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THEORY OF CUMULATIVE FUEL CONSUMPTION BY LPG POWERED CARS

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Abstract

Theory of cumulative fuel consumption is shown for the first time in [1] and [2]. The theory of cumulative fuel consumption has been presented also in this work. The example of LPG car research results have shown the way of getting to mathematical model of cumulative fuel consumption and the intensity of cumulative fuel consumption. In this case, the studies were carried out 16 cars whose engines were powered by LPG. The vehicles are operated in a small fleet of vehicles, which run in city traffic. Data on outcomes of exploitation and operation fuel consumption is acquired from the accounting documents of the company. Very good results prediction mathematical model of operational data are obtained. The high value of prescience quotients (in this case R-sq > 9.999) are similar to the values that were obtained in various other cases. Conversance of mathematical model of cumulative fuel consumption allows carrying out comprehensive analysis of this significant exploitative parameter. The presented theory must not only be regarded as a theory of cumulative fuel consumption but also can be seen more broadly as a theory of cumulative energy consumption. Research is being conducted on the application of the theory to evaluate energy consumption by hybrid vehicles. The results can be very interesting.

Keywords: *fuel consumption, theory, applications*

1. Theory of cumulative fuel consumption

Fuel is injected into the combustion chamber in a “quantum”. Quanta are the fuel delivery. Fuel quanta have a random size.

Total volume (or weight) of quanta of fuel supplied to the engine is called fuel consumption.

Fuel quantum summation leads to determining the cumulative fuel consumption. Fuel consumption caused by the time t of the engine work, can be designated as:

$$Q_{sp}(t) = \sum_{i=1}^{n(t)} \nu_i = n(t) \cdot \overline{\nu}(t)\,, \tag{1}$$

where:

ν_i – i-th quantum of spent fuel (e.g. fuel delivery per engine revolution)
$\overline{\nu}(t)$ – the average size of the quantum of fuel consumed to the time t,
$n(t)$ – quantum number of the fuel consumed to the time t,
$Q_{sp}(t)$ – the cumulative fuel consumption at time t.

To know the cumulative fuel consumption to time t it should be familiar with the average size of the quantum-supplied fuel and the number of quanta of fuel consumed to that time.

T_p is a random variable representing the time between successive doses of fuel. The distribution of the random variable is:

$$F_p(t) = P_p\{T_p < t\}\,, \tag{2}$$

where P_p $\{T_p < t\}$ is the probability that T_p assumes smaller values of t; t is any amount of time.

Derivative of the cumulative

$$\frac{\mathrm{d}F_p(t)}{\mathrm{d}t} = F_p'(t) = f_p(t)\,, \tag{3}$$

is the density of random variable T_p.

If we assume further that P_p $\{t, t + \Delta t\}$ is the probability that in the time interval Δt, and thus a period of time $(t, t + \Delta t)$ there will be no fuel supply, and that the fuel was also administered over a time period $(0, t)$, according to Bayes rule

$$P_p\{t,\, t+\Delta t\} = \frac{P_p\{T_p \geq t+\Delta t\}}{P_p\{T_p \geq t\}} = \frac{R_p(t+\Delta t)}{R_p(t)}\,, \tag{4}$$

where:

$$P_p\{T_p \geq t\} = 1 - P_p\{T_p < t\} = 1 - F_p(t) = R_p(t)\,. \tag{5}$$

If there is

$$P_p^d\{t,\, t+\Delta t\} = 1 - P_p\{t,\, t+\Delta t\}\,, \tag{6}$$

a probability that in the time interval there will be dose Δt fuel, provided that there was no fuel supply in the time period $(0, t)$ then:

$$P_p^d\{t,\, t+\Delta t\} = P_p\{t \leq T_p \leq t+\Delta t\}\,, \tag{7}$$

$$P_p^d\{t,\, t+\Delta t\} = 1 - P_p\{t,\, t+\Delta t\} = 1 - \frac{R_p(t+\Delta t)}{R_p(t)}\,. \tag{8}$$

After dividing both sides of equation (8) through Δt, there is obtained

$$\frac{P_p^d\{t,\, t+\Delta t\}}{\Delta t} = \frac{1}{\Delta t}\left(1 - \frac{R_p(t+\Delta t)}{R_p(t)}\right) = \frac{R_p(t) - R_p(t+\Delta t)}{\Delta t}\cdot\frac{1}{R_p(t)}\,. \tag{9}$$

