), were determined as follows:

$$h = 1.2 \cdot e^{(3.285 + 1.57 \cdot 10^{-3} T)} \quad (1)$$

$$k_{rad} = 19.72 \cdot \left(1 - e^{\left(-\left(\frac{T}{131.79} \right)^{0.89} \right)} \right) \quad (2)$$

3. MODELLING THE COIL HEATING

Governing equations The three dimensional transient heat transfer problem of coil heating in the furnace can be written in Euclidean coordinates as:

$$\rho c_p \frac{\partial T}{\partial t} = \nabla(k \nabla T) \quad \text{in } (t_{start}, t_{end}) \times \Omega \quad (3)$$

where: T (K) is the temperature, the matrix $k = \{k_{ij}\}_{ij=1,\dots,3}$ (W/m K) contains the thermal conduction according to spatial direction, ρ (kg/m³) is the density, and c_p (J/kg K) is the specific heat capacity. The axial conductivity is the conductivity of the magnesium whereas the radial conductivity adapts the layered structure of the coil. The fitting was performed by means of an inverse method based on the experimental data, which were written during the trial of coil heating in the furnace through thermocouples. The position of the thermocouples was chosen that it could be possible to describe the heat flow in radial as well as in axial direction. The contact conditions between layers are involved in the radial component of the conductivity coefficient. Furthermore, the anisotropy is modelled by the transformation from the cylindrical system to the Cartesian system. Note that the matrix k is not diagonal as it is in the formulation in the cylindrical coordinates. Moreover, the symmetry with respect the x-y-plane is used in the model, cf. Fig. 1.

Note that the temperature $T = T(t, x)$ depends from the time and the spatial variable, while the coefficients can depend from the temperature and hence change in time and space too. To shorten the notation, we will drop these dependencies throughout this paper, except we need this explicitly in the notation.

The domain $\Omega \subset \mathbb{R}^3$ is modelled as a hollow cylinder with inner radius r_{inner} , outer radius r_{out} and width d . We assume the magnesium coil as a continuum and model the layer structure by using a non-isotropic formulation in the equation, c.f. the remark about the coefficient matrix k . In contrast to most former works we consider the heat transfer problem

in Cartesian coordinates instead of the, in this case, more usual cylindrical coordinates, cf. e.g. [3]. The reason for solving this problem in Cartesian coordinates is the further simulation of the whole magnesium hot strip rolling process. The rolling process of strip will be considered by modelling in connection with uncoiling/coiling process in Cartesian coordinates. Thereby, the transformation from Cylinder to Cartesian coordinates should not be executed.

The boundary conditions are generalised Neumann type and differ from boundary segment to boundary segment. The boundary of the hollow cylinder Γ is separated into three segments:

- (i.) the inner mantle Γ_1 ,
- (ii.) the outer mantle and the top of the hollow cylinder Γ_2 , and
- (iii.) the bottom of the hollow cylinder Γ_3 .

The boundary conditions related to these three parts are:

$$\begin{aligned}\vec{n} \cdot (k\nabla T) &= -h_1(T - T_1) && \text{on } \Gamma_1 \\ \vec{n} \cdot (k\nabla T) &= -h_2(T - T_2) && \text{on } \Gamma_2 \\ \vec{n} \cdot (k\nabla T) &= 0 && \text{on } \Gamma_3\end{aligned}$$

The initial temperature is given by $T(t_{start}) = T_{start}$.

