Reduction of railway bridge vibration with groundhook mass damper

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The aim of this research is to compare effectiveness of an ordinary mass damper with a more complex one equipped with an extra spring element connecting the damper mass with the ground. For both simple- and nonlinear- primary structure models and for stable load state, theoretically efficiency of both types of dampers is just the same therefore their efficiencies in bridge structures subjected to non-stationary load are investigated.

Keywords and phrases: mass damper, bridge dynamics, non-stationary load.

Introduction

As nowadays conventional high-speed trains and railways are constructed worldwide, problems bound to specific behavior of this means of transport are getting more essential. From economical point of view it is desired to design lightweight bridge structures, but from engineering point of view it is necessary to be sure, that each bridge can satisfy both — the Ultimate Limit States conditions and the Serviceability Limit States conditions. For train bridges dynamical excitation may affect both states severely, so in order to suppress these effects, tuned mass dampers can be applied. In this paper efficiency of a groundhook damper is investigated. For this reason two models are researched to determine optimal damping parameters.

Mechanical model of damper for simple primary structure

The mass damper considered in this paragraph is a generalized version of the den Hartog's damper — it contains two springs (with stiffness — respectively k_1 and k_2) instead of just one. The first spring connects the damper mass body with the primary structure M and the second one connects the damper mass body with ground (point with blocked movements in all directions) as shown in Fig. 1. Just as proposed by den Hartog, the primary structure is assumed to be subjected to a periodic load.

Classical way of searching optimal parameters of mass damper was applied, as presented in [3]. Differential equations describing behavior of such a mechanical system:

$$\begin{cases} M \cdot \ddot{x}_1 + (k_1 + K)x_1 - k_1 \cdot x_2 = P_0 \sin \omega t, \\ m \cdot \ddot{x}_2 + (k_1 + k_2) \cdot x_2 - k_1 \cdot x_1 = 0. \end{cases} \tag{1}$$

It is known that solution can be found for x_1 and x_2:

$$\begin{cases} x_1 = a_1 \sin \omega t, \\ x_2 = a_2 \sin \omega t. \end{cases} \tag{2}$$

Finally relative amplitudes of vibration subjects following relation:

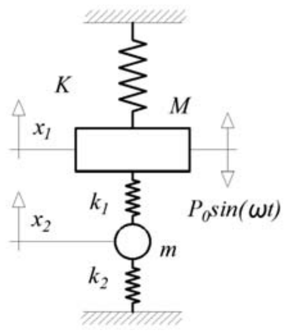

Fig. 1.

$$\begin{cases} \dfrac{a_1}{x_{st}} = \dfrac{-\dfrac{\omega^2 \cdot m}{k_1} + 1 + \dfrac{k_2}{k_1}}{\left(-\dfrac{\omega^2 \cdot m}{k_1} + 1 + \dfrac{k_2}{k_1}\right)\left(-\dfrac{\omega^2}{\Omega_n^{\,2}} + \dfrac{k_1}{K} + 1\right) - \left(\dfrac{k_1}{K}\right)}, \\ \dfrac{a_2}{x_{st}} = \dfrac{1}{\left(-\dfrac{\omega^2 \cdot m}{k_1} + 1 + \dfrac{k_2}{k_1}\right)\left(-\dfrac{\omega^2}{\Omega_n^{\,2}} + \dfrac{k_1}{K} + 1\right) - \left(\dfrac{k_1}{K}\right)}. \end{cases} \tag{3}$$

From (3) one may deduce that for this simple model the primary structure is stationary if:

$$k_1 + k_2 = m \cdot \omega^2 \qquad \textit{and} \qquad k_1 > 0 \tag{4}$$

Mechanical model of damper for nonlinear primary structure

The difference between nonlinear and simple models lies in definition of the primary structure — unlike the damper presented in previous paragraph, this one is assumed to suppress vibration of a mass supported by two nonlinear spring elements as shown below.

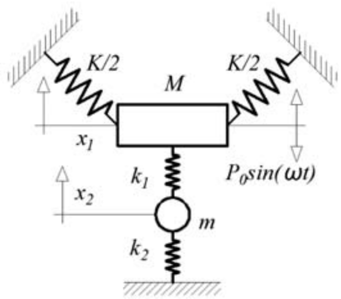

Fig. 2.