The limes of the expression (9) with $\Delta \to 0$ is:

$$\lim_{\Delta t\to 0}\frac{P_p^d\{t,\, t+\Delta t\}}{\Delta t} = \lim_{\Delta t\to 0}\left(-\frac{R_p(t+\Delta t) - R_p(t)}{\Delta t}\right)\cdot\frac{1}{R_p(t)} = -\frac{R_p'(t)}{R_p(t)}\,. \tag{10}$$

Limes (10) can be determined by $\lambda_p(t)$. It represents the intensity of the dosing fuel at time t:

$$\lambda_p(t) = -\frac{R_p'(t)}{R_p(t)}\,. \tag{11}$$

The number of quanta given fuel to the engine until t can be defined as:

$$n(t) = \int_o^t \lambda_p(t)\ \mathrm{d}t = \int_o^t\left(-\frac{R_p'(t)}{R_p(t)}\right)\mathrm{d}t = -[\ln R_p(t) + C\,]\Big|_0^t = \left(\ln\frac{1}{R_p(t)} - C\right)\Big|_0^t\,. \tag{12}$$

At time $t = 0$ is not given any fuel, and no amount of fuel – from the definition of $R_p(t = 0) =$ $= 1 - F_p(t = 0) = 1 - 0 = 1$,

$$0 = \ln\frac{1}{R_p(t=0)} - C = \ln(1) - C = -C\,, \tag{13}$$

further means that $C = 0$ and the number of doses given to the time t is expressed as:

$$n(t) = \ln \frac{1}{R_p(t)} \cdot \tag{14}$$

Further, it is necessary to know the distribution of the random variable T_p and therefore the distribution of time intervals between successive doses of fuel.

The size of individual quantumvi fuel has a random size. This quantity can be described as a two-dimensional statistical distribution with density $f(\nu, t)$.

The average size of the quanta of fuel fed to the engine over a period $(0, t)$ can be determined generally as:

$$\bar{\nu}(t) = \int_0^\infty \int_0^t \nu \cdot t \cdot f(\nu, t) \, \mathrm{d}\nu \, \mathrm{d}t \, . \tag{15}$$

The limits of integration, the amount of fuel doses not change from zero to infinity but at a certain interval to (15), it can be written as:

$$\bar{\nu}(t) = \int_{\nu_{\min}}^{\nu_{\max}} \int_0^t \nu \cdot t \cdot f(\nu, t) \, \mathrm{d}\nu \, \mathrm{d}t \, . \tag{16}$$

Taking the above into account, it is in accordance with (1) and (14) to write:

$$Q_{zsp}(t) = \bar{\nu}(t) \cdot n(t) = \bar{\nu}(t) \ln \frac{1}{R_p(t)} = \bar{\nu}(t) \ln \frac{1}{1 - F_p(t)} \cdot \tag{17}$$

In the case of the Poisson distribution $F(t) = 1 - \mathrm{e}^{-\lambda t}$.

In this equation, the specific operation of the engine and Rp (t) are unknown. Both of these values can be determined in operational research. It was assumed that these intervals could describe the type of statistical distribution of the Poisson distribution function.

There is also a well-known form of the statistical distribution of the dose of fuel. Because it is a function of time, however, it can be assumed approximately $\bar{\nu}(t) = \bar{\nu} \cdot t^a$.

Based on both assumptions and taking into account (1) and (14) there can be written:

$$\begin{aligned} Q_{zsp}(t) &= \bar{\nu}(t) \cdot n(t) = \bar{\nu}(t) \cdot \ln \frac{1}{1 - F(t)} = \bar{\nu} t^a \cdot \ln \frac{1}{1 - (1 - \mathrm{e}^{-\lambda t})} = \\ &= \bar{\nu} t^a \cdot \ln \frac{1}{\exp(-\lambda t)} = \bar{\nu} t^a \cdot \ln \, \exp(\lambda t) = \bar{\nu} t^a \lambda \, t. \end{aligned} \tag{18}$$

Because assumptions, $\bar{\nu} \equiv \text{const}$ and $\lambda \equiv \text{const}$, also:

$$\bar{\nu} \cdot \lambda = c = \text{const} , \tag{19}$$

Then:

$$Q_{zsp}(t) = c \cdot t^a \cdot t = c \, t^{a+1} \, . \tag{20}$$

This is a relatively simple relationship describing the cumulative fuel consumption as a function of time.