Weak formulation of heat transfer equation The idea of the finite element method (FEM) is to bring Eqn. (3) into the variation or weak form by multiplying Eqn. (3) with a suitable test function $v \in V$, where V is the space of test function, i.e. the space of integrable function with integrable derivative $H^1(\Omega)$. Doing this, we obtain the variation equation

$$\int_{\Omega} \rho c_p \frac{\partial T}{\partial t} v \, dx = \int_{\Omega} \nabla(k\nabla T) v \, dx \quad \forall v \in V. \quad (4)$$

Applying formally Green's formula to Eqn. (4), and inserting the boundary conditions we obtain

$$\begin{aligned}\int_{\Omega} \rho c_p \frac{\partial T}{\partial t} v \, dx &= - \int_{\Gamma_1} h_1(T - T_{\Gamma_1}) v \, ds - \int_{\Gamma_2} h_2(T - T_{\Gamma_2}) v \, ds \\ &\quad - \int_{\Omega} k \nabla T \nabla v \, dx \quad \forall v \in V,\end{aligned} \quad (5)$$

where the integral over Γ_3 disappears.

To find a discrete approximation of Eqn. (5), we first approximate Ω by $\Omega_{\Delta} = \cup_{i=1}^{n_e} \Omega_{\Delta}^i$, where n_e is the number of elements. Since our geometry is symmetric with respect to the z-axis, we choose Ω_{Δ}^i prisms elements with respect to the z-axis.

In a second step, we choose a finite subspace \tilde{V} of $H^1(\Omega)$. We have given the discrete temperature as $T_{\Delta}(t, x) = \sum_{i=1}^{np} \tau_i(t) \phi_i(x)$ where the functions $\phi_i(x)$ are the basis of \tilde{V} and

$\tau_i(t)$ are the time dependent coefficient functions with respect to $T_\Delta(t, x)$. In the same manner we replace also T_1 and T_2 as well as k by their discrete approximations. Since $\{\phi_i\}_{i=1 \dots np}$ is a basis of \tilde{V} , it is sufficient to test Eqn. (5) only with the basis functions of \tilde{V} . After evaluating all integrals we end up at the system.

$$\begin{aligned} \frac{d}{dt} D\vec{T}(t) &= - \left(H_1(\vec{T}(t)) + H_2(\vec{T}(t)) + K(\vec{T}(t)) \right) \vec{T}(t) + G_1(\vec{T}(t)) + G_2(\vec{T}(t)) \\ \vec{T}(t_{start}) &= \vec{T}_{start} \end{aligned} \quad (6)$$

Eqn. (6) describes a n_p -dimensional, non-autonomous, nonlinear system of ordinary differential equations. The order of the matrices and vectors in Eqn. (6) corresponds to the order of the sums over the integrals in Eqn. (5), where K is the temperature depending stiffness matrix, H_1 and H_2 corresponds together with vectors G_1 and G_2 to the boundary integrals. The matrix D is the mass matrix to ρc_p .

The main theorem of integration theory gives $\vec{T}(t)$ as the solution of the integration problem

$$D\vec{T}(t) = D\vec{T}_{start} + \int_{t_s}^t - \left(H_1(\vec{T}(\theta)) + H_2(\vec{T}(\theta)) + K(\vec{T}(\theta)) \right) \vec{T}(\theta) + G_1(\vec{T}(\theta)) + G_2(\vec{T}(\theta)) d\theta,$$

where: $t \in (t_{start}, t_{end})$. Now we discretise also the time by $t_{start} = t_0, t_1, \dots, t_{nt} = t_{end}$, end consequently we construct the iterative process

$$\begin{aligned} D\vec{T}(t_{n+1}) &= D\vec{T}_n + \int_{t_n}^{t_{n+1}} - \left(H_1(\vec{T}(\theta)) + H_2(\vec{T}(\theta)) + K(\vec{T}(\theta)) \right) \vec{T}(\theta) + G_1(\vec{T}(\theta)) \\ &\quad + G_2(\vec{T}(\theta)) d\theta \\ \vec{T}(t_{start}) &= \vec{T}_{start} \end{aligned} \quad (7)$$

where: $n=0, 1, \dots, nt$. Note that Eqn. (10) is still nonlinear, but for sufficiently small $\delta t_n := t_{n+1} - t_n$ we can assume that $Q_1(\vec{T}(\theta)) \approx Q_1(\vec{T}(t_n))$ for $\theta \in (t_n, t_{n+1})$. In the same way we can assume that we can approximate the matrices Q_2 , K and the vectors G_1 and G_2 by their values at the last time step. The problem