Differential equations describing behavior of such a mechanical system:

$$\begin{cases} K \cdot (a - x_1) - K \cdot (a - x_1)\sqrt{\dfrac{2 \cdot a^2}{a^2 + (a - x_1)^2}} + k_1 \cdot (x_2 - x_1) + P_0 \sin \omega t = M\ddot{x}_1, \\ -k_2 \cdot x_2 + k_1(x_1 - x_2) = m\ddot{x}_2. \end{cases} \tag{5}$$

First two elements of the first equation can be expanded to Maclaurin series:

$$K \cdot (a - x_1)\left(1 - \sqrt{\frac{2 \cdot a^2}{a^2 + (a - x_1)^2}}\right) \approx -0.5 \cdot K \cdot x_1 + 0.375 \cdot K \cdot \frac{1}{a} \cdot x_1^2 \tag{6}$$

For $x_1 = 0.01a$ error of the nonlinear term is estimated at level of 1%. As for optimal tuning of the damper the value of x_1 is zero it was assumed that taking into account only the linear term is satisfying.

$$\textit{for} \qquad K \cdot (a - x_1)\left(1 - \sqrt{\frac{2 \cdot a^2}{a^2 + (a - x_1)^2}}\right) = -0.5 \cdot K \cdot x_1 \tag{7}$$

$$\begin{cases} -\dfrac{K}{2} \cdot x_1 + k_1 \cdot x_2 - k_1 \cdot x_1 + P_0 \sin \omega t = M \cdot \ddot{x}_1, \\ -k_2 \cdot x_2 + k_1 \cdot x_1 + k_1 \cdot x_2 = m \cdot \ddot{x}_2. \end{cases} \tag{8}$$

Finally:

$$\frac{a_1}{P_0} = \frac{k_1 + k_2 - \omega^2 \cdot m}{\left(\dfrac{K}{2} + k_1 - \omega^2 \cdot M\right) \cdot \left(k_1 + k_2 - \omega^2 \cdot m\right) - k_1^2} \tag{9}$$

Optimal tuning condition is just the same as in previous paragraph (4).

It is assumed that for a more complex system (a beam with non-stationary load) the same relations should give satisfying suppression of vibrations, with elimination of the resonant growth of amplitude of deflections.

The goal of this research is finding optimal relation of k_1 to k_2.

Mechanical models of the bridge and of carriages

The Euler-Bernoulli simple beam governing equation with constant values of $m(x)$, $C_y(x)$, $EI(x)$:

$$m\frac{\partial^2 y(x,t)}{\partial t^2} + C_y \frac{\partial y(x,t)}{\partial t} + EI\frac{\partial^4 y(x,t)}{\partial x^4} = P_b(x,t) \tag{10}$$

where: m — the beam mass per unit length,
$y(x,t)$ — the beam deflection,
C_y — the beam damping,
EI — the beam bending stiffness,
$P_b(x,t)$ — load.

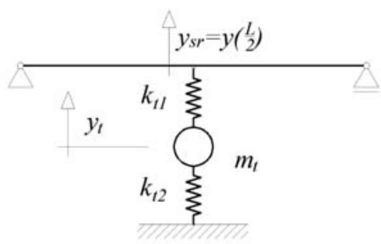

Fig. 3.

Schematic model of the bridge-damper system is shown in Fig. 3.

Equation (11) presents total load acting on the Euler beam, consisting of train load and damper reaction.

$$\begin{aligned} P_b(x,t) = &-\sum_{k=1}^{N_v} p_{ki} \cdot \delta\left[x - v \cdot (t - t_k)\right] \cdot H_0(t, t_k) - \\ &-\delta\left[x - \frac{L}{2}\right] \cdot \left[(y_{sr} - y_t) \cdot k_{t1}\right] \end{aligned} \tag{11}$$

$$H_0(t, t_k) = U(t - t_k) - U\left[t - \left(t_k + \frac{L}{v}\right)\right] \tag{12}$$

$$\delta(x)=\begin{cases}\infty & \text{for } x=0\\ 0 & \text{for } x\neq 0\end{cases} \quad \text{and} \quad \int_{-\infty}^{\infty}\delta(x)dx=1 \qquad (13)$$

$$U(t)=\begin{cases}1 & \text{for } t\geq 0\\ 0 & \text{for } t<0\end{cases} \qquad (14)$$

where: N_v — number of carriages,
t_k — the k — carriage time of approach of the beginning of the bridge,
L — the bridge length,
v — speed of moving load.

