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ABOUT SOME PROBLEMS RELATED TO DETERMINATION OF THE E.G. BIOT COEFFICIENT FOR ROCKS

Use of the poroelasticity theory by Biot in the description of rock behaviour requires the value of the e.g. Biot coefficient α to be determined. The α coefficient is a function of two moduli of compressibility: the modulus of compressibility of the rock skeleton K_s and the effective modulus of compressibility K. These moduli are determined directly on the basis of rock compressibility curves obtained during compression of a rock sample using hydrostatic pressure.

There is also a concept suggesting that these compressibility moduli might be determined on the basis of results of the uniaxial compression test using the fact that, in the case of an elastic, homogeneous and isotropic material, the modulus of compressibility of a material is a function of its Young modulus and its Poisson ratio.

This work compares the results obtained from determination of the Biot coefficient by means of results of compressibility test and uniaxial compression test. It was shown that the uniaxial compression test results are generally unsuitable to determine the value of the coefficient α. An analysis of values of the determined moduli of compressibility shows that whereas the values of effective moduli of compressibility obtained using both ways may be considered as satisfactorily comparable, values of the relevant rock skeleton moduli of compressibility differ significantly.

Keywords: poroelasticity, Biot medium, Biot coefficient, effective compressibility modulus, rock skeleton compressibility modulus

1. Basic terms and definitions

During the physical and subsequent mathematical search for computational models of rocks, the rock is initially approximated as a single-phase, continuous medium, usually brittle-elastic. In the case of a more complex rock mass structure its anisotropy is considered (for example,

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by introducing the e.g. transversal isotropy), as well as possible emergence of ductile behaviour (usually, by introducing the Coulomb-Mohr or the Hoek-Brown boundary balance condition).

Thus, designed models fail when it becomes necessary to consider the fact that rocks are not continuous materials, and in addition to the e.g. solid phase, they also include voids of various shapes and dimensions (pore space), which may be filled with pressurised pore fluid. In this case, a rock becomes a bi- or even a triphasic medium including a solid phase, namely the solid skeleton of rock (hereinafter the abbreviated term "skeleton" will be used), and fluid – liquid and/or gas – filling the pore space. The interaction between the pore fluid and the rock may be purely mechanical, including stress/strain changes in the rock caused by rock fluid pressure variations, or physico-chemical and even chemical. In the most complex situation, we are dealing with an interaction being a combination of the aforementioned factors.

The subject of consideration presented in this work is a linearly elastic (i.e. subject to Hooke's Law), isotropic and homogeneous, porous medium, the pore space of which is filled completely with pore fluid subjected to pore pressure p_p. It is assumed that the medium is subjected to macroscopic stress σ_{ij}. It is postulated that relationships between stress and strains in the considered medium shall be formulated not for the quantity pair: macroscopic stress – pore pressure, but for a certain substitute stress σ'_{ij}, hereinafter referred to as *effective stress*. It is also assumed that the effective stress is a certain function of the macroscopic stress and the pore pressure, namely that:

$$\sigma'_{ij} = f\left(\sigma_{ij}, p_p\right) \tag{1}$$

With thus formulated assumptions, Maurice Anthony Biot gave the e.g. poroelasticity equation system ([1,2]), used by Amos Nur and James D. Byerlee [3] to subsequently derive equation (1) in the form of:

$$\sigma'_{ij} = \sigma_{ij} - \alpha\, p_p\, \delta_{ij} \qquad i, j = 1, 2, 3 \tag{2}$$

This equation, in which δ_{ij} is the so called Kronecker symbol and α is a dimensionless factor, is called the effective stress equation (or law).

It is easy to notice that the key to practical use of equation (2) is to specify the value of the α coefficient, often referred to as the Biot coefficient in the literature. The aforementioned article [3] showed that in the case of a poroelastic medium, this coefficient may be calculated using the formula:

$$\alpha = 1 - \frac{K}{K_s} \tag{3}$$

where K is the e.g. effective modulus of rock volumetric compressibility (common name: bulk modulus of compressibility), and K_s is the modulus of volumetric compressibility of its skeleton (the following terms are used further in the text: K – modulus of rock compressibility, K_s – modulus of rock skeleton compressibility). The modulus of compressibility K is usually determined for a rock sample in an air-dry state, while when determining the modulus K_s, the pore space of the sample should be filled with an inert fluid.

It should be noted here that equation (2) can be treated as an effective stress law also when the subject of the analysis is not the deformation of the entire medium but only its pore space.

Then, however – as it already has been showed in [3] – the Biot coefficient α is given by the relationship:

$$\alpha = 1 - \frac{nK}{K_s - K} \tag{4}$$

where n is the porosity of the medium. The analysis of this particular relationship is not included in this work.

2. Definitions of moduli of compressibility and their determination methods

The modulus of compressibility K is usually determined for a rock sample in air-dry state, while when determining the modulus K_s, the pore space of the sample should be filled with an inert fluid. We assume that while determining the modules K and K_s the following conditions are met:

a) the temperature T during the experiment is constant at any time t throughout the sample volume ($T(t) = const.$),
b) the pore space of the rock is open and completely filled with pore fluid (drained conditions),
c) during the experiment, the changes in the pore pressure p_p in the sample occur in such a way that it can be assumed that the pore pressure is the same at any point in the sample pore space,
d) it is assumed that the tested material is linearly elastic and the stress in the sample is a superposition of the residual stress in the skeleton and the pressure of the pore fluid.

In the studies described in this work, the K and K_s moduli were determined using two methods, named as "standard" and "computable" method.