$$\begin{aligned} D\vec{T}(t_{n+1}) &= D\vec{T}(t_n) + \int_{t_n}^{t_{n+1}} - \left(H_1(\vec{T}(t_n)) + H_2(\vec{T}(t_n)) + K(\vec{T}(t_n)) \right) \vec{T}(\theta) + G_1(\vec{T}(t_n)) \\ &\quad + G_2(\vec{T}(t_n)) d\theta \\ \vec{T}(t_{start}) &= \vec{T}_{start} \end{aligned} \quad (8)$$

is linear in every sub interval (t_n, t_{n+1}) . The integral can now be evaluated by a suitable integration scheme, e.g. an implicit Runge-Kutta or BDF schema. This technique is called semi implicit approach.

Implementation The modelling of the coil heating has been carried out in MATLAB[®] Software by means of the OOPDE [1] toolbox provided by the Institute of Numerical Mathematics and Optimisation of TU Bergakademie Freiberg. OOPDE is a MATLAB code that provides classes for solving, pre and post process stationary and transient partial differential equations in one, two and three spatial dimensions. For special issues it offers 3D prism elements, 3D bilinear finite elements as well as ODE solvers that use iterative methods like precondition conjugate gradient method (pcg) to solve the linear problems appearing e.g. in Eqn. (12).

We model the coils as a three dimensional hollow cylinder where the inner radius is fixed and the outer radius as well as the width of the coil can vary. In order to simplify the geometrical model, the symmetry of the coil in both the x-y plane and the z-axis is used. A typical discrete model of the coil consists of 135865 prism elements with 85668 nodes.

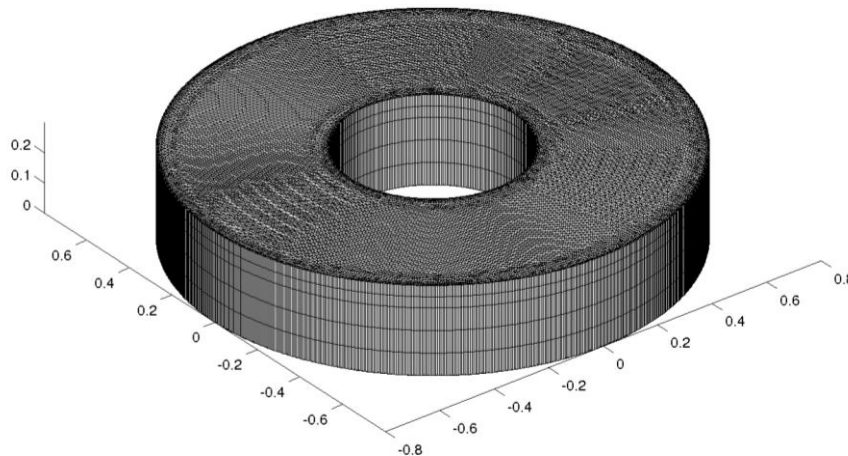


Fig. 1. Geometry model of the coil

4. EXPERIMENTAL WORK

At the Institute of Metal Forming the experimental trials were carried out, on the one hand, for validation of the developed model and, on the other hand, for the determination of important thermal coefficients. For this purpose, the coil was equipped with thermocouples ALMEMO[®]. The Temperatures were measured at the interior of the coil during coil heating in the furnace and air cooling. Fig.2 shows the position of thermocouples in the model.

The overall dimensions of the coil were 800mm (outer diameter) x 300mm (inner diameter) x 600mm (width of strip) and the thickness of the strip was 5mm. Additional temperature measurements were made also in the furnace chamber. The program for the coil

heating in the furnace was selected typical for magnesium coil. At the beginning of coil heating the furnace is being heated up to about 450°C for 1 h. Afterwards, the furnace temperature is holding at 450°C for 10 h. Thereby the temperature in the coil is achieved uniformly. At the end of the coil heating the furnace temperature is going down to 370°C for 4 h.

