JACEK JUZWA
IRENA KUCIARA
WIESŁAW PIWOWARSKI
KAZIMIERZ SICIŃSKI

Analyses of parabolic processes to assess mapping stability of mining area ground dislocations in the INGEO system

The article features deliberations concerning the analysis of the following in the homomorphism of processes: deformations corresponding to the medium (rock mass) vibrations which generate physical threats in the subarea of topological transformations. Here, the basic issue is deformation mapping applied to model the dislocation processes related to the paraseismic process. Time dependencies are characterized by structure and dynamics of the processes. The damage of the part of the rock mass near the exploited deposit causes deformations and, most frequently, topological transformation of successive layers. Quite often rock bursts are generated, which is related mainly to the exceeded boundary states of the medium. Here it is very important to have measuring information about the medium transformations. In addition, it is necessary to define parameters and measures that characterize the anisotropy of the rock mass structures. The research within the INGEO project was focused on solutions based on the adaptation of the parabolic differential description supported by monitoring a concrete physical dislocation process. The mapping state of the process trajectory was distinguished in the deformation space by means of mathematical algorithms. Numerical modelling of deformation fields was supported by GPS sensors (innovative direct monitoring), on-line GNSS technology, and compaction sensors with a view to measuring complex dislocation fields. This solution is a new technology. A parametrically optimized model adequately illustrates a standard (measurement results) layout of vertical dislocations.

Key words: *parametric estimation, parabolic model, on-line measurements, deformation process, trajectories, random phenomena*

1. INTRODUCTION

Underground exploitation of a deposit breaches the original state of stresses in the rock mass surrounded by excavations. As a result of that, the roof layers get damaged causing dislocations in the medium located above the roof. In extreme cases the process results in energy accumulation and release in the rock mass as well as rock bursts which may often have devastating consequences. For many years Polish coal mines have used intrinsically safe regularly improved systems to assess rock bursts hazards, such as ARAMIS and ARES, described in [4], [5], and [6]. These systems make use of the analysis of vibrations registered in the rock mass.

The innovative INGEO system, which has been designed to assess rock burst hazards too, has an extra feature which enables to register deformations of excavations in the exploited area and to register, with precision, surface deformations in the area above the advancing longwall.

Processes are empirical facts which reflect changes in the successive stages of the phenomenon development and are an important source supporting scientific research. Geodetic observations of dislocations of particular points of the ground surface are the basis to estimate transformations of the subarea. However, they are not enough, as they often do not describe satisfactorily the complexity of the rock mass destruction processes [12].

The basic procedure to come to general physical laws is reasoning by interpolation of the experiment results [2]. When analyzing the transformation process of the medium, generated by underground exploitation, the most frequent procedure is to follow the scheme: hypothesis – problem formulation – model. The subject of deliberations is modelling the process of the rock mass points dislocation in the area impacted by underground exploitation.

Fast or short-term processes are usually subject to monitoring methods (goafs, rock bursts). Slow and long-term processes can be both monitored and modelled as post-mining dislocation processes. These procedures allow to carry out T-optimization to select a model that would well illustrate the analyzed process.

It is an exaggeration to think that the model would suit the data perfectly – due to many reasons. Even if the model suits the data, it still does not mean it is an adequate one. The dislocation field observed in terms of quantitative characteristics (measurements) is non-linear and randomly disturbed. Thus for different subsets of the experiment results the global minimum of the loss function is not fulfilled. The essence is an emergent (multiple) complexity here [9].

2. PROBLEM CHARACTERISTICS

The objective of the project was to make use of state-of-the-art achievements of today's engineering to create a base of modern solutions in the range of multiple analyses of environment transformations oriented towards sustainable development and improvement of the analyzed subspace quality.

Thus the authors presented techniques for monitoring and modelling environmental changes in the conditions of associated hazards: vibrations and dislocations. The issue of monitoring was described thoroughly, e.g. in [4]. The work [13], in turn, features medium dislocation processes with state co-ordinates as a transformation of one state into another through the medium destruction. In general, the knowledge is provided by the so called Reynolds transport theorem. The whole information about the vector field is carried by the n number of the X_i functions whose value encodes the value of the vector co-ordinates

$$X(x) = X_i\,(x) \cdot \frac{\partial}{\partial x_i}$$

Modern technological development requires more and more precise mathematical models along with more and more precise information about phenomena that surround us. Adequate knowledge in this respect will allow to predict the process and consequences of the analyzed phenomenon in different conditions, controlled by humans or uncontrolled. If physically located points of the subarea are not stable, in order to illustrate this movement we make use of a common topological fact that continuous and injective mapping of the metric compact space is homeomorphism onto its own image (measurement) [1]. Then, if one of the matrix eigenvalues has a positive real part, the stationary point x_0 of the analyzed subarea is not Lyapunov stable.

The scientific and research operations, most of them innovative, carried out in the project improve the efficiency of modelling the rock mass destruction processes. Innovative technologies for measuring the state of the medium (the real scene) allow to adapt the models, in a sense of providing optimal new definitions and adjusting existing solutions for the assessment of the environmental hazard. The measurements of the real scene are understood in a wider sense than just acquiring numbers. After all, these are quite complicated processes investigated as a mixture of clear situations. As a result of that we come to the terms of eigenstates, eigenvalues and eigenfunctions.

Formally, it is possible to write a measurement mapping f_y as a compound of the following states:

$$f_y = k \circ m \circ s$$

where:

k – function mapping X into the space Ω of objects of cognition.
m – function describing the actual measurement, i.e. assigning $\omega \in \Omega$ to objects of cognition
s – scaling function.