2.1. Determination of moduli of compressibility through the results of compressibility test (hereinafter the standard method)

The standard method used to determine the modulus of compressibility of rock includes a test performed on a rock sample, an e.g. compressibility test. It includes compression of a rock sample using hydrostatic pressure, with simultaneous measurement of changes to its volume; this test has to be a drained one, which means that the porous space of the rock may not be isolated from the environment. The result is a curve depicting the relationship between the hydrostatic pressure p and the volumetric strain of the sample e. If we assume that: V_0 – initial sample volume at time t_0, V – sample volume at any time t and sample volume change $\Delta V = V - V_0$, then the volumetric strain e is given by the formula:

$$e = \frac{\Delta V}{V_0} \tag{5}$$

During the test used to determine the effective rock compressibility modulus K, the sample is separated from the medium providing hydrostatic pressure by using a flexible screen, while the porous space of the rock is connected to air under atmospheric pressure. In this case, it may be stated (assuming that the filtration rate is high enough; the correctness of this assumption is not discussed in this paper) that any changes of volume of the porous space caused by rock deformation do not translate into a change of pore pressure p_p, which remains constant and equal to atmospheric pressure. The compressibility curves obtained using this method are usually denoted by label "$p_p = 0$", because the air pressure can be neglected in comparison with applied confining pressures p. In this case modulus K may be defined as:

$$K = \left(\frac{dp}{de}\right)_{t,\, p_p = 0} \tag{6}$$

Taking into account the assumptions of Biot's theory and the condition d) formulated above, it should be stated that the modulus K is determined for the rectilinear part of the relation $p(e)$, i.e. that it can be written:

$$K = \left(\frac{\Delta p}{\Delta e}\right)_{t,\, p_p = 0} \tag{7}$$

However, if the compression test is used to determine the rock skeleton compressibility modulus K_s, the sample is placed in a high-pressure chamber without shields, such that contact between the porous space of the rock with the medium providing hydrostatic pressure is ensured. It is assumed in this case, that pore pressure within the sample remains equal to the confining pressure, and sample deformation is caused by deformation of the rock skeleton only, the porous space of which does not deform. Thus, the obtained compression curves are denoted by label "$p_p = p$". So, the K_s modulus may be defined in the following way:

$$K_s = \left(\frac{dp}{de}\right)_{t,\, p_p = p} \tag{8}$$

and then, as it has already been mentioned above for the K modulus

$$K_s = \left(\frac{\Delta p}{\Delta e}\right)_{t,\, p_p = p} \tag{9}$$

Now, three important methodological remarks regarding the $p_p = p$ test should be made:

1) The fluid used in these experiments must be an inert fluid, i.e. physicochemically indifferent to the rock. It is important that the interaction between the pore fluid and the rock is only mechanical.
2) The entire pore space of the sample should be filled with the pore fluid. To achieve this, it is best to pre-place an air-dry sample "under vacuum" to remove air and moisture from its pore space, then pour pore fluid over it and check the level of saturation by weight.
3) The rate of change of the confining pressure dp/dt must be selected so that the pore pressure p_p must be able to "follow" the confining pressure p. If the dp/dt is too high, some cracks may close prematurely and the condition of equal hydrostatic and pore pressures will not be met, which will make the obtained result worthless.

Schemes of "$p_p = 0$" and "$p_p = p$" curves are shown in Figure 1. However, examples of compressibility curves obtained for both igneous and sedimentary rocks can be found in the following works: [4-7].

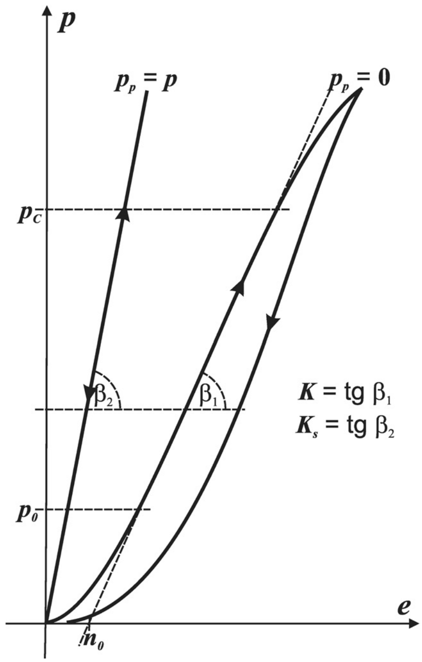

Fig. 1. Determination method of volumetric compressibility moduli K and K_s using compressibility curves; p_0, n_0 – crack closing pressure and crack porosity respectively [10], p_C – pressure of rock consolidation ([8,9])

The desired rock compressibility moduli K and K_s are, respectively, tangents of β_1 and β_2 angles of "e" slopes of the linear parts of curves, respectively, "$p_p = 0$" and "$p_p = p$" (Fig. 1). It is obvious that the $K_s \geq K$ relationship applies, wherein $K_s = K$ is true, when rock porosity meets the condition of $n = 0$.

More detailed information related to execution of compressibility tests and to interpretation of their results may be found in the works of Jerzy Gustkiewicz ([6,8,9]).

2.2. Determination of moduli of compressibility through the results of uniaxial compression test (hereinafter the computable method)

The compressibility test – although not very complicated in theory – requires a high-pressure cell and adequate sensors to measure deformation inside the cell. Such equipment is usually expensive and difficult to access. Therefore, the question arises whether there is no other way to determine the values of the rock's volumetric compressibility modulus.

If the material under study is an isotropic and homogeneous Hooke material, it has four elastic constants characterising its deformation-related properties: modulus of volumetric compressibility – K, shear modulus (Kirchhoff modulus) – G, longitudinal elasticity modulus (Young modulus) – E, and Poisson's ratio – v. In this case only two of these four constants are independent, which means that only two of them have to be determined (for example by means of laboratory tests), and the other two may be calculated using the relevant formulae.

In particular, if Young's modulus E and Poisson's ratio v are known, the volumetric compressibiliy modulus K is related to the constants E and v by the following formula ([11] – page 108):

$$K = \frac{E}{3\left(1-2v\right)} \tag{10}$$

except that – according to [12] (page 33) – the elasticity constants E and v should be determined in the drained conditions at the pore pressure $p_p = 0$ MPa.

Let us now assume that the formula (10) is universal, that is, it may be used both to calculate the rock compressibility modulus K and the rock skeleton compressibility modulus K_s. In this case the desired modulus of compressibility calculated using that formula depends on the method used to determine the Young modulus and Poisson's ratio. These constants can be calculated from the results of the well-known uniaxial compression test, the methodology of which was finally ordered by [13] however in order to obtain both compressibility moduli (K and K_s), a significant change should be introduced in this methodology.

