Digital filter tuned according to spectral analysis of direct drive to reduce mechanical resonance effect

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Review of the filter structures and spectral analysis algorithms was done. Digital filters allow filtering the speed signal in control loop of direct drive with a mechanical resonance. The analogue filter equations were transformed into digital filters to enable tune them in real time. Spectral analysis with additional algorithm allows to determine the mechanical resonance frequency of multi-mass system. Determination of resonance frequencies allow for proper tune the digital filter. This paper presents selected algorithms of determine signal spectrum. In order to determine the mechanical resonance frequency from the whole spectrum, it is require to use an additional algorithm that was omitted from the consideration. Filters were tuned according to frequency characteristics of the object.

1. Introduction

Mechanical systems with direct drive is now days commonly used in practice. In this case, precise control is required. Direct drive system is less robust to change of load. It is required to design a control system that take into account multi-mass object [2]. Direct drive control requires a proper filter. Digital filter must be tuned to cancel the mechanical resonance part of system. For this purpose can be used frequency or time-frequency analysis [5]. The mechanical resonance frequency can be obtained from spectrum of the object [6]. It is important that the algorithm of spectral analysis could be performed quickly enough in control loop to identify resonance frequency. This will allow to keep track of changes in frequency characteristics system with direct drive. The use of appropriate adaptive algorithm allow to tune the filters in real time. Other methods that allow the correct speed control require a reduction of control loop bandwidth below mechanical resonance frequency or use the Rigid-Body Luenberger Observer [1].

2. Mathematical model of direct drive

The single mechanical resonance block is described by the continuous transfer function:

$$G_{r,i} = \frac{s^2 + 2\xi_{ar,i}\omega_{ar,i}s + \omega_{ar,i}^2}{s^2 + 2\xi_{r,i}\omega_{r,i}s + \omega_{r,i}^2} \cdot \frac{\omega_{r,i}^2}{\omega_{ar,i}^2} \tag{1}$$

where $\omega_{ri} = 2\pi f_{ri}$ and $\omega_{ari} = 2\pi f_{ari}$ are the resonance and anti-resonance angular frequency for i-th block, $\xi_{r,i}$ and $\xi_{ar,i}$ are the resonance and anti-resonance damping coefficients.

The transfer function from electromechanical torque T_{em} to motor velocity ω_m is:

$$\frac{\omega_m}{T_{em}} = \frac{1}{J_d} \cdot \frac{1}{s} \cdot \prod_{i=1} G_{r,i} \tag{2}$$

where J_d is the total moment of inertia and R is the number of the resonance block.

Simulations were performed for the data contained in Table 1. The frequency response of simulated direct drive was shown in Fig. 1. Spectrum of velocity signal contains information of the mechanical resonance frequency. Spectral analysis allows the extraction of this information.

Table 1. Parameters of the model

J_d (kg·m^2)	i -	$f_{ar,i}$ (Hz)	$\xi_{ar,i}$ *10^{-4}	$f_{r,i}$ (Hz)	$\xi_{r,i}$ *10^{-4}
0.753	1	60	10	120	90
	2	150	5	250	100
	3	300	10	350	90