The train was modeled as a series of N_v carriages moving along the beam with a constant speed v. One carriage consists of a rigid body with two degrees of freedom, supported in two points on suspensions, each consisting of coupled spring and damper as shown in Fig. 4.

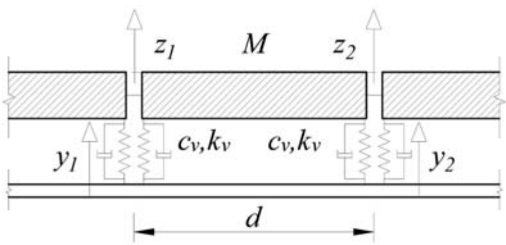

Fig. 4.

$$p_{ki}=0.5\cdot M\cdot g+k_v\cdot(y_k-z_k)+c_v\cdot(\dot{y}_k-\dot{z}_k) \quad \text{and} \quad k=1,2 \qquad (15)$$

where: g — gravitational acceleration,
M — mass of the carriage,
z_k — vertical displacement of the rigid bogie mass k point,
y_k — current deflection of beam under the k point,
k_v — stiffness of the carriage suspension,
c_v — damping of the carriage suspension.

Each carriage subjects to following system of differential equations:

$$\begin{cases}\ddot{z}_2+\ddot{z}_1=\dfrac{2}{M}\cdot(k_v\cdot(y_1-z_1)+k_v\cdot(y_2-z_2)+c_v\cdot(\dot{y}_1-\dot{z}_1)+c_v\cdot(\dot{y}_2-\dot{z}_2))\\ \ddot{z}_2-\ddot{z}_1=\dfrac{d^2}{I}\cdot(-k_v\cdot(y_1-z_1)+k_v\cdot(y_2-z_2)-c_v\cdot(\dot{y}_1-\dot{z}_1)+c_v\cdot(\dot{y}_2-\dot{z}_2))\end{cases} \qquad (16)$$

$$I=\frac{M\cdot d^2}{12}$$

where: d — length of a carriage.

Numerical model

$$y(x,t)=\sum_{j=1}^{N}\varphi_j(x)\cdot\eta_j(t)=\Phi^T(x)\cdot\mathrm{H}(t) \qquad (17)$$

where: $\Phi(x)$ — vector of shape functions of first natural modes of the beam,
N — number of natural modes taken into account.

For a simple beam vector of N first natural modes consists of following terms:

$$\Phi(x)=\left\{\sin\frac{\pi\cdot v}{L},\sin\frac{2\pi\cdot v}{L},\ \ldots\sin\frac{N\pi\cdot v}{L}\right\} \qquad (18)$$

Putting (17) into (10), multiplying both sides by Φ(x), integrating with respect to x variable and coupling with the damper governing equation gives:

$$\begin{cases}\left(\int_0^L\Phi(x)\cdot m\cdot\Phi^T(x)dx\right)\ddot{\mathrm{H}}(t)+\left(\int_0^L\Phi(x)\cdot C_y\cdot\Phi^T(x)dx\right)\dot{\mathrm{H}}(t)+\\ \left(\int_0^L\Phi(x)\cdot EI\cdot\Phi^T(x)^{IV}dx\right)\mathrm{H}(t)=\\ =-\sum_{k=1}^{Nv}p_k\cdot\Phi[v(t-t_k)]\cdot H_0(t,t_k)-\Phi\left(\dfrac{L}{2}\right)\cdot((y_{sr}-y_t)\cdot k_{t1})\\ m_t\cdot\ddot{y}_t(t)+(k_{t1}+k_{t2})\cdot y_t=k_{t1}\cdot y_{sr}\end{cases} \qquad (19)$$

In a simplified form:

$$\begin{cases}[M_b]\cdot\ddot{\mathrm{H}}(t)+[C_b]\cdot\dot{\mathrm{H}}(t)+[K_b]\cdot\mathrm{H}(t)=\\ =F_b(t)-\Phi\left(\dfrac{L}{2}\right)\cdot((y_{sr}-y_t)\cdot k_{t1})\\ m_t\cdot\ddot{y}_t(t)+(k_{t1}+k_{t2})\cdot y_t=k_{t1}\cdot y_{sr}\end{cases} \qquad (20)$$