A diagram depicting a suitable uniaxial compression test is presented below (Fig. 2). The core of this test includes execution of an unload-load loop along the linear section of the stress-strain relationship. The aim of this loop is as follows: it is assumed that, during the initial unload phase, sample cracks remain closed and the measured sample deformation includes deformation of its skeleton only. The tangent of angle γ_2, between the x-axis and the linear part of the unload curve $\sigma_1 \sim \varepsilon_1$, is known as the intrinsic Young modulus (denoted as E_{int}), wherein the ratio of transverse deformation increase $\Delta\varepsilon_{3\text{-}int}$, determined for the linear part of the unload curve $\sigma_1 \sim \varepsilon_3$, to the longitudinal deformation increase $\Delta\varepsilon_{1\text{-}int}$, determined for the linear part of the unload curve $\sigma_1 \sim \varepsilon_1$, is known as the intrinsic Poisson's ratio v_{int} (see Fig. 2). Assuming, that formula (10) is universal – as it has already been mentioned above – we may define an intrinsic compressibility modulus as:

$$K_{int} = \frac{E_{int}}{3\left(1-2v_{int}\right)} \tag{11}$$

In a similar way, we can then determine the values of the effective Young's modulus E_{ef} and the Poisson's ratio v_{ef}, the determination of which includes pore deformation. The effective Young modulus E_{ef} is the tangent of angle γ_1, an angle of slope measured towards the x-axis for the linear part of the $\sigma_1 \sim \varepsilon_1$ curve. On the other hand, the effective Poisson's ratio v_{ef} is a ratio of (shown in Fig. 2) the deformation increase $\Delta\varepsilon_{3\text{-}ef}$ to the deformation increase $\Delta\varepsilon_{1\text{-}ef}$, measured for linear parts of curves, respectively, $\sigma_1 \sim \varepsilon_3$ and $\sigma_1 \sim \varepsilon_1$. Having the values of the constants E_{ef} and v_{ef} determined in this way, we can define the effective modulus of compressibility K_{ef} as:

$$K_{ef} = \frac{E_{ef}}{3\left(1-2v_{ef}\right)} \tag{12}$$

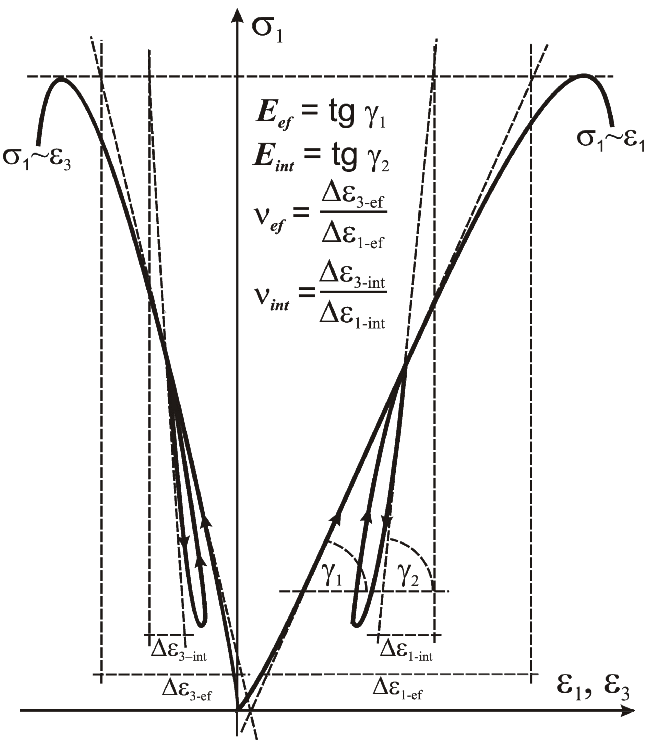

Fig. 2. Determination of effective and intrinsic material constants on the basis of an uniaxial compression test: E_{ef}, E_{int} – elasticity modulus (effective and intrinsic respectively), v_{ef}, v_{int} – Poisson's ratio (effective and intrinsic respectively)

This work was created as an attempt at an experimental answer to the question if, in the case of rock, it can be assumed that:

$$K = K_{ef} \text{ and } K_s = K_{int} \tag{13}$$

where K and K_s as in Fig. 1 and K_{ef} and K_{int} as in Fig. 2, and consequently, can we use to calculate the α coefficient the formula (3) in the form:

$$\alpha = 1 - \frac{K_{ef}}{K_{int}} \tag{14}$$

3. Experimental procedures

3.1. Compressibility test

The compressibility tests, the results of which are discussed in this paper, were performed with the use of the GTA-10 device located in the Rock Deformation Laboratory of the Strata

Mechanics Institute of the Polish Academy of Sciences in Krakow. This device consists of a cell that allows the performance of conventional triaxial compression tests according to the methodology described by [14], a set of hydraulic hydrostatic and pore pressure amplifiers, and a power supply system which is based on high-pressure pump supplying the piston travel system and the amplifiers.

The maximum hydrostatic pressure obtained in the chamber and the maximum pressure of the pore fluid are equal and amount to 400 MPa. The maximum loading force generated by the piston is 1500 kN. The diagram of the GTA-10 device is shown in Fig. 3, and detailed information on how to use it is provided by [15] and [16]. Hydrostatic pressure and pore pressure are induced by kerosene, which for the tested rocks can be considered as an inert liquid.