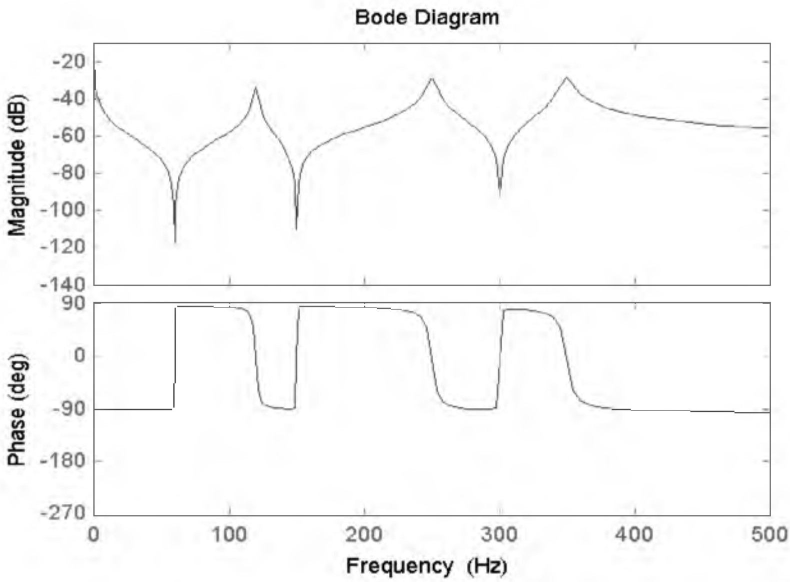

Fig. 1. Bode characteristic of direct drive

3. Algorithms of spectral analysis

3.1. Direct method

The Fourier transform is a mathematical operation that decomposes a function into its constituent frequencies. Fourier transformation of the signal *f(t)* is defined by the integral:

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)\, e^{-j\omega t} dt \tag{3}$$

where the variable $\omega \in (-\infty;+\infty)$.

Discrete Fourier transform (DFT) of signal *x(n)* of length N is defined as:

$$X(k) = \frac{1}{N}\sum_{n=0}^{N-1} x(n) W_N^{kn} \tag{4}$$

$$W_N = e^{-j2\pi/N},\ k = 0, 1, 2, ..., N-1$$

Invers discrete Fourier transform is defined as:

$$x(n) = \sum_{k=0}^{N-1} X(k) W_N^{-kn},\ n = 0, 1, 2, ..., N-1 \tag{5}$$

Computational complexity of the direct method is $O(N^2)$.

3.2. Goertzel algorithm

Designating the entire spectrum of the signal is not always necessary. Where it is necessary to designate a few frequency of signal Goertzel algorithm can be used [4]. The algorithm can be represented as the digital filter (6)-(8). Bi-quad sections that implements the filter is shown in Fig. 2. Execution of the algorithm requires N multiplications for real numbers and one complex multiplication for $n=N$. The presented algorithm has lower computational complexity than the direct method. In the case of track changes of the mechanical resonance frequency it is possible to register that frequency change occurred.

$$y_k(n) = y_k(n-1) W_N^{-k} + x(n),\quad n = 0, 1, 2, ..., N-1 \tag{6}$$

$$X(k) = y_k(n)\big|_{n=N} \tag{7}$$

$$\frac{X(z)}{Y(z)} = \frac{1}{1 - W_N^{-k} z^{-1}} = \frac{1 - W_N^{k} z^{-1}}{1 - \left(W_N^{-k} + W_N^{k}\right) z^{-1} - z^{-2}} =$$

$$= \frac{1 - W_N^{k} z^{-1}}{1 - 2\cos\frac{2\pi k}{N} z^{-1} - z^{-2}} \tag{8}$$

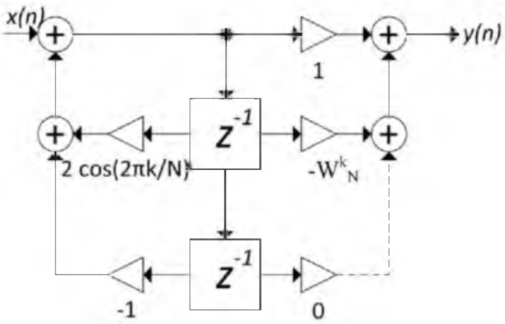

Fig. 2. Implementation of Goertzel algorithm

3.3. Radix-2 algorithm

The use of fast Fourier transform algorithm radix-2 requires a N=L·M=2[l] number of samples [3, 8]. The signal can be divided into even and odd samples. DFT take the form:

$$X(k)=\frac{1}{N}\left(\sum_{n=0}^{N/2-1}x(2n)W_N^{k(2n)}+\sum_{n=0}^{N/2-1}x(2n+1)W_N^{k(2n+1)}\right) \tag{9}$$