If a point of the medium (surface) in R^{2+1} (X, Z; t) moves due to the developing exploitation, then, phenomenologically, something like a moving vector appears. Is it possible to determine (with no references to the measurement) the movement of the point if the movement trajectory is not unequivocally determined? This is an open issue both in terms of differential geometry and field theory [12], [14].

Point dislocation trajectories $w_i(x,t)$ or $u_i(x,t)$, determined by means of measurements, are different for significantly different points in space with the probability approaching 1. The measurement results determine “narrow” number intervals while the transformation functions “f” are regular. If the U field is adequately smooth (e.g. of C^1 class), the initial problem is well set locally. This means that for each point $x \in M$ there is locally a single integral curve $f_x(t)$, i.e.

$$\langle -\varepsilon, \varepsilon \rangle \supset t \to f_x(t) \in M$$

starting from this point, i.e. fulfilling the initial condition $f_x(0) = x$.

The distribution of stresses in the rock mass where exploitation is carried out depends on the conditions existing in the primary state (before mining works began) and on geo-mechanical conditions of the conducted exploitation, such as the neighbourhood of goafs or the direction of the exploitation advance with respect to quasi stable zones. Therefore the mapping state of the process trajectory can be distinguished in space by means of mathematical algorithms.

3. MODELLING AND ANALISYS OF DISLOCATION PROCESS

The research concerned solutions based on the adaptation of the monitoring-supported differential description to the mapping of a concrete physical process. The authors worked out models and algorithms describing the rock mass deformations in the underground exploitation area.

Information about the state of the rock mass destruction is provided by a mathematical model whose parameters are known in detail and information acquired from measuring devices is precise too. The model formula and measurement errors make it necessary to characterize the modelling quality [7]. Modelling a process in relation to the location algorithm of a certain (satisfactory) number of measuring devices allows to obtain enough knowledge to define the deformation subarea. Thus the solutions were oriented towards the support of environmental revitalization operations and security of the people.

Let $\Omega \in topR^n$ denote the space where the process of mining dislocations takes place, while ω stands for the surface which limits the given space. The elementary stream of released energy of the balanced system runs through the $d\vec{\omega}$ elements [14], i.e.:

$$du = -\Phi d\vec{\omega}$$

$d\vec{\omega}$ – surface element treated as vector $\perp$ to ω $d\vec{\omega}$

The local measure of the dislocation field heterogeneity is its gradient Φ defined by Fourier’s law:

$$\Phi = a \cdot \nabla u$$

The dislocation field, generated by mining exploitation, can be described [14] with the use of differential formulas, i.e.:

$$\left.\begin{aligned} \frac{\partial u(x,t)}{\partial t} = \Im\left(x,t,u,\frac{\partial u}{\partial x_1},\frac{\partial u}{\partial x_2},\frac{\partial^2 u}{\partial x_1^2},\frac{\partial^2 u}{\partial x_2^2},\Theta \right) \\ f\left(x,t,u,\frac{\partial u}{\partial x_1},\frac{\partial u}{\partial x_2},\Theta \right) = 0 \\ u(x,0) = u_0(x) \end{aligned}\right\} \tag{1}$$

where:

$\Omega \in (a,b) \times (0,T)$, $\partial'\Omega$

Ω – two dimensional coherent and limited area $\Omega \in R^2$,

$u = u(x,t)$ – state variable,

$\Theta = (\theta_1,\ldots,\theta_m)$ – vector of parameters; $\Theta \in R^m$.

The estimation accuracy of parameters Θ depends on the location of observation points. Here it is important to select observation spots of a system with spatiotemporal dynamics [8]. The observed dislocation field is non-linear and is randomly disturbed due to the heterogeneity of the rock mass structure. So the field of the subarea dislocation $u(x,t) = u^{pom}(x,.)$ observed with respect to measurements is loaded.

$$\left.\begin{aligned} E\left\{\varepsilon\left(x^j,t\right)\right\} = 0 \\ E\left\{\varepsilon\left(x^j,t\right)\cdot\varepsilon\left(x^j,\tau\right)\right\} = \sigma^2\cdot\delta_{ij}\cdot\delta\left(t-\tau\right) \end{aligned}\right\}$$

$\delta_{ij}\cdot\delta(t-\tau)$ – respectively Kronecker delta and Dirac delta,

$\left\{\varepsilon\left(x^j,t\right)\right\}$ – measurement noise.

Estimators of parameters:

$$\left.\begin{aligned} \hat{\Theta} = \arg\min_{\theta\in\Theta}\sum_{j=1}^{m}\sum_{i=1}^{n}\left[y_i^j - \hat{u}\left(x^j,t_i;\theta\right)\right]^2 \text{discreet measurement} \\ \hat{\Theta} = \arg\min_{\theta\in\Theta}\sum_{j=1}^{m}\int_0^t\left[y^j - \hat{u}\left(x^j(t),t;\theta\right)\right]^2 dt \text{ continuous measurement} \end{aligned}\right\} \tag{2}$$