The deformation of the sample was measured with a coreless inductive sensor (see [17]) attached to the bands placed at the ends of the sample. A sample with attached sensor is shown in Fig. 4. Examples of the results of relevant experiments for sandstones are shown in Fig. 5 (corresponding curves for opuka stone are presented in chapter 5, Fig. 12). The volumetric

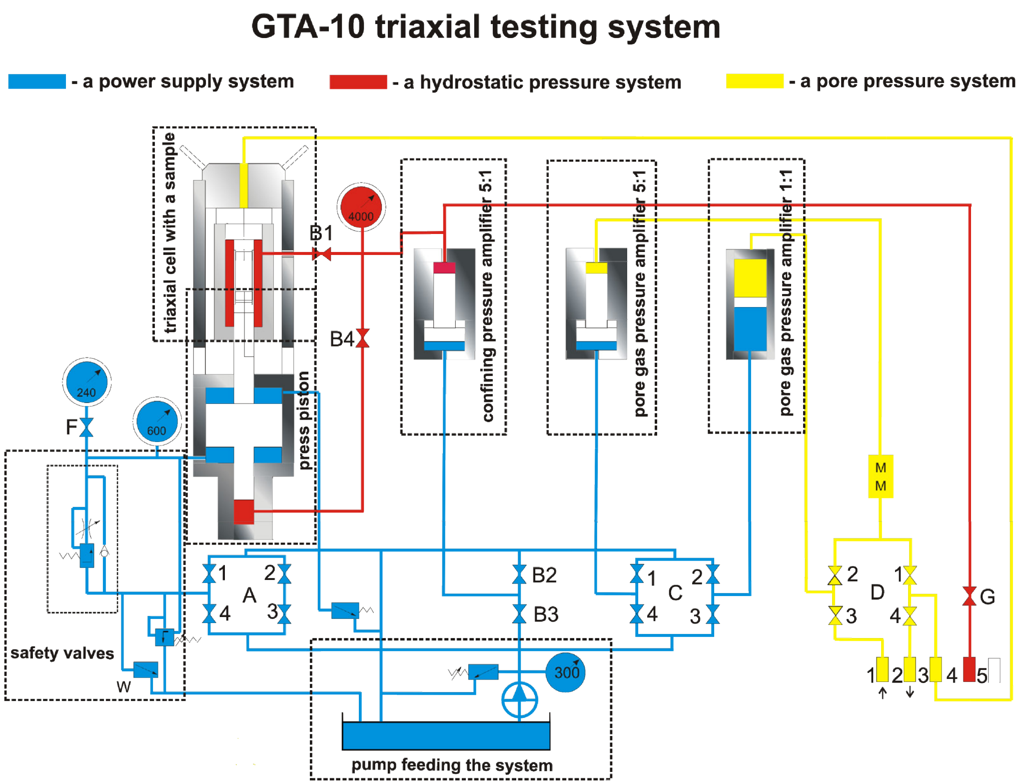

Fig. 3. Diagram of the GTA-10 device: A1, A2, A3, A4 – valves controlling the movement of the press piston, B1, B4 – valves that control the flow of liquid in the high pressure cell, B2, B3 – valves controlling liquid pressure in a triaxial cell through a pressure amplifier with a ratio of 5:1, C1, C2, C3, C4 – valves controlling pore pressure in a sample through a low pressure amplifier with a ratio of 1:1 (on the right) and high pressure amplifier with a ratio 5:1 (on the left), D1, D2, D3, D4 – valves controlling the flow of pore gas to and from inside the tested sample

deformation of the sample e was calculated from linear part of the measured deformation assuming homogeneity and isotropy of the tested material.

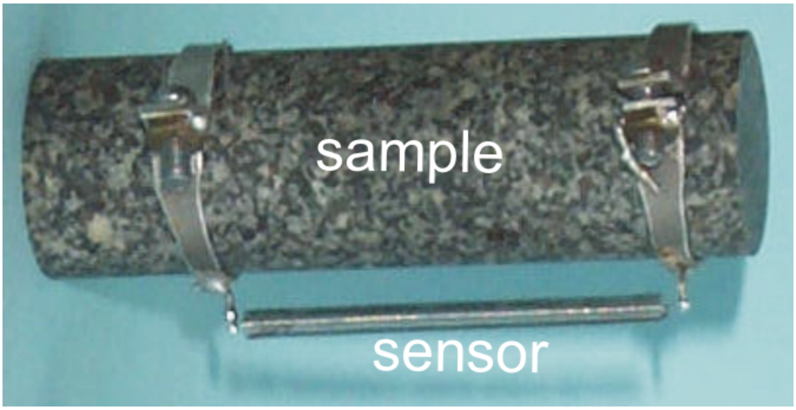

Fig. 4. The coreless inductive sensor attached to the rock sample (the "$p_p = p$" test)

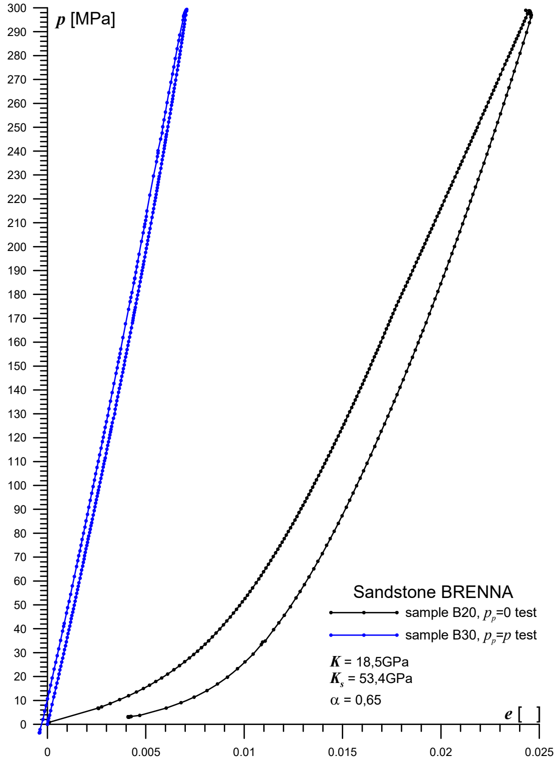

Fig. 5. Compressibility test for sandstones, results of the $p_p = p$ and $p_p = 0$ tests (an example); α – Biot coefficient, p – hydrostatic pressure, e – change of volume

3.2. Uniaxial compression test

The uniaxial compression tests were carried out with the use of the Instron 8500 Rock Testing System hydraulic strength testing machine (see Fig. 6), located at the Rock Deformation Laboratory of the Strata Mechanics Research Institute of the Polish Academy of Sciences. This device consists of a frame with a very high stiffness (10 MN × mm^{-1}) and a press actuator that enables the application of a maximum compressive force of 5 MN. The actuator control system is the e.g. *feedback loop control system* that allows to perform experiments with controlling the force loading the sample or its vertical or circumferential deformation.