After the heating the coil was removed from the furnace and was left to cool down on the air. The recording of coil cooling down on the air was made for 6 hours.

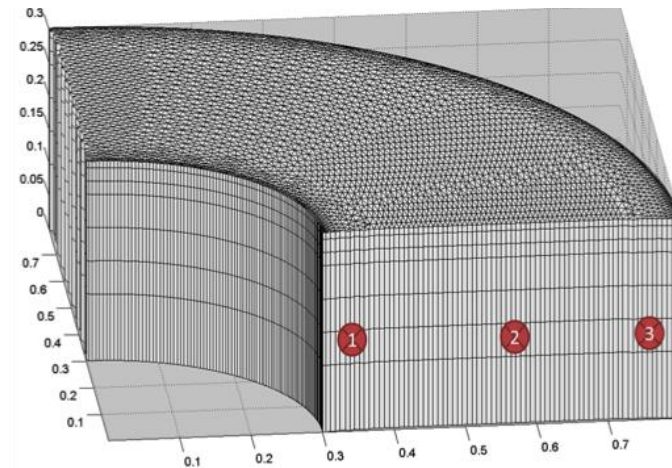


Fig. 2. The position of thermocouples in the coil cross section

5. NUMERICAL RESULTS AND DISCUSSION

The temperature distribution in the coil was calculated after the heating in the furnace and the cooling down on the air by means of the worked out model in MATLAB[®] Software. Fig. 3 exhibits the temperature distribution of the coil after heating in the furnace. It should be noted that the maximum difference between the hottest and coolest place in the coil is about 10-15°C, i.e. the temperature of the coil is nearly homogeneous distributed. On the other hand this temperature distribution seems to be correct from a physical perspective. Fig. 4 shows the temperature distribution of the coil after 6 hours cooling down on the air. The temperature after 6 hours cooling down was dropped till about 115-130°C.

Furthermore, the model's results were analysed regarding measured temperature through thermocouples in order to validate them. In addition, the numerical results were compared with the measured temperature curves at the given positions of thermocouples, cf. Fig. 2. The furnace temperature was for the calculation also analytically reproduced. The comparison of the predicted and measured temperatures in the middle of the coil cross section is shown in Fig. 5 and Fig. 6. Note that the difference of the temperatures curves occurs mostly during the heating and soaking process. The reason for these errors could be the inaccurate determination or modelling of the thermal coefficients in the furnace or the approximation of the furnace temperature. We should also take into account numerical errors, which are affected by e.g. the mesh quality, time increments etc.

On the other hand, the constitutive factor for the model accuracy is the calculation of the radial thermal conductivity between the layered structures of the coil, which depends significantly on the contact pressure between the layers. It is also clear that the pressure varies both spatially and in time. The increasing of the model's accuracy can be achieved by means of a thermo-mechanical coupled model. However, the behaviour of the temperature curves during the coil heating shows sufficient adjustment. So the model can be applied for analyse the temperature development and distribution within the coil during heating or cooling.