Finally as matrices:

$$\begin{bmatrix}[M_b] & [0]\\ [0]^T & m_t\end{bmatrix}\cdot\begin{bmatrix}\ddot{\mathrm{H}}(t)\\ \ddot{y}_t(t)\end{bmatrix}+\begin{bmatrix}[C_b] & [0]\\ [0]^T & 0\end{bmatrix}\cdot\begin{bmatrix}\dot{\mathrm{H}}(t)\\ \dot{y}_t(t)\end{bmatrix}+$$
$$+\begin{bmatrix}[[K_b]+[K_{dod}]] & -\Phi\left(\dfrac{L}{2}\right)\cdot k_{t1}\\ -\Phi\left(\dfrac{L}{2}\right)\cdot k_{t1} & k_{t1}+k_{t2}\end{bmatrix}\cdot\begin{bmatrix}\mathrm{H}(t)\\ y_t(t)\end{bmatrix}=\begin{bmatrix}F_b(t)\\ 0\end{bmatrix} \qquad (21)$$

Where [0] is zero vector.

This differential equation was solved using the Finite Differences Method.

Resonant load

According to [2] a beam bridge responds with resonance to a series of moving loads if one of following two conditions is satisfied:

$$v=\frac{\omega_j\cdot d}{2\cdot\pi\cdot n} \qquad (22)$$

$$v=\frac{\omega_j\cdot L}{\pi\cdot n} \qquad (23)$$

where: ω_j — bridge natural frequency,
d — load spacing.

The (23) condition is much less possible to occur, because it refers to very high speeds — practically not achieved. Therefore it is assumed that the damper should be tuned for load caused by train moving with the first resonant speed according to relation (22).

Analysis results and conclusions

Values of train mass and dimensions were set to imitate a TGV train. This paper deals with a bridge-train system with following parameters:

$$N_{vv} = 10 \quad \textit{number of carriages}$$
$$d = 18m$$
$$EI = 22e10\,\mathrm{m}^4\mathrm{N/m}^2$$
$$L = 40m$$
$$m = 60000kg/m$$
$$M = 60000kg$$
$$\omega_1 = \pi^2\sqrt{\frac{EI}{m \cdot L^4}} = 11.812\ rad/\sec$$

Thus the resonant load occurs for speed:

$$v = 33.84m/s \tag{24}$$

Then the load frequency ω:

$$\omega = \frac{2 \cdot \pi \cdot v}{d} = 11.812 rad/s \tag{25}$$

The simulation was carried out in three ceses differing in the damper mass m_t. The mass m_t was set as 0.5%, 1% and 1.5% of the total mass of the bridge. Finally, in order to eliminate the resonance effect, the damper had to be tuned according to (4) as follows:

Case 1:

$m_t = 12000kg$
$k_1+k_2 = \omega^2 \cdot \mathrm{m}_t \approx 1.67e6\ N/m$

Case 2:

$m_t = 24000kg$
$k_1+k_2 = \omega^2 \cdot \mathrm{m}_t \approx 3.349e6\ N/m$

Case 3:

$m_t = 36000kg$
$k_1+k_2 = \omega^2 \cdot \mathrm{m}_t \approx 5.023e6\ N/m$

Excitation of the structure was investigated in two points. In the first approach, in order to analyze influence of the k_2/k_1 ratio on the dynamic behavior of the beam bridge, deflection in mid-span was calculated for models with and without mass damper, in three above-mentioned damper cases. In this approach deflection was estimated only for the first natural mode of the modal composition, as it quite accurately refers to the one degree of freedom model of the primary structure. For optimal k_2/k_1 ratios from the first approach, the second approach was applied — deflection under each moving axis was calculated and compared with results for model without damper. For each calculation deflection of these dynamical systems were compared with equivalent static deflection.

Results prove that the higher efficiency rate is achieved

Table 1. Maximal deflection in the bridge midspan in three cases.

	case			
$k_2/(k_1+k_2)$	1	2	3	without damper
0	8,356	7,387	6,939	13,42
0,1	8,661	7,659	7,131	13,42
0,2	9,072	7,936	7,398	13,42
0,25	9,286	8,133	7,565	13,42
0,3	9,536	8,349	7,736	13,42
0,4	10,121	8,831	8,16	13,42
0,5	10,875	9,471	8,719	13,42
0,6	11,705	10,349	9,521	13,42
0,7	12,407	11,493	10,657	13,42
0,8	12,944	12,492	12,056	13,42

Table 2. Maximal deflection under moving loads in three cases for $k_2/k_1 = 0$.