The subject of the experiments were samples with a diameter d equal to 32 mm and 35 mm and a height h equal to 64 mm and 70 mm, respectively. The samples were compressed in the axial strain control mode in such a way as to maintain the axial strain rate at level of 5×10^{-5} s^{-1}. Longitudinal and circumferential deformations of the sample were measured by means of resistive strain gauges with a measuring base length of 10 mm, glued in pairs, on opposite sides of the sample (see Fig. 7).

Fig. 6. Instron 8500 Rock Testing System hydraulic strength testing machine


Fig. 7. Rock sample in uniaxial compression test (an example)

The elasticity constants, which were then used to calculate the values of the compressibility modulus K_{ef} and K_{int}, were determined on the basis of the uniaxial compression test results. An example of such a result is shown in Fig. 8. The test was performed according to the methodology described by [13] included in the collection of rock mechanics research methods recommended by International Society for Rock Mechanics and edited in [18].

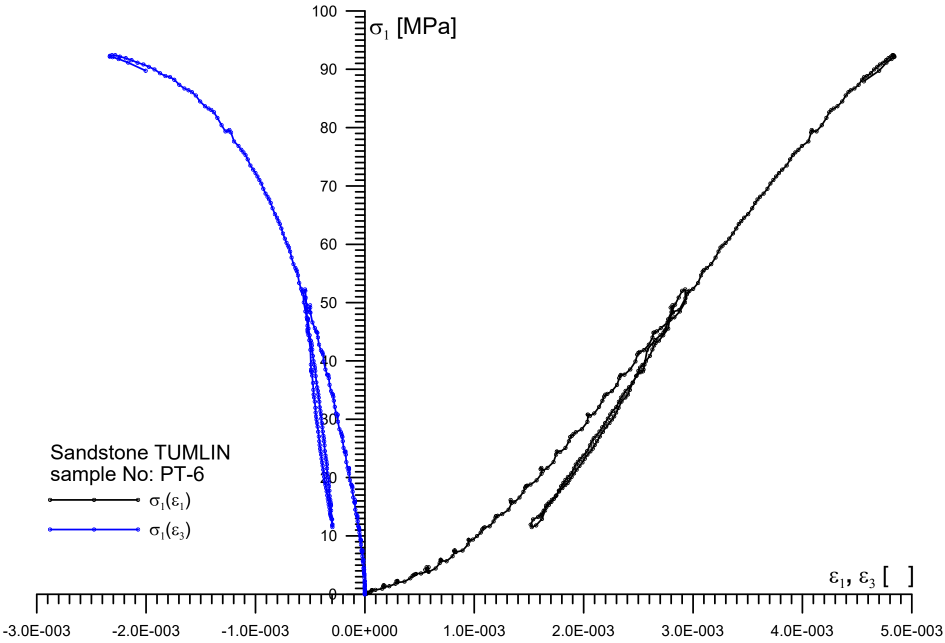

Fig. 8. Result of the uniaxial compression test with the unload-load loop (an example), sandstone "Tumlin", sample no. PT-6; σ_1 – axial stress, ε_1 – axial/longitudinal strain, ε_3 – transverse strain

4. Experimental results

The studies were performed on samples made of three materials: two Polish sandstones – named "Brenna" and "Tumlin", as well as for the rock obtained from the quarry in the village Bénatký near Litomyšl in Czech Republic and named opuka stone. The opuka stone is identified by Czech petrologists as "calcitic spongilite" or "marlstone" (see [19]).

Biot coefficient values were determined by means of standard method, on the basis of results of 11 compressibility tests: four for the "Brenna" sandstone (2 tests for K, 2 tests for K_s), three for the "Tumlin" sandstone (2 for K, 1 for K_s), and three for the opuka stone (2 for K, 1 for K_s). Test results are presented in Table 1.

During analysis of the values of the Biot coefficient presented in Table 1, it should be remembered that determination of one α value requires the results of two – performed independently and for different samples – compressibility tests: one used to determine K (col. 2, Tab. 1) and one to determine K_s (col. 3, Tab. 1). Thus, values of α coefficients presented in column 5 of Table 1 were obtained by inserting K and K_s value pairs into formula (3), as described in column 5, Table 1. The meaning of the notation is as follows: if, in the case of the "Brenna" sandstone, the line corresponding to the B10 sample indicates a B10 & B30 notation in column 4, this means that the K value obtained for the sample B10 (19.0 GPa) and the K_s value obtained for the sample B30 (53.4 GPa) were used in formula (3). The number of independently obtained (i.e. for different samples) K and K_s values allows four Biot coefficient values to be determined for the "Brenna" sandstone, two for the "Tumlin" sandstone, and two for the "Bénatký" rock.

TABLE 1

Values of Biot coefficient determined by means of the standard method

		sample number	compressibility modulus K [GPa]	compressibility modulus K_s [GPa]	Biot coefficient: a pair of samples	Biot coefficient: α [ ]	Biot coefficient: α average value [ ]
		1	2	3	4	5	6
sandstone	"Brenna"	B10	19,0		B10&B30	**0,64**	**0,68**
		B20	18,5		B10&B50	**0,70**	
		B30		53,4	B20&B30	**0,65**	
		B50		64,4	B20&B50	**0,71**	
	"Tumlin"	T30	21,8		T30&T50	**0,54**	**0,56**
		T40	21,0		T40&T50	**0,56**	
		T50		47,8			
opuka stone		O10	23,4		O10&O60	**0,76**	**0,76**
		O20	23,7		O20&O60	**0,76**	
		O60		99,3			

Biot coefficient values were determined using the computable method, analysing the results of five uniaxial compression tests: two tests for the “Brenna” sandstone (samples PB-2 and PB-3), two for the “Tumlin” sandstone (PT-5 and PT-6), and one for the opuka stone (OL-7). In this case determination of the α coefficient required results of just one experiment, and the results were analysed according to the procedures described above in chapter 2.2. The values of the coefficient α were calculated using formula (14) and the calculations results are shown in Table 2.