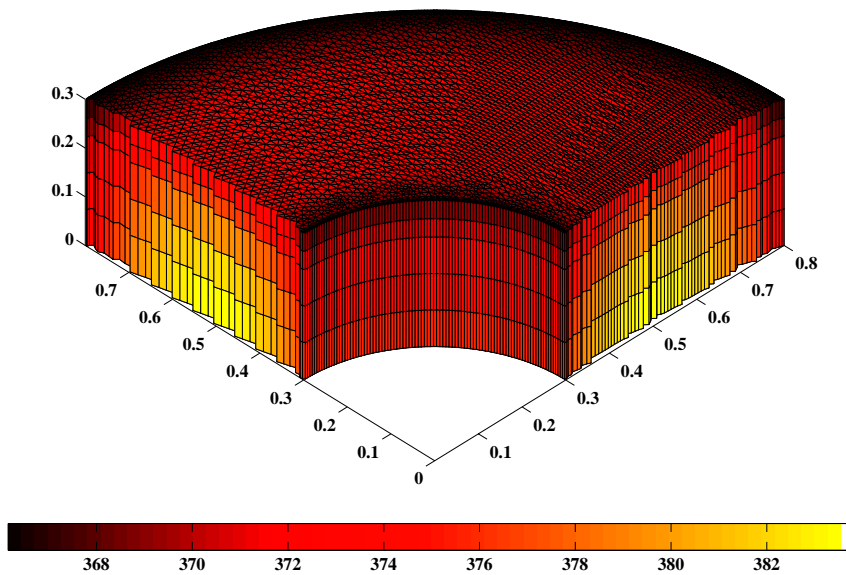


Fig. 3. Temperature distribution after heating

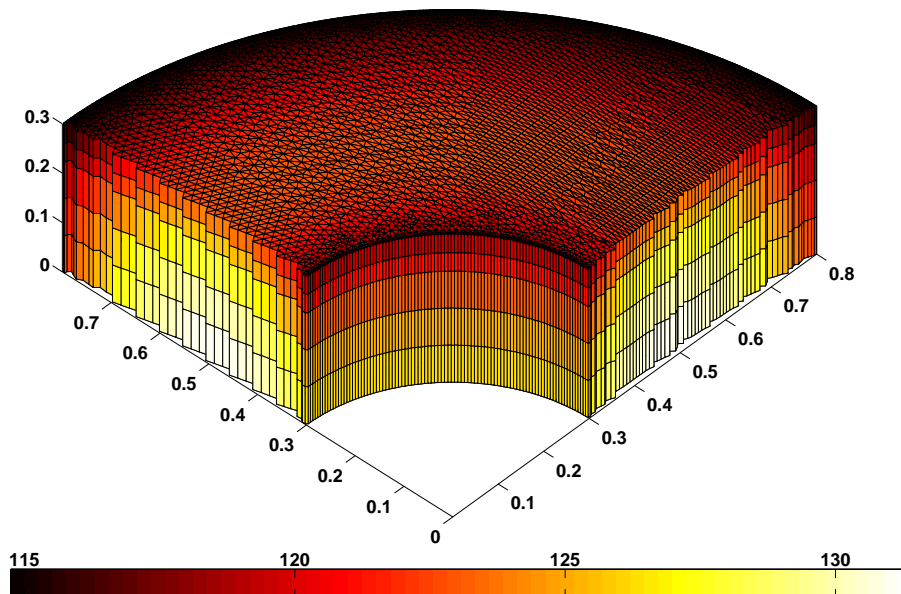


Fig. 4. Temperature distribution after cooling

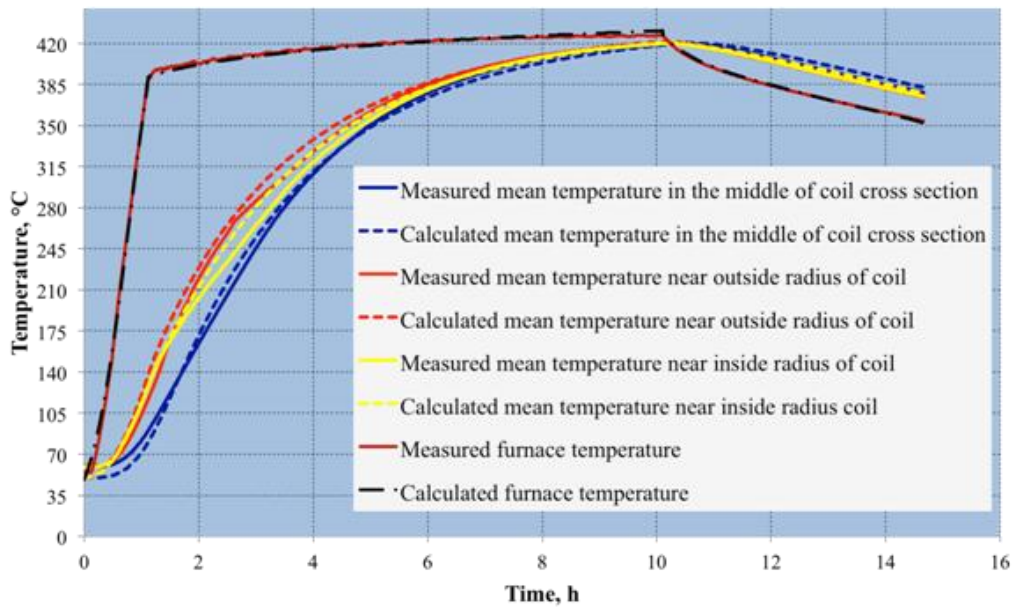


Fig. 5. Comparison of the predicted and measured temperatures in the middle of the coil cross section during heating

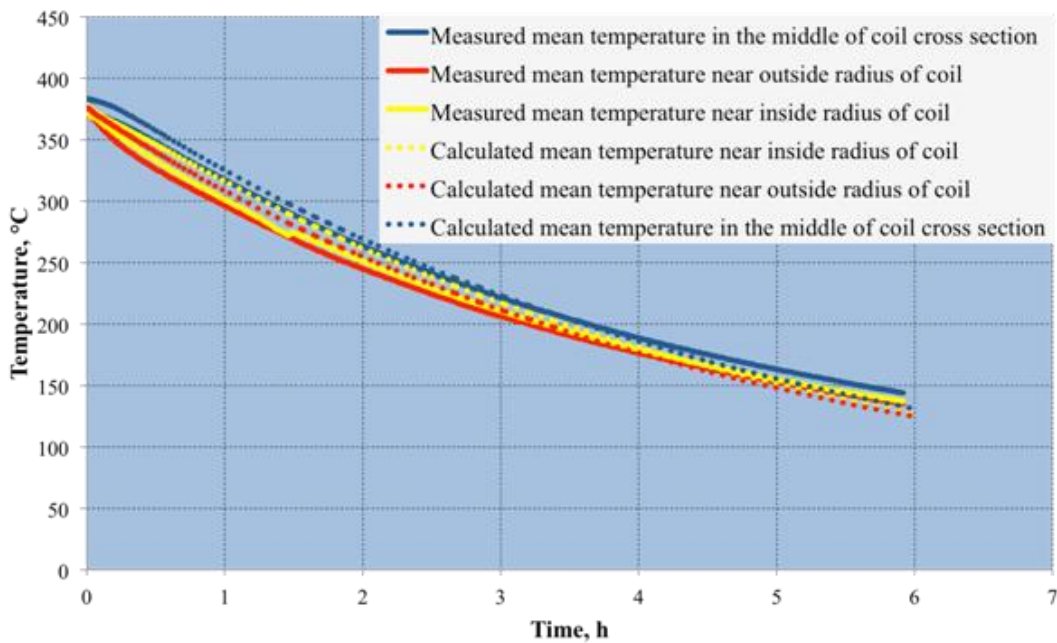


Fig. 6. Comparison of the predicted and measured temperatures in the middle of the coil section during cooling

6. CONCLUSION AND FUTURE WORK

The worked out model shows a good capability of being applied to estimate the temperature history of the coil during heating and cooling dependent on an initial temperature profile, coil geometry, boundary conditions and thermal parameters in the

production of magnesium strips. The experimental trials were carried out with a coil after TRC, which was equipped with thermocouples. The yielded results of trials were used for the estimation of the significant thermal coefficients, such as heat transfer coefficient and radial conductivity for the simulation and its validation. The work might be extended through the working out of a structural model. Thereby it should be increased the result's accuracy due to the taking into account the stress condition between the coil layers.

The application of this model enables to estimate the temperature distribution of the coil on the heating stage and at the beginning of the rolling stage. Thereby the first model for the technology chain of production magnesium strip was developed and evaluated.

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