The estimate $\hat{\theta}$ depends on the location of sensors x^j. This fact justifies the need for optimal selection of the sensors location that would maximize the accuracy of the received parameter estimates. The work [7] says that with sufficiently long observation the reverse FIM (Fisher Information Matrix) is an asymptotic matrix of the estimated parameters covariance – i.e. the measure of the estimates dispersion around the vector of the parameters real values. The sequence $\{x^1,...,x^l\}$ signifies different places of measurements and a series of corresponding values $\{r_1,...,r_l\}$. We are building a dot matrix of the plan carrier. Then the matrix is defined as a set of the following pairs:

$$\xi = \begin{bmatrix} x^1, x^2, ..., x^l \\ p_1, p_2, ..., p_l \end{bmatrix}$$

where $p_i = \frac{r_i}{l}$

Let us assume that the state space is composed of a finite number of points $(x_0 < x_1 < ... < x_N) \subset R$ (discreet states). Differentiation is based on the fact that a process comes to the successive state. The points x_i can also describe selected groups of states. The model controlling the quality of the observed dislocation field distribution, also called multi-compartmental, has the following form [8]:

$$\frac{dm_i(t)}{dt} = a_i(t) \cdot m_i(t) + b_{i-1}(t) \cdot m_{i-1}(t) - b_i(t) \cdot m_i(t) \quad (3)$$

where:

m_i – number of points in the subarea corresponding to the state x_i,

a_i – coefficient of population growth in the subarea i due to divisions inside the subarea

b_i – differentiating rate.

As it was mentioned before, the estimation accuracy of the model depends on the location of measurement points in the given area – a non-linear loaded field. As measurement points are, as a rule, stationary, while observations are made in a finite number of measurement sessions $t_1,…,t_k$, the following takes place:

$$u_k^j = u\left(x^j, t_k : \hat{\theta} \right) + \varepsilon(x^j, t_k) \quad \begin{matrix} j=1,...,n \\ k=1,...,K \end{matrix} \qquad (4)$$

$\varepsilon(x,t)$ – measurement noise.

Numerical modelling of deformation fields emission was supported with the use of sensors – GPS receivers [3], [11], which were adapted to direct transmission – a new GNSS on-line technology, and with the use of compaction sensors for the measurement of complex dislocation fields, which is an innovative technology for determining the impact of particular factors as a whole of the dislocating rock mass structures. The conditions of consistency apply here: if there is no formula A, such that both A and the negation of A can be drawn from the axiom of the given theory by means of the related deduction system.

Algorithmic modelling procedures state the following:

- each state of exploitation development is a projection of the defining process,
- there is a transition between the states,
- previous state $\xleftrightarrow{prefix(operator)}$ state of the successive event,
- initial state $l \neq 0$ is a process instance in the specification clause,
- final state l_N → END with no definition of successive actions.

4. MAPPING OF DISLOCATION PROCESS

In order to describe the process quantitatively according to the presented methodology and computational algorithms in the INGEO project, several computing experiments were conducted. The tests were related to a typical mining and geological situation, i.e. longwall exploited with a caving. The verification had a multi-thread character and referred to the assessment of the algorithms usefulness, description accuracy and impact of extra (boundary and initial) conditions on the mapping quality. Additionally, for each active process an edge of the dislocation field was generated.

The comparison of numerical simulations results and experimental measurements results is the basic phase of updating computational models [9].

4.1. Dislocation process model

Let (X, Σ_1) and (X, Σ_2) stand for sigma-algebras

where:

X, Y – measurable sets,

Σ_1, Σ_2 – σ – algebras.

The statistics T: $(X, \Sigma_1) \to (X, \Sigma_2)$ is sufficient if and only if there are F – measurable functions g_θ ($\theta \in \Theta$) and Σ_2 measurable function h such that:

$$p_\theta(x) = g_\theta(T(x)) \cdot h(x)$$

The statistics T is sufficient if and only if the distribution density of the sample $(X_1, X_2,…, X_n)$ can be presented in the following form:

$$f_\theta(x_1,\ldots,x_n)=g_\theta(x_1,\ldots x_n)\cdot h(x_1,\ldots,x_n)$$

i.e. as a product of the function h dependent on the sample value but independent of the parameter Θ and the function g_θ which, in turn, is dependent on the parameter Θ and on the sample, but only through the value of the statistics T (factorization theorem).

Modelling is finding one and the best approximation of the process:

$$O_X = F(X,\Theta) \tag{5}$$

Increasing the accuracy of integration in time (1, 6) does not require extra restrictions with respect to time and space. It is possible thanks to the use of the midpoint method (calculating a central derivative in 2 points which gives much higher precision) – Crank-Nicolson method.

$$\frac{\partial u(x,t)}{\partial t}=\eta\cdot\frac{\partial^2 u(x,t)}{\partial x^2} \tag{6}$$

Approximation of the solution

$$u=u_0\left[\frac{x}{l}+\sum_{i=1}^{\infty}\frac{2}{i\cdot\pi}(-1)^i\cdot\sin\left(\frac{i\cdot x}{l}\right)\cdot\exp\left(\frac{-i^2\cdot\pi^2\cdot k\cdot t}{l}\right)\right] \tag{7}$$

The probability of generating the observation vector O_t by the model M is carried out as follows:

$$P(O_t|\mathrm{M})=P(O_{t1}|\mathrm{M})\cdot P(O_{t2}|\mathrm{M}),\ldots,\cdot P(O_{ts}|\mathrm{M}) \tag{8}$$

4.2. Deformation process – 3D mapping

3D presentation of modelling results ensures the following restitution: a set of 3D projections $\xrightarrow{restitution}$ *object* with the possibility to analyze its geometric quantities with respect to improper elements which constitute the projective space. Each 3D object consists of many surfaces – digital edition of a picture. Proper mapping is difficult since it must ensure that the axes of rectangular spatial coordinates are mapped onto any concurrent straight lines – this allows to select any axonometric system $\xleftarrow{Pohlke's\ theorem}$ (dimetric projection).