TABLE 2

Values of Biot coefficient determined by means of the indirect method

		sample number	effective constants ν_{ef} [ ]	effective constants E_{ef} [GPa]	intrinsic constants ν_{int} [ ]	intrinsic constants E_{int} [GPa]	compressibility moduli K_{ef} [GPa]	compressibility moduli K_{int} [GPa]	Biot coefficient α [ ]
		1	2	3	4	5	6	7	8
sandstone	"Brenna"	PB-2	0,30	19,9	0,18	32,7	16,6	17,0	**0,03**
		PB-3	0,25	19,3	0,18	33,9	12,9	17,7	**0,27**
	"Tumlin"	PT-5	0,29	31,5	0,21	47,1	25,0	27,1	**0,08**
		PT-6	0,21	23,9	0,13	38,7	13,7	17,4	**0,21**
opuka stone		OL-7	0,21	29,3	0,23	41,5	16,8	25,6	**0,34**

The values of the calculated Biot coefficients presented in col. 8 of table 2 are significantly different from that obtained by means of compressibility test. Chapter 5 of this paper will be devoted to the analysis and discussion of these results.

5. Analysis of obtained test results

A comparison of α values presented in Tables 1 and 2 shows that Biot coefficient determination methods described in chapter 2 yielded significantly different results. It is thus necessary to look for answers to two main questions:

– what caused the observed differences?
– which of the analysed Biot coefficient determination methods should be considered as correct?

The search for answers to these questions should begin with a deeper analysis of the laboratory test results recorded in Tables 1 and 2.

Firstly, it should be noted that formulas (3) and (14) used for calculating the α coefficient for standard and computable method respectively use the values of the volumetric compressibility moduli determined in different ways. A comparison of values for these constants obtained using both methods shows that whereas these values may be considered similar for the K and K_{ef} moduli (see Tab. 1, col. 2 and Tab. 2, col. 6), the differences for the K_s and K_{int} moduli are quite substantial (cf. Tab. 1, col. 3 and Tab. 2, col. 7). Moreover, whereas in the case of the standard method, the differences between K and K_s are very clear – $K_s \approx (2 \div 3)K$ – the computable method yields $K_{int} \approx (1 \div 1.5)K_{ef}$. It thus seems that the main problem lies in the determination of the value of the modulus of compression of rock skeleton K_s or K_{int}.

In the case of the standard method, the K_s modulus is determined for a sample subjected to hydrostatic compression and fully saturated with a liquid. The pressure of liquid is identical inside the sample in the porous space and around the sample, inside the high-pressure chamber. It leads us to conclusion that the measured deformation of the sample includes only the deformation of its solid phase while the pore space of the tested sample, which is filled with pore liquid, is not deformed (this statement is correct for non-isolated pores. The, so called, isolated pores, namely pores that do not have contact with other pores and thus cannot be saturated with the pore liquid, of course may deform themselves).

During the uniaxial compression test, the pore space of the sample is filled with air under atmospheric pressure, which means that the pores can deform freely. In principle, isometric pores do not deform until the load capacity of the pore wall is exceeded and the rock skeleton is destroyed, which is accompanied by permanent deformations. On the other hand, cracks – due to their specific shape – "work", i.e. they can close or open depending on the magnitude of the stresses induced by the load. This process is generally reversible and does not affect the rock skeleton, the structure of which remains intact, however it should also be noted that the material may contain cracks oriented with respect to the stress direction in such a way, that they do not "work" but their edges become stress concentrators inducing further fractures of the rock skeleton and ultimately permanent sample deformations. We can speculate that this orientation of cracks may significantly influence the results of deformation measurements in both directions in case of uniaxial compression test.

Those considerations show that in the case of the computable method, in which the sample is compressed uniaxially the sample deformations consist not only rock skeleton deformations but may also consist deformations of pore space and even permanent deformations caused by rock skeleton destruction. So in consequence, the K_{int} modulus could be "too small".

Let us now look at the relationship between the axial stress and the corresponding strains obtained during uniaxial compression tests for the tested sandstones (Figs. 9 and 10).

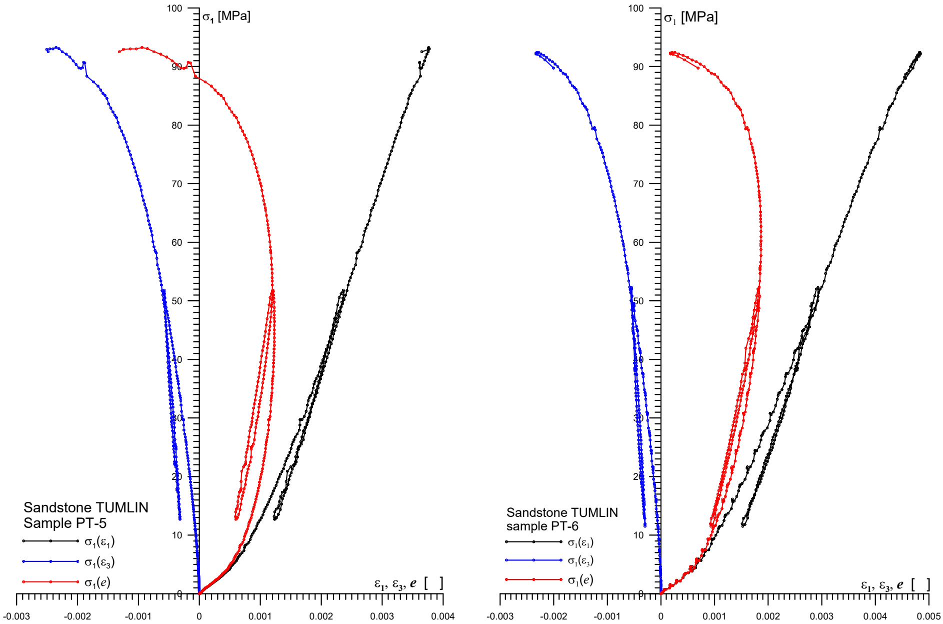

Fig. 9. Uniaxial compression test: relationship between axial stress and longitudinal strain – $\sigma_1(\varepsilon_1)$, transverse strain – $\sigma_1(\varepsilon_3)$ and volumetric strain – $\sigma_1(e)$; sandstone "Tumlin"