$$W_N^{k(2n)}=e^{-\frac{j2\pi k(2n)}{N}}=e^{-\frac{j2\pi kn}{N/2}}=W_{N/2}^{kn} \tag{10}$$

$$W_N^{k(2n+1)}=W_N^{k(2n)}W_N^k=W_{N/2}^{kn}W_N^k \tag{11}$$

$$X(k)=\frac{1}{N}\left(\sum_{n=0}^{N/2-1}x(2n)W_{N/2}^{kn}+W_N^k\sum_{n=0}^{N/2-1}x(2n+1)W_{N/2}^{kn}\right) \tag{12}$$

where $k=0,1,2,...,N-1$. Normalization N should be applied at the end of the calculation. Knowing that $W_{N/2}^{kn}$ has period N/2 can be determined (13)-(17).

$$W_{N/2}^{(k+N/2)}=W_{N/2}^{kn}W_{N/2}^{knN/2}=W_{N/2}^{kn}e^{-\frac{j2\pi knN/2}{N/2}}=$$
$$=W_{N/2}^{kn}(\cos(2\pi kn)-j\sin(2\pi kn))=W_{N/2}^{kn} \tag{13}$$

$$X(k)=X_{2n}(k)+W_N^kX_{2n+1}(k),\quad k=0,1,2,...,N-1 \tag{14}$$

$$X\left(k+\frac{N}{2}\right)=X_{2n}(k)+W_N^{(k+N/2)n}X_{2n+1}(k) \tag{15}$$

$$W_N^{(k+N/2)}=W_N^ke^{-\frac{j2\pi}{N}\frac{N}{2}}=W_N^k(\cos\pi-j\sin\pi)=-W_N^k \tag{16}$$

$$\begin{bmatrix}X(k)_i\\X\left(k+\frac{N}{2}\right)_i\end{bmatrix}=\begin{bmatrix}1&1\\1&-1\end{bmatrix}\begin{bmatrix}X(k)_{i-1}\\W_N^kX\left(k+\frac{N}{2}\right)_{i-1}\end{bmatrix},\quad i=1,2,...,\log_2(N) \tag{17}$$

Fig. 3 present the structure of the calculation block (17) calculating basic radix-2 algorithm decimation in time called butterfly. Number of stages amounts i.

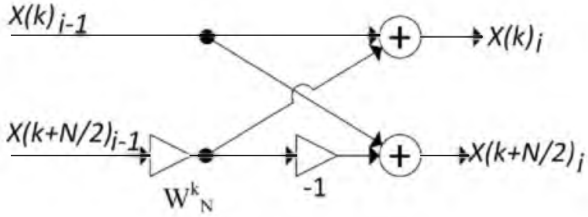

Fig. 3. Generalized butterfly

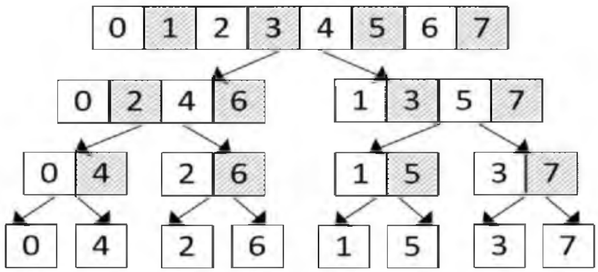

Fig. 4. Interlaced decomposition

Example of decomposition of the signal of length N=8 is shown in Fig. 4. Decomposition of the signal on the even and odd elements is carried out until the one-element sets will be obtained. Frequency spectrum of a one-point time domain signal is equal to itself (4) when $W_N^{kn}\big|_{k=0,n=0} = 1$. Submission of spectra should be done on individual even and odd samples according to (17) in $i=log_2(8)=3$ steps.

Computational complexity of the radix-2 method is $O(N/2 \cdot log_2(N))$.