Figures and diagrams below feature sample modelling results, also in 3D.

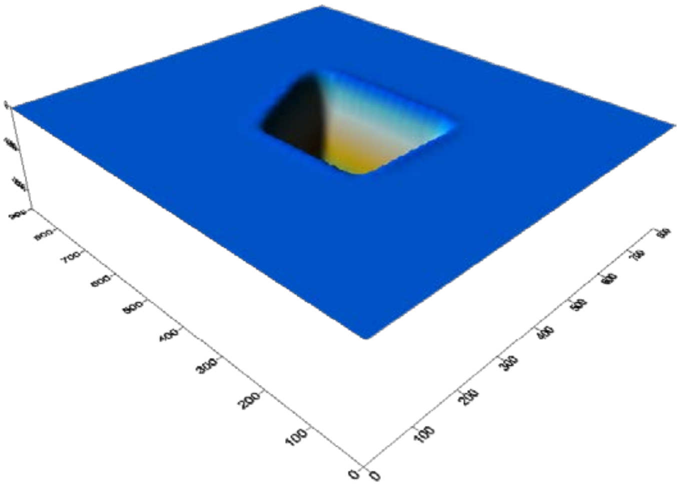

Fig. 1. 3D vertical dislocation – rock mass (40 m above the deposit)

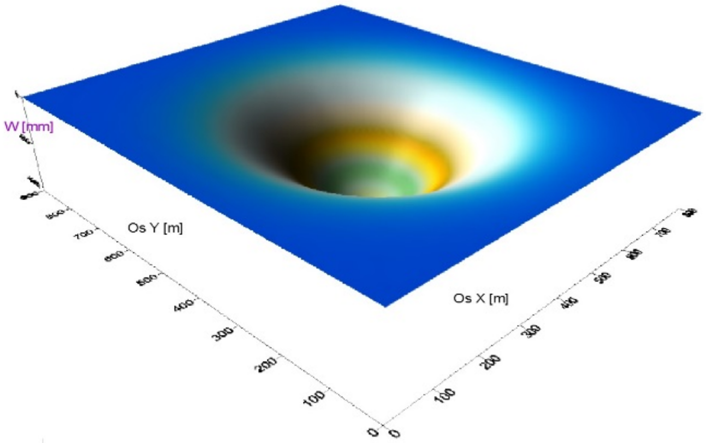

Fig. 2. 3D vertical dislocation – land surface

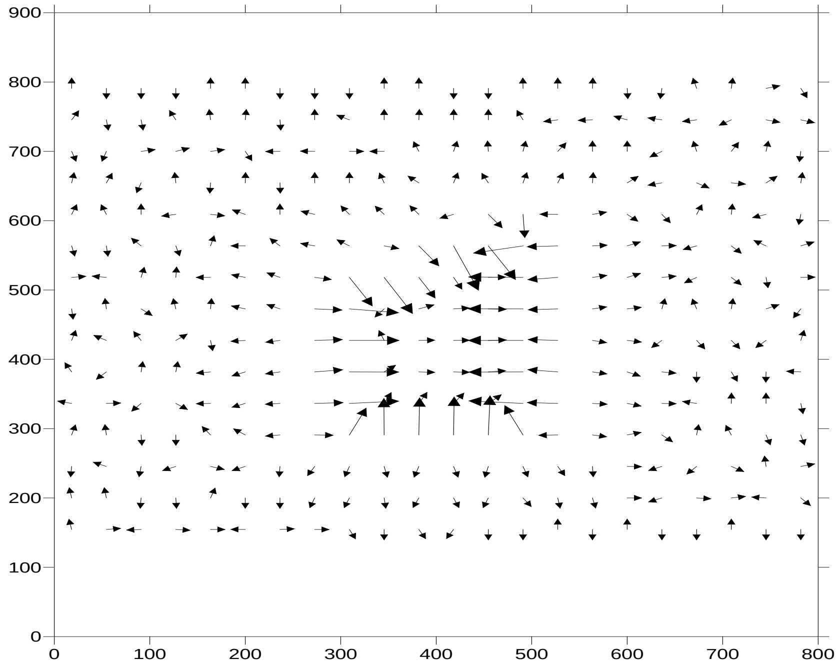

Fig. 3. Vectors of 2D horizontal dislocations – rock mass (40 m above the deposit)