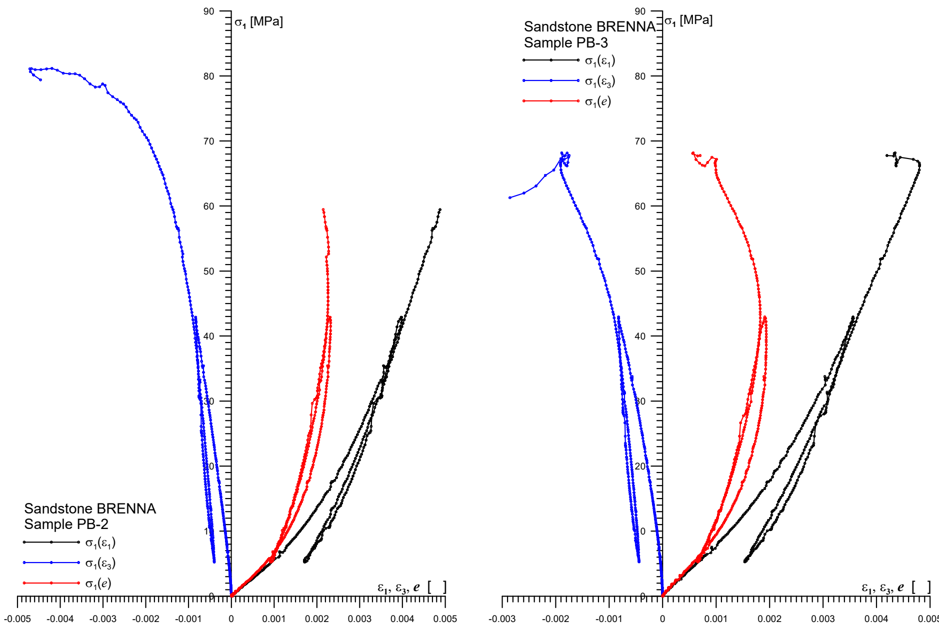

Fig. 10. Uniaxial compression test: relationship between axial stress and longitudinal strain – $\sigma_1(\varepsilon_1)$, transverse strain – $\sigma_1(\varepsilon_3)$ and volumetric strain – $\sigma_1(e)$; sandstone "Brenna"

The results of the experiments shown in these figures have some common features:

(i) all shown relationships between the stress in the sample σ_1 and its volume change e are clearly nonlinear (the smaller range of the curves $\sigma_1(\varepsilon_1)$ and $\sigma_1(e)$ for the PB-2 sample is caused by early destruction of the longitudinal strain gauge during the experiment),

(ii) the relationships between the axial stress σ_1 and the transverse deformation ε_3 of the specimen show a problem with clear identification of the linear section, which is required to determine the value of Poisson's ratio,

(iii) the unload-load loops practically do not show the elastic hysteresis and their angle of inclination to the abscissa axis suggests the appearance of permanent deformations in the sample.

The above observations lead to the emergence of a fundamental doubt: is there a stress range for the studied sandstones for which the system of Hooke's equations could be considered a constitutive equation? This doubt is important if we realize that the system of constitutive equations of Biot's medium is in fact a system of Hooke's equations supplemented with components taking into account the volumetric deformation of the pore space (see [1]). Therefore, it seems reasonable to expect that for a given material Biot deformation constants will be the same as Hooke's deformation constants with the additional condition that they will meet the e.g. constitutive equation of the pore fluid formulated by Biot.

In the case of opuka stone, the corresponding relationships, showed in Fig. 11 are slightly different. In all stress-strain curves, it is possible to distinguish linear sections, and the shape and slope of the unload-load loops to the abscissa axis practically exclude the appearance of permanent deformations. Therefore, it seems that the application of Hooke's mathematical model in this case, and consequently Biot's mathematical model, should not result in large errors. Unfortunately, it does not look so good, and the Biot coefficient for the opuka stone determined by this method is only about 50% of the value of the average Biot coefficient determined by the standard method (0,34 vs. 0,73).

It follows from the formulas (3) and (8) that the Biot coefficient is closely related to the values of the respective compressibility moduli, and these, may be calculated on the basis of the values of the effective and the intrinsic elasticity constants using formulas (11) and (12). The formulas (11) and (12) are rational functions of elasticity constants E_{ef}, v_{ef}, E_{int} and v_{int} wherein v_{ef} and v_{int} are present in the denominator. Values of such a function are particularly "susceptible" to changes to variable values located near zero points of the denominator (small changes to the function argument result in large value changes, e.g. the $y = \mathrm{tg}(x)$ function near $x = 0{,}5\pi$, or $y = x^{-1}$ near $x = 0$), which in this case means that small changes to the v_{ef} and v_{int} values may cause large changes of K_{ef} and K_{int} This is particularly important in the context of problems with correct determination of Poisson's ratio values familiar to everyone performing laboratory tests on rocks.

At this point, it is worth returning to compressibility tests. Analysing the deformation process of a rock sample loaded by hydrostatic pressure Jerzy Gustkiewicz ([6,8,9]), following considerations of Rychlewski [20], pointed out, that hydrostatic load may induce in the rock skeleton deviatoric-state stress that may cause the destruction of the rock skeleton and, finally, its permanent deformations. Let's look from this angle at the results of compressibility test carried out for opuka stone (see Fig. 12).

These results differ from the curves for sandstone shown in Fig. 5. The $p_p = p$ curve shown in Fig. 5 (sample B30) is very close to the model curve from Fig. 1 and does not indicate the appearance of permanent deformations in the tested sandstone. The same curve in Fig. 12

(sample O60) shows that – however it is linear in whole range of applied hydrostatic pressure – the deformation of the sample is not an elastic one and shows the appearance of permanent volumetric deformations of the opuka stone skeleton. The same can be said for $p_p = 0$ curves. The one shown in Fig. 5 (sample B20) corresponds well to the model curve from Fig. 1. The $p_p = 0$ curves in Fig. 12 (samples O10 and O20) differ from the corresponding model curve in Fig. 1 in the shape of the part that reflects the loading process.