4. Discretization method

Each continuous transfer function can be converted to discrete form using Tustin - bilinear transformation [9]. Bilinear transformation maps s-plane to the unit circle better than forward or backward differentiation method. Tustin method with modification described below have exactly the same frequency response at one or more specified frequencies of the original continuous-time system and discrete-time system. It was important in digital filter design from analog model. Tustin discretization method used approximate integration. The integration is performed using the area under the curve - approximate trapezoidal area. Following relationship was obtained:

$$s \approx 2f_z \frac{1-z^{-1}}{1+z^{-1}} \tag{18}$$

The following symbols were introduced $s = j\omega$, $z = e^{j\,\Omega T}$, $T = \dfrac{1}{f_z}$, $\Omega = 2\,\pi\, f_d$ where f_z is sampling frequency and f_d is digital frequency. From (18) can be determine the relationship between the ω analog angular frequency and Ω digital angular frequency. That relationship is important for designing filters with desired frequency response.

$$j\omega = 2f_z \frac{1-e^{-j\Omega T}}{1+e^{-j\Omega T}} = 2f_z \frac{-\dfrac{e^{2(-j\Omega T/2)}-1}{2\,e^{-j\Omega T/2}}}{\dfrac{e^{2(-j\Omega T/2)}+1}{2\,e^{-j\Omega T/2}}} = 2f_z \frac{-\sinh\left(\dfrac{-j\Omega T}{2}\right)}{\cosh\left(\dfrac{-j\Omega T}{2}\right)} =$$

$$= 2f_z \frac{-\left(-j\sin\left(\dfrac{-j\Omega T}{2}j\right)\right)}{\cos\left(\dfrac{-j\Omega T}{2}j\right)} = 2f_z \frac{j\sin\left(\dfrac{\Omega T}{2}\right)}{\cos\left(\dfrac{\Omega T}{2}\right)} = 2f_z j\tan\left(\frac{\Omega T}{2}\right) = 2f_z j\tan\left(\frac{\pi f_d}{f_z}\right) \tag{19}$$

Every transfer function can be decomposed into the product of the bi-quad sections. The continuous transfer function can be expressed as:

$$T_F(s) = \frac{b_2\left(\dfrac{s}{2f_z}\right)^2 + b_1\dfrac{s}{2f_z} + b_0}{a_2\left(\dfrac{s}{2f_z}\right)^2 + a_1\dfrac{s}{2f_z} + a_0} \tag{20}$$

Determination of discrete transfer function requires use of substitution (18). Coefficient $2f_z$ has been reduced as a result of substitution (21).

$$T_F(z) = T_F(s)\Big|_{s=2f_z\frac{1-z^{-1}}{1+z^{-1}}} =$$

$$= \frac{(b_0+b_1+b_2)+2(b_0-b_2)z^{-1}+(b_0-b_1+b_2)z^{-2}}{(a_0+a_1+a_2)+2(a_0-a_2)z^{-1}+(a_0-a_1+a_2)z^{-2}} =$$

$$= \frac{B_0'+B_1'\,z^{-1}+B_2'\,z^{-2}}{A_0'+A_1'\,z^{-1}+A_2'\,z^{-2}} = \frac{B_0+B_1\,z^{-1}+B_2\,z^{-2}}{1+A_1\,z^{-1}+A_2\,z^{-2}} \tag{21}$$

Coefficients are described by the following relations:

$$B_0' = b_0+b_1+b_2,\quad B_1' = 2(b_0-b_2),\quad B_2' = b_0-b_1+b_2,$$

$$A_0' = a_0+a_1+a_2,\quad A_1' = 2(a_0-a_2),\quad A_2' = a_0-a_1+a_2,$$

$$B_0 = \frac{B_0'}{A_0'},\quad B_1 = \frac{B_1'}{A_0'},\quad B_2 = \frac{B_2'}{A_0'},\quad A_1 = \frac{A_1'}{A_0'},\quad A_2 = \frac{A_2'}{A_0'}.$$