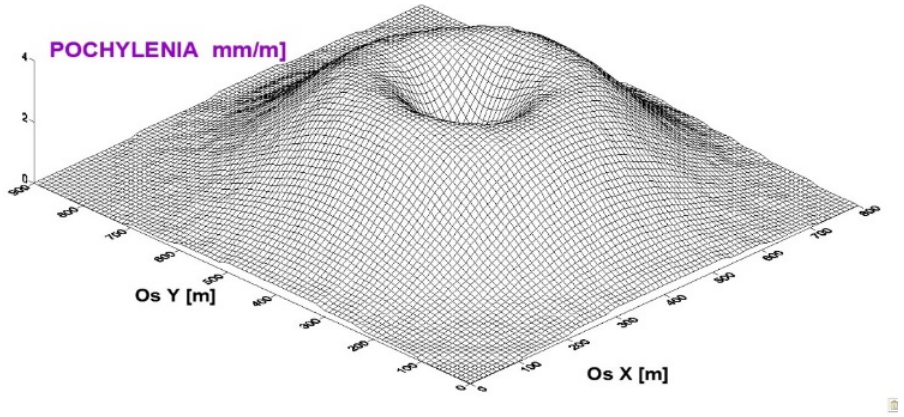

Fig. 4. 3D decline of the land - surface

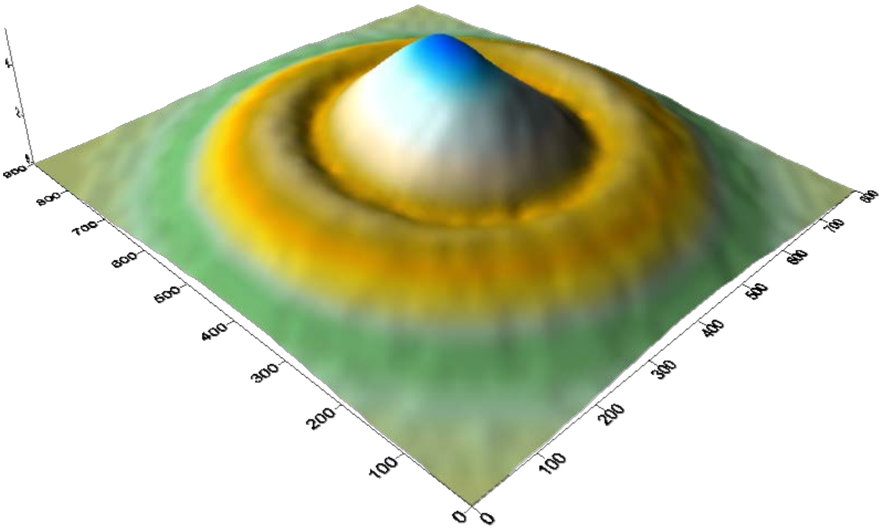

Fig. 5. 3D horizontal deformations – land surface

4.3. Geodetic observations of the mining land dislocation field

Geodetic observations within the INGEO project included the analysis of height changes of land surface points, for the points located within the mine exploitation area, in the zone of underground exploitation impact.

Due to a very small impact of the rheological factor in the deformation process, the issue of describing unidentified subsidence troughs can be assigned to the mapping of identified subsidence troughs with respect to the impact of the exploitation direction and succession on their shape.

The installed measurement network enables to acquire information about the mining land deformations. The spots were located in a quasi-optimal manner – there were some points selected which have characteristic location in the given subarea. This allowed later estimation of the process in any point of the analyzed area.

The designed measurement grid enabled to observe the dislocation field. Measurement points were distributed about 25 m from one another. The location of measurement points in the grid and their density was laid out to enable the following:

- to determine the spatial variability of the deformation field,
- to estimate dislocations beyond measurement points and in other moment of time,
- to determine the probability of assigning a given point or area to a certain threat category of the mining area as well as the probability of exceeding the threshold value.

In order to detect the edge of the deformation subarea, an analysis of local derivatives is conducted. The first and the second derivative of the field mapping were used to detect the edge, The gradient of picture I in the point (x, y) is a vector determined by the following formula:

$$\nabla I(x,y)=\begin{bmatrix}G_x\\ G_y\end{bmatrix}=\begin{bmatrix}\dfrac{\partial U(x,y)}{\partial x}\\ \dfrac{\partial U(x,y)}{\partial y}\end{bmatrix} \qquad (9)$$

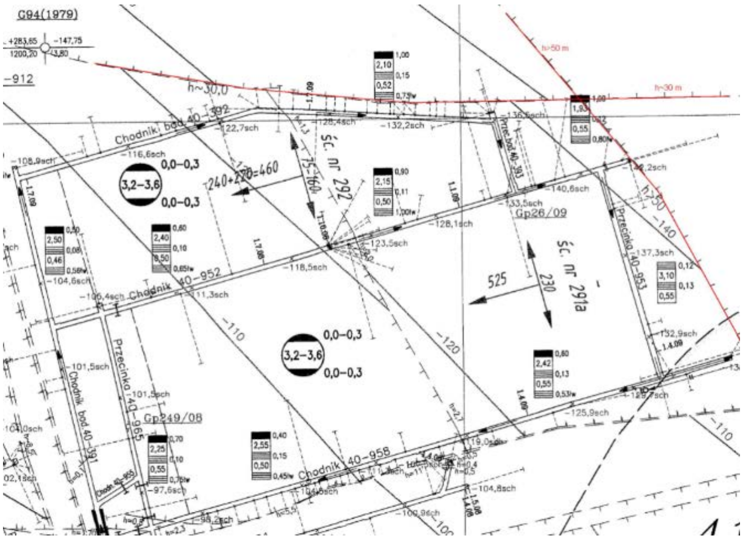

Fig. 6. Section of a mining excavations map, measurement and mapping of dislocations