		Deflection under moving loads [mm] Axis serial number										
		1	2	3	4	5	6	7	8	9	10	11
case	1	3,687	5,011	6,499	7,173	7,639	7,844	7,773	7,453	6,957	6,432	4,414
case	2	3,643	4,959	6,31	6,795	7,001	6,885	6,571	6,293	6,366	6,389	4,717
case	3	3,602	4,911	6,144	6,523	6,585	6,435	6,318	6,317	6,36	5,857	3,551
without damper		3,731	5,106	6,715	7,685	8,619	9,499	10,323	11,087	11,796	12,439	11,11

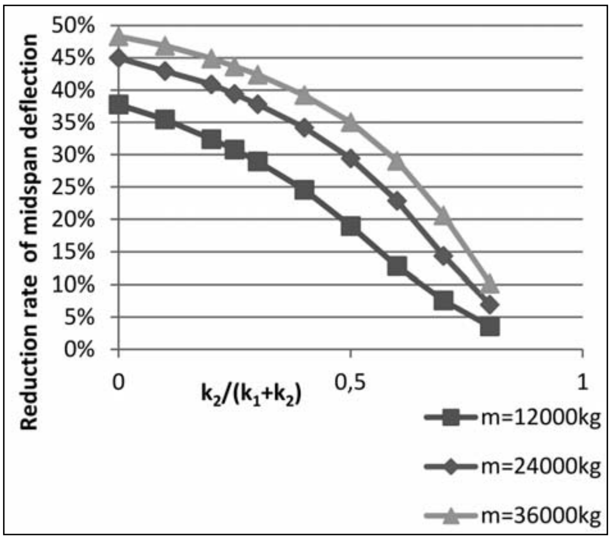

Fig. 5. Reduction of vibration in the bridge midspan in three cases.

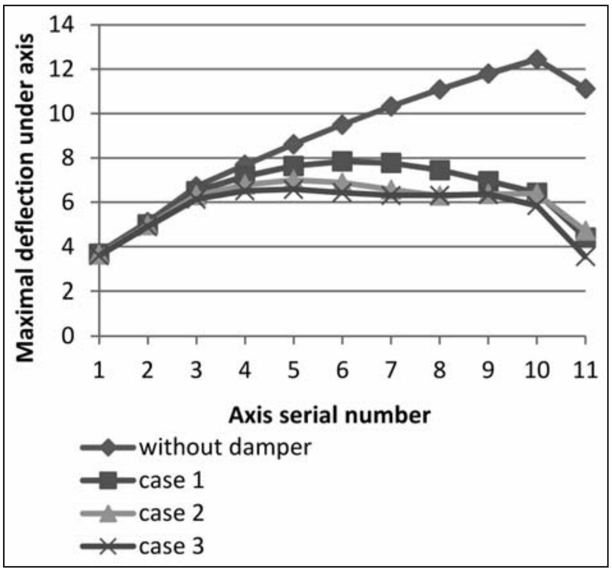

Fig. 6. Reduction of vibration under moving loads in three cases.

for zero stiffness value of the additional spring element, and it can be observed that the higher the participation of the extra spring stiffness in the total stiffness of the damper, the higher drop of effectiveness. Applying the extra spring element makes the system slower reacting to non-stationary load, which results in bigger increase of deflections in initial phase of the train-bridge interaction. The damper mass also affects effectiveness of a damper, which is higher for heavier damper mass, but growth of effectiveness drops with rise of the mass. A significant suppression was observed for deflection in the midspan — the maximal deflection was reduced from 13.4 mm to 8.4 mm in the first case, 7.4 mm in the second case and 6.9 mm in the third case. Deflections were reduced by 37,7%, 45,0% and 48,3% respectively.

Calculations show considerable reduction of maximal deflection under moving axes, but bigger suppression was noted for axes above 5th, as the damper began accumulating energy and when values of deflection were bigger. Simulation results for optimal system with zero value of the k_2/k_1 ratio are presented below.

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