5. Digital structures of filters

5.1. Notch filter

A notch filter can be used to reduce the mechanical resonance frequency [7]. A notch filter allows the suppression of selected frequency $\omega_0 = 2\pi f_0$ with the width of Q. Determination of discrete transfer function requires substitution (18) into:

$$T_F(z) = \left. \frac{s^2 + \omega_0^2}{s^2 + s\dfrac{\omega_0^2}{Q} + \omega_0^2} \right|_{s=2f_z\frac{1-z^{-1}}{1+z^{-1}}} = \left. \frac{\dfrac{s^2}{\omega_0^2} + 1}{\dfrac{s^2}{\omega_0^2} + \dfrac{s}{\omega_0}\dfrac{1}{Q} + 1} \right|_{s=2f_z\frac{1-z^{-1}}{1+z^{-1}}} \tag{22}$$

Taking into account (19) nonlinear relationship between the analog angular frequency and digital angular frequency yield:

$$\omega_0 = 2f_z \tan\left(\frac{\pi f_0}{f_z}\right), \qquad \omega_0' = \frac{\omega_0}{2f_z} = \tan\left(\frac{\pi f_0}{f_z}\right) \tag{23}$$

Coefficients of continuous transfer function:

$$b_2 = (\omega_0')^{-2}, \quad b_1 = 0, \quad b_0 = 1,$$
$$a_2 = (\omega_0')^{-2}, \quad a_1 = (\omega_0')^{-1} Q^{-1}, \quad a_0 = 1.$$

Determined discrete transfer function coefficients:

$$B_0' = 1 + (\omega_0')^{-2},$$
$$B_1' = 2\left(1 - (\omega_0')^{-2}\right),$$
$$B_2' = 1 + (\omega_0')^{-2},$$
$$A_0' = 1 + (\omega_0')^{-1} Q^{-1} + (\omega_0')^{-2},$$
$$A_1' = 2\left(1 - (\omega_0')^{-2}\right),$$
$$A_2' = 1 - (\omega_0')^{-1} Q^{-1} + (\omega_0')^{-2}.$$

5.2. Bi-quad filter

A bi-quad filter reduces the i-th block of multi-mass system (1). Filter design parameters must be identified in order to properly tune the filter. Determination of discrete transfer function requires substitution (18) into:

$$T_F(z)=\prod_{i=1}^{L}\frac{s^2+2\zeta_{r,i}\hat{\omega}_{r,i}s+\hat{\omega}_{r,i}^2}{s^2+2\zeta_{ar,i}\hat{\omega}_{ar,i}s+\hat{\omega}_{ar,i}^2}\cdot\frac{\hat{\omega}_{ar,i}^2}{\hat{\omega}_{r,i}^2}\Bigg|_{s=2f_z\frac{1-z^{-1}}{1+z^{-1}}}=\prod_{i=1}^{L}\frac{\frac{s^2}{\hat{\omega}_{r,i}^2}+2\zeta_{r,i}\frac{s}{\hat{\omega}_{r,i}}+1}{\frac{s^2}{\hat{\omega}_{ar,i}^2}+2\zeta_{ar,i}\frac{s}{\hat{\omega}_{ar,i}}+1}\Bigg|_{s=2f_z\frac{1-z^{-1}}{1+z^{-1}}} \tag{24}$$

where $\hat{\omega}_{r,i}=2\pi\hat{f}_{r,i}$ is determined resonance frequency and $\hat{\omega}_{ar,i}=2\pi\hat{f}_{ar,i}$ is determined anti-resonance frequency of the mechanical system.