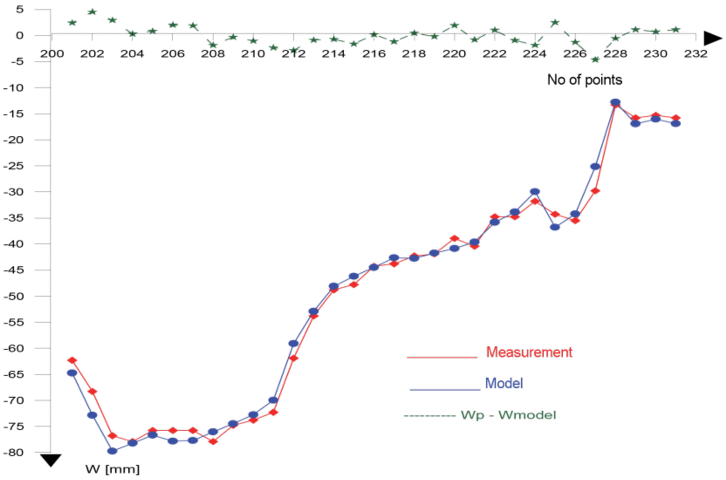

Fig. 7. Comparing the results of modelling and monitoring the process of vertical dislocations

5. VISUALIZATION OF RESULTS OF GNSS ON-LINE DISLOCATION MONITORING

Geodetic measurements on mining areas are the basic source of information about land deformations, including dislocations. The monitoring process of the mining area should be understood as an innovative control and measurement project with a view to verifying the deformation state of a concrete mining subarea. In addition, it is the basis to analyze topological transformations of the given region [3], [11] to be verified by the INSAR method [10].

Observation equation of the phase has the following form:

$$\Phi = \frac{f \cdot d}{c} + f\,(dt - dT) - \frac{f}{c}(d_{trop} - d_{jons}) + N + \varepsilon_{\Phi} \qquad (10)$$

where:

Φ – measured phase,
N – initial indeterminacy of the phase measurement,
f – carrier wave frequency,
ε_{Φ} – phase measurement error,
d – distance between receiver antenna and satellite,
dt – offset of satellite clock,
dT – offset of receiver clock,
d_{jons} – ionospheric delay,
d_{trop} – tropospheric delay,
c – light speed,
e_p – pseudorange measurement error.

Pseudorange observation equation has the following form:

$$p = d + c \cdot (dt - dT) + d_{trop} + d_{jons} + \varepsilon_p \qquad (11)$$

where:

p – measured pseudorange,
d – distance between receiver antenna and satellite,
ε_p – pseudorange measurement error.

Office software Trimble was used during the experiments.

The operations of the monitoring system are executed by the following modules:

- GNSS (GPS) data post-processing module for short and long vector solutions (base lines),
- Trimble RTK Engine module for RTK vector solutions up to 35 km,
- Network Motion Engine module for monitoring the GPS receivers network with the use of multistand determination,
- Rapid Motion Engine module which detects precisely the positions of antennas in the network with a view to observing small and slow deformations and rare violent movements (e.g. landslides, bursts).

The most important results of GPS-based dislocation monitoring in the INGEO project are the following visualization forms of results sets:

- a complete view of satellites in the sky, accessible to be observed from available antennas,
- diagrams of 3-directional dislocations (deformation monitoring),
- history of dislocations of points with installed GPS antennas in a horizontal plane, with respect to the observation point (Scatter Plot).

The presented diagrams feature 3D visualizations of dislocations increase on tower tops with respect to the location of the installed GPS antennas at the moment when the results of dislocations measurements, made in the GPS technology, start to be registered.

3D dislocation diagrams were assigned the following colours:

- Δ Northing – northern direction of dislocation,
- Δ Easting – eastern direction of dislocation,
- Δ Height – vertical direction of dislocations (change along the height of the tower).

Let the trajectory of dislocation between the neighbouring moments of time be described by a general expression:

$$u = U_0 \ln\left(\frac{t}{\beta}\right) \qquad (12)$$

while between points $u_i(t_i)$ and $u_{i+1}(t_{i+1})$ let the trajectory of dislocation be a linear approximation:

$$u^l = \frac{(t - t_i)\cdot(u_{i+1} - u_i)}{(t_{i+1} - t_i)} \qquad (13)$$

The average error resulting from the adopted linear approximation from the real value on the section between u_i and u_{i+1} is as follows:

$$dt_i = \frac{1}{(u_{i+1} - u_i)} \cdot \int_i^{i+1} \left(u - u^l\right) d\tau \qquad (14)$$

Please note that it is possible to extend the monitoring system of the reference station with respect to the ASG Eupos station through buying one licence of the T 4D Control software.

An important element of GPS-based dislocation monitoring is its confirmed utility, i.e. continuous visualization of the mining subarea dislocations, as well as the utility in the form of the GPS RTK supported measurements on the measurement lines of local geodetic networks located in mining areas.

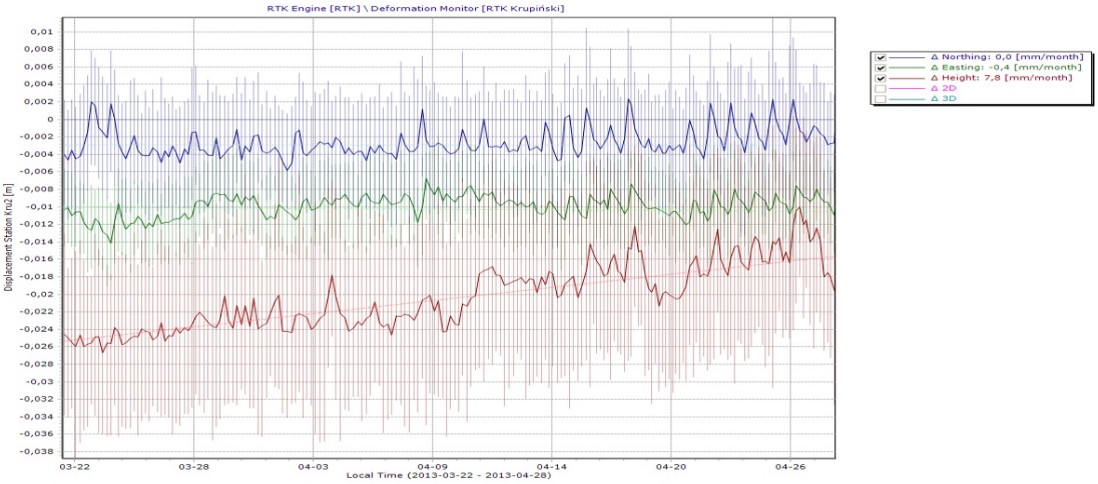

Fig. 8. Dislocation trajectories: ΔN; ΔE; ΔH of a point in the period 22.03.2016 ÷ 28.04.2016