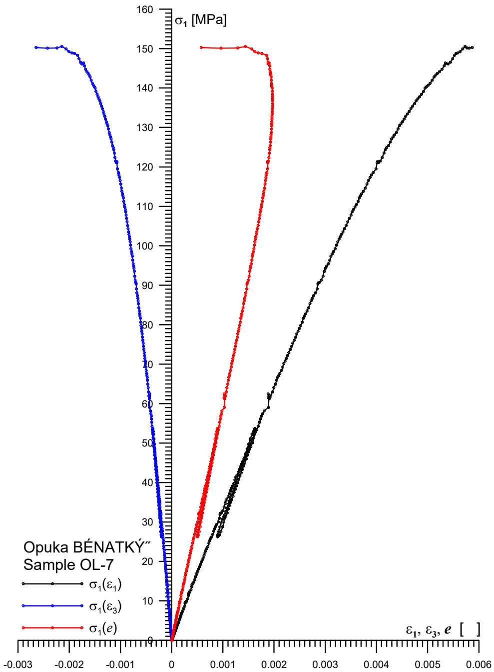

Fig. 11. Uniaxial compression test: relationship between axial stress and longitudinal strain – $\sigma_1(\varepsilon_1)$, trans-verse strain – $\sigma_1(\varepsilon_3)$ and volumetric strain – $\sigma_1(e)$; opuka stone

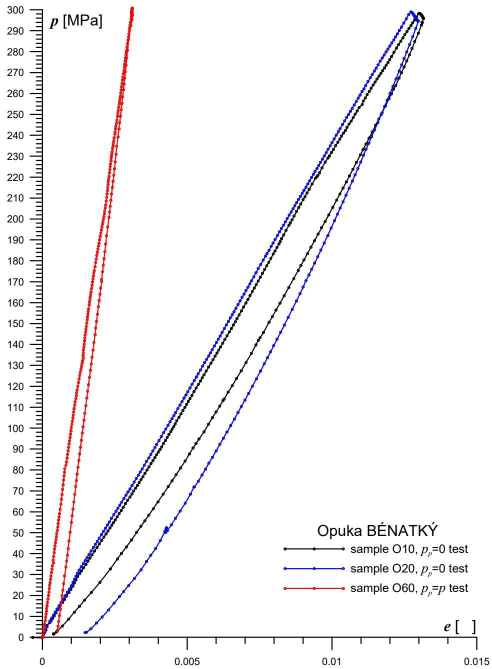

Fig. 12. Compressibility tests, opuka stone; p – hydrostatic pressure, e – change of volume

The observed differences cannot be explained by an error in the measuring deformations method because this method was identical for all discussed cases. It is also necessary to exclude incomplete opuka stone sample saturation with kerosene (due to the large amount of isolated pores in the pore space), because in this case the $p_p = p$ curve would not be rectilinear and the linear shape of the load curves $p_p = 0$ of the opuka stone samples suggests that the crack porosity of that rock is negligible.

Ultimately, it seems that the reasons for the appearance of permanent deformations should be found in the geological structure of the studied material. Either some minerals that are included in opuka stone (e.g. microfossils) was permanently deformed, or the calcite, which is a component of this rock, experienced a phase transformation into aragonite during loading, which was then resulted in local damage to the structure and permanent deformations.

6. Concluding remarks

Professor Maurice Anthony Biot, formulating in the year 1941 – for the purposes of soil mechanics – his theory of an elastic porous medium, made the following assumptions:

> *"The following basic properties of the soil are assumed: (1) isotropy of the material, (2) reversibility of stress-strain relations under final equilibrium conditions, (3) linearity of stress-strain relations, (4) small strains, the water contained in the pores is incompressible, (6) the water may contain air bubbles, (7) the water flows through the porous skeleton according to Darcy's law."* [1].

Meanwhile, considering all that was written above (especially in chapter 5), it should be stated that:

a) to determine the appropriate modulus of compressibility of a rock (regardless of the adopted test method), the key is to correctly measure the deformation of the tested sample,
b) it is extremely difficult to select the stress range in which the rock can be considered as a material that meets the assumptions of Biot's theory; in particular, stress concentrations caused by the structure of the material (e.g. granular nature of sandstones, local inhomogeneities) may cause deformations that cannot be considered "small" and reversible neither in the uniaxial compression test nor in the compressibility test,
c) the difficulties mentioned in point b) they are magnified by phenomena resulting from deformation of the rock pore space; they can contribute to the growth of both permanent deformation (destruction of isometric pores, material destruction at the edges of cracks) and reversible deformation (opening and closing of cracks),
d) the results of strain measurement (for transverse deformations in particular) during uniaxial compression tests depend on numerous factors – the number and type of deformation sensors, their measurement basis and the method of installation on the sample, the dimensions and precision of sample preparation for testing (parallel bases and sides), and, finally, on the method used to affix the sample inside the resistance testing machine (the presence or absence of a spherical seat, friction between the sample bases and the press plates). These are known issues, already described many times in scientific literature,

Considering all of that it seems reasonable to ask the question: Can the rock be treated as a Hooke/Biot material, and if so, to what extent? Of course, people performing measurements of rock mass deformation *in situ* are certainly able to provide many situations in which the forecast of such deformation prepared on the basis of Hooke's model was adequately accurate, thus somewhat automatically undermining the purpose of the aforementioned question. Nevertheless, the conformity of calculation and measurement results does not necessarily mean that the accepted calculation method is physically correct. In view of the results presented above, however it should be mentioned that in the case of the rocks studied by the authors, the Hooke/Biot physical model is an incorrect one, *ergo*, its deformation properties may not be described using the elasticity constants of this model. Of course, this result by no means disqualifies the usefulness of Hooke's model in the rock and rock mass mechanics and only points out the difficulties inherently present in studies in this field.

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