Taking into account (19) nonlinear relationship between the analog angular frequency and digital angular frequency yield:

$$\hat{\omega}_{ar,i}=2f_z\tan\left(\frac{\pi\hat{f}_{ar,i}}{f_z}\right),\qquad \omega'_{ar,i}=\frac{\hat{\omega}_{ar,i}}{2f_z}=\tan\left(\frac{\pi\hat{f}_{ar,i}}{f_z}\right),$$

$$\hat{\omega}_{r,i}=2f_z\tan\left(\frac{\pi\hat{f}_{r,i}}{f_z}\right),\qquad \omega'_{r,i}=\frac{\hat{\omega}_{r,i}}{2f_z}=\tan\left(\frac{\pi\hat{f}_{r,i}}{f_z}\right).$$

Coefficients of i-th block of continuous transfer function:

$$b_{2,i}=\left(\omega'_{r,i}\right)^{-2},\quad b_{1,i}=2\zeta_{r,i}\left(\omega'_{r,i}\right)^{-1},\quad b_{0,i}=1,$$

$$a_{2,i}=\left(\omega'_{ar,i}\right)^{-2},\quad a_{1,i}=2\zeta_{ar,i}\left(\omega'_{ar,i}\right)^{-1},\quad a_{0,i}=1.$$

Discrete transfer function coefficients determined for the i-th block:

$$B'_{0,i}=1+2\zeta_{r,i}\left(\omega'_{r,i}\right)^{-1}+\left(\omega'_{r,i}\right)^{-2},$$

$$B'_{1,i}=2\left(1-\left(\omega'_{r,i}\right)^{-2}\right),$$

$$B'_{2,i}=1-2\zeta_{r,i}\left(\omega'_{r,i}\right)^{-1}+\left(\omega'_{r,i}\right)^{-2},$$

$$A'_{0,i}=1+2\zeta_{ar,i}\left(\omega'_{ar,i}\right)^{-1}+\left(\omega'_{ar,i}\right)^{-2},$$

$$A'_{1,i}=2\left(1-\left(\omega'_{ar,i}\right)^{-2}\right),$$

$$A'_{2,i}=1-2\zeta_{ar,i}\left(\omega'_{ar,i}\right)^{-1}+\left(\omega'_{ar,i}\right)^{-2}.$$

6. Results of identification

Identification of the mechanical resonance frequency is possible providing as a input signal to open-loop system a constant frequency sinusoid. Then turn the frequency change at the interval with a specific step. Goertzel algorithm was used for calculating the spectrum corresponding to input signal. Identification took a long time change the frequency range of 1 Hz to 500 Hz in steps of 1 Hz. Each input signal took 1 second. Another input signal is the sinusoidal signal with increasing frequency (25). This signal is called chirp. The results of the simulations of open-loop system for the parameters in Table 1 for the input signal (25) was shown in Fig. 5. Both input signals allow to find the mechanical resonance frequencies, however the first method allows to find the mechanical antiresonance frequencies. In the case of simulation found frequencies correspond to the simulation parameters.

$$x(k) = \sin\left(2 \cdot \pi \cdot \left(f_0 + \frac{f_1 - f_0}{t_1} \cdot k \cdot \tau_s\right) \cdot k \cdot \tau_s\right) \quad (25)$$

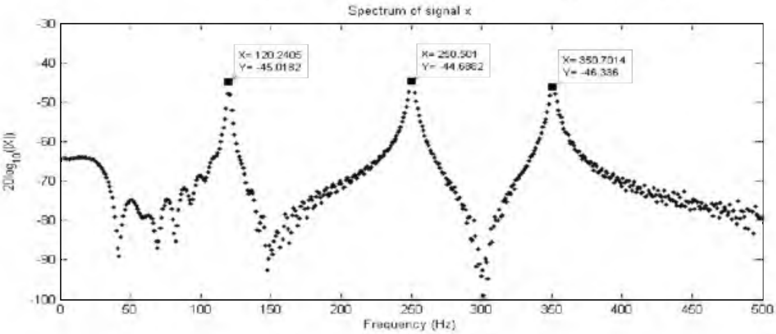

Fig. 5. Frequency analysis of simulated object - (25) was used as input signal

7. Tune digital filters

In a perfect solution, the best filter is the filter of the inverted section of mechanical resonance block (1), which reduces the resonant block. However, the use of this type of filter requires proper identification of damping factors and frequency of mechanical resonance and anti-resonance. In case of errors, some frequencies will be gain.