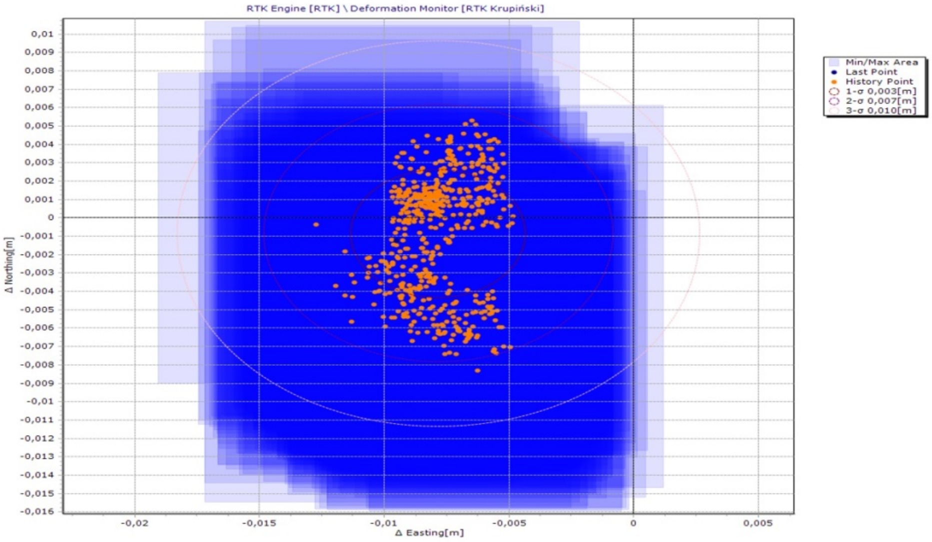

Fig. 9. 2D dislocations diagram (E,N) of the GPS antennas installation point in the period 24.09.2015 ÷ 28.04.2016

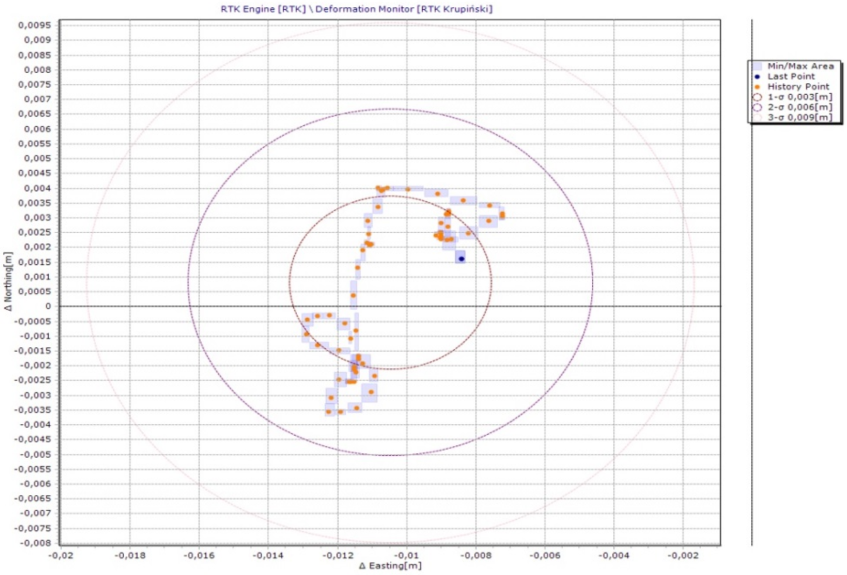

Fig. 10. Sample 2D dislocations diagram (E,N) for the tested shaft – 28.04.2016 (with synchronization timing Δt= 0.25 h)

2D registrations made within 2 months (24.02 – 28.04.2016) show that the range of trajectory field (the tower top of the considered shield) comprised the following area: towards N = 11 mm, towards E = 9 mm.

6. CONCLUSIONS AND REMARKS

For the real (dislocation) process U with càdlàg trajectories we formulated and analyzed the issue of modelling a process whose trajectories should be uniformly close to the trajectories of the U_p (observed) process.