In Fig. 6-7 where shown the frequency characteristics of filters tuned properly according to the characteristics of the object (2).

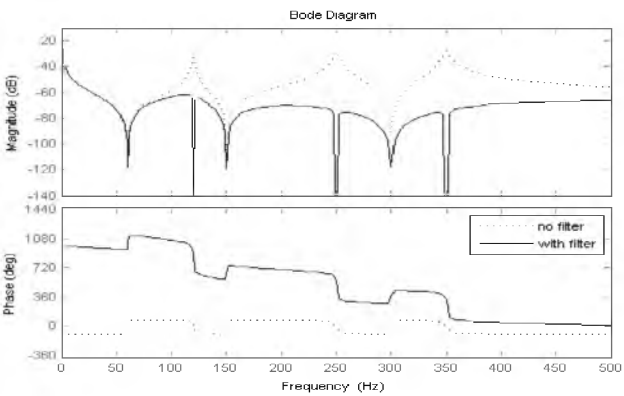

Fig. 6. Frequency characteristic of mechanical system with Notch filters

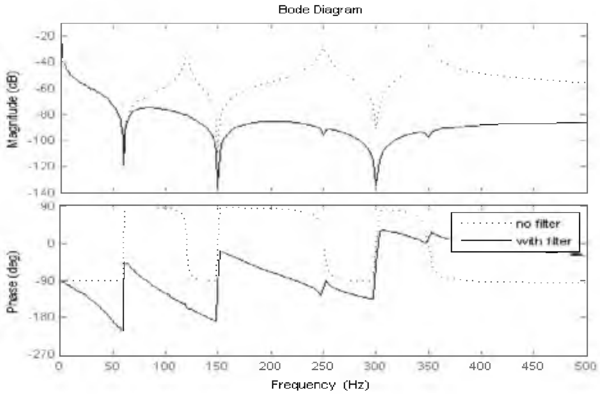

Fig. 7. Frequency characteristic of mechanical system with bi-quad filters

8. Conclusion

This paper presents the algorithms for the determination of a discrete spectrum from discrete signal. Selection of an appropriate algorithm for spectral analysis has allowed to realize it in real system. Presented was follow algorithms for the spectrum determination: direct method, Goertzel and radix-2. According to literature analysis radix-2 and Goertzel algorithm was proposed. Radix-2 was chosen because of simplified construction of a butterfly. This allows to keep track changes of the identified direct drive system. Rather than perform the analysis of the entire spectrum Goertzel allow to analysis the changes of already identified mechanical resonance frequencies.

The use of the Goertzel algorithm allowed for accurate identification in the frequency domain with a sinusoidal input signal with different frequency. Using chirp input signal with radix-2 algorithm allow to fast identification of object. Designation of the entire spectrum should be performed in case of change direct drive system. Fast frequency designation allows for proper execution of algorithm that tune filter to attenuate mechanical resonance frequency.

In the last part this article presents an analytical determination of digital filters. Digital filters are implemented in cascade as a single discrete square sections. Presented digital transfer functions allow for simple change the parameters of the filter. Additional algorithms designating the mechanical resonance frequency will allow to appropriate tune the filter [6].

More complicated filters structures such as analog biquad filter of L blocks is easy to implement. It was design as L second-order sections. Each of them will be executed one after another. Implementation of filters with higher orders was obtained.

Using spectral analysis can be identify the resonance frequency of mechanical part of object. The last chapter is devoted to the presentation of digital filters tuned in accordance with the earlier identification of the object. Tuned digital filters attenuate the mechanical resonance frequency in feedback signal of the control system which does not strengthen them. Therefore, control system does not stimulate mechanical resonance frequency of the object.

The presented method can be used for the control of electric machines with direct drive with low requirements such as washing machines and fans. Precise control is required in DVD application and turn tables of CNC machines.

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