Based on the conducted geodetic measurements, GNSS on-line technology and multiple formal analyses the following conclusions can be drawn:

1. The formulated model describing the evolution of post-mining dislocations operates on a decoupled physical field – here linearization operations are used (integration, discretization, etc.). The solution does not take into account non-monotonic effects (upthrusts, discontinuities) typical of coupled fields. The model, optimized in terms of parameters, accurately maps the model (measurement results) distribution of vertical dislocations. Around the $\partial\Omega$ edge of the dislocation field the mapping results can be burdened with forced evolution of the “free” edge.
2. Mapping quality. Having different parameters of the distribution function and a proper model it is possible to estimate the data credibility. The validation was conducted of the software developed for the purposes of the analyses conducted in the INGEO project, comprising the modelling of deformation processes with the consideration of random disturbances. The validation confirmed the convergence of the achieved results with the data obtained due to the monitoring process of the rock mass located above the exploitation area.
3. The monitoring of the mining area conducted in the INGEO project should be understood as a state-of-the-art control and measurement undertaking for the verification of the deformation state of a concrete mining subarea. This is also the basis to analyze topological transformations of the given area. The monitoring was based on the solutions and modifications of the GNSS on-line measurement technology. The measurement technologies Time-To-First-Fix (TTFF) enable to improve the availability of the satellite signal in areas with environmental difficulties. This enables to depict trajectories and to search for a certain time-dependent spatial measure. The presented modelling and monitoring methodology enables T-optimization in order to select a model that would approach the analyzed process.
4. An important new element of on-line dislocation monitoring in the GPS technology is its confirmed basic utility, i.e. on-line visualization of mining subarea dislocations, as well as the utility in the form of supporting the GPS RTK measurement of single objects and measurement lines of geodetic networks located in mining areas.
5. Currently, there are works conducted to analyze the results of research made with the use of a full stochastic model. The purpose is to acquire experiences helping to determine if it is possible to use the modelling and the developed models (considering random character of tested processes) for the prediction of threats caused by random phenomena, such as rock bursts. To get reliable results it is necessary to conduct observation for a longer time in relation to monitoring results with respect to the parameters of the observed bursts.
6. An attempt to associate dislocation and vibration processes is an innovative approach to model incomplete information about the processes preferences. The trade-off relation can be used for theoretical analysis of the decision making process with respect to the so called coalition processes.

Literature

1. Adams R., Fournier J.: *Sobolev Spaces*. Elsevier/Academic Press, Amsterdam, 2003.
2. Allemang R.J., Brown D.L.: *A Correlation Coefficient for Modal Vector Analysis*. Proceedings of the 1st International Modal Analysis Conference, s. 110 -116, Orlando, 1982.
3. Bosy J., Figurski M.: *Problematyka opracowania obserwacji satelitarnych GPS w precyzyjnych sieciach lokalnych*. (Satellite GPS observations in precise local networks), Wroclaw Agricultural Academy Publishing, Wrocław, 2003.
4. Isakow Z.: *Safecomine intrinsically safe system for monitoring of hazards in mines related to disturbance of the strata and environment equilibrium.* Proceedings of 7th International Symposium on Rockburst and Seismicity in Mines, Dalian, Chiny, 2009, pp. 1045-1056.
5. Isakow Z.: *Systemy do oceny zagrożeń sejsmicznych* w kopalniach. *Cz. 1. (Systems for assessment of seismic threats in mines, Part. 1).* Mechanizacja i Automatyzacja Górnictwa, 2004, 4/399, pp. 5-18.
6. Isakow Z.: *Systemy do oceny zagrożeń sejsmicznych w* kopalniach. *Cz. 2. (Systems for assessment of seismic threats in mines, Part. 2).* Mechanizacja i Automatyzacja Górnictwa, 2004, 5/400, pp. 9-25.
7. Khuri A. I., Cornell, J. A.: *Response Surfaces, Design and Analyses.* 2nd ed. Statistics: Textbooks and Monographs. Marcel Dekker, New York, 1996.
8. Marciniak-Czochra A., Stiehl T., Ho A. D., Jäger W. and Wagner W.: *Modeling asymmetric cell division in hematopoietic stem cells - regulation of self-renewal is essential for efficient repopulation.* Stem Cells Dev., 2008.
9. McLachlan G. J.: *Discriminant Analysis and Statistical Pattern Recognition*. Wiley Interscience, Nowy Jork, 1992.
10. Mirek K., Isakow Z.: *Preliminary analysis of InSAR data from south-west part of Upper Silesian Coal Basin.* Gospodarka Surowcami Mineralnymi, 2009, 25/3, pp. 239-246.

11. Narkiewicz J.: *Globalny system pozycyjny GPS. Budowa, działanie, zastosowanie*. (Global Position System. Structure, operations, application) Wyd. WKIŁ, 2008.
12. Piwowarski W.: *Estimation of the missing results of the surveys concerning a non-stationary post-mining dislocations field*. Geodezja i Kartografia 52(3), Wydawnictwa PAN, Warszawa 2003.
13. Sweatt David J.: *Mechanisms of memory*. Elsevier, 2003.
14. *Uciński* D.: *Optimal Measurement Methods for Distributed-Parameter System Identification*. CRC Press, Boca Raton, FL, 2005.

The article was prepared as a result of the INGEO project: Innovative methods and system to assess rockburst hazard based on probabilistic analysis of the fracturing process and online geo-tomography. The project was co-financed by the National Centre for Research and Development within the Applied Research Programme, agreement No PBS2/B2/8/2013.

JACEK JUZWA
IRENA KUCIARA
KAZIMIERZ SICIŃSKI
{J.Juzwa,K.Sicinski,I.Kuciara}@ibemag.pl
Institute of Innovative Technologies EMAG
ul. Leopolda 31, 40-189 Katowice

WIESŁAW PIWOWARSKI prof. dr hab. inż.
piwowar@agh.edu.pl
Faculty of Mining Surveying and Environmental Engineering
AGH University of Science and Technology
al. A. Mickiewicza 30, 30-059 Kraków