The cumulative fuel consumption, based on the engine run time, is the intensity of the cumulative fuel consumption after a given period of engine operation.

$$\frac{\mathrm{d}Q_{zsp}}{\mathrm{d}t} = c(a + 1) \, t^a \, . \tag{21}$$

The intensity of the cumulative fuel consumption takes values infinitely large when $a < 0$ and where $t \to 0$, and so almost immediately after the start of the engine (but rapidly decreases with increasing length of the distance travelled).

2. A method of determining the cumulative fuel consumption on the basis of experimental data

Running a mathematical model of consumption, which is the equation (18), and at the assumptions, equation (20), is known when these factors are known as *c* and *a*.

Their values are determined, e.g. the mathematical study of experimental results.

Equation (20) can be converted to a convenient form in the first-degree polynomial because after taking the logarithm on both sides thereof is replaced with:

$$\ln Q_{zsp}(t) = \ln (c\,t^{a+1}) = \ln c + (a+1)\ln t\,. \tag{22}$$

After substituting:

$$\ln Q_{zsp}(t) = y\,, \quad \ln c = b_0\,, \quad (a+1) = b_1\,, \quad \ln t = x\,, \tag{23}$$

(22) is converted into:

$$y = b_0 + b_1\,x\,. \tag{24}$$

This is the equation of a straight line.

The cumulative fuel consumption is considered in the time domain. To meet this requirement it should be given time instance – moto hours. So given the working time machine for example agricultural tractors. In the transport vehicle uptime adopted administered as their course. However, the course is not the same as working time vehicle engines. When the vehicle is stationary and the engine is, running there is fuel consumption without increased mileage.

For these reasons, it is impossible to directly apply the model type (18), namely (20), because no data is collected relating to the fuel consumption in the time domain, understood and as is apparent from the need to model the cumulative fuel consumption. In further exploitation, the time of the vehicle is expressed in kilometres. With this assumption:

$$t \leftrightarrow km\,, \tag{25}$$

where: *km* – mileage.

The adoption of this assumption makes that the model (20) is in the form

$$Q_{zsp}(km) = c(km)^{a+1}\,. \tag{26}$$

The assumption (25) can lead to errors because fuel consumption is recorded, and the life of the engine is not because it is recorded mileage.

Details of the method of determining the mathematical model of the cumulative fuel consumption and statistical verification of his prediction, and a particular example of the results of investigations results LPG powered cars, are given below.

It was investigated the fuel consumption of fourteen LPG powered cars. The results of these tests were used to determine the mathematical model of the cumulative fuel consumption. An example of the results is given in Tab. 1.

Table 2 shows the coefficients of a mathematical model.

Table 3 shows the results of the analysis of the quality of the mathematical model. Achieved parameters are exceptionally good. This further confirms the chart in Fig. 1.

Its course is distinctive and is repeated in the case of research of many cars.

Table 4 contains the model coefficients of cumulative consumption LPG fuel of all tested cars.

Prediction parameters of models are very high. R-squared for cumulative fuel consumption models of all vehicles is not less as 0.999.

3. Conclusions

1) The theory of cumulative fuel consumption has been presented.

Tab. 1. The measurement and calculation data of LPG fuel consumption of the engine of the car (Renault Traffic 3)

	Mileage	Cumulative fuel consumption	Mileage (logarithmically)	Cumulative fuel consumption (logarithmically)	Cumulative fuel consumption (model)	Cumulative fuel consumption (difference)	Intensity of cumulative fuel consumption
	km	dm^3	ln(km)	ln(dm^3)	dm^3	%	dm^3/km
1	2 343	213	7.759187	5.3612922	219	–2.60	0.090240
2	6 792	612	8.823479	6.4167323	612	0.01	0.087168
3	10 826	969	9.289701	6.8762646	961	0.85	0.085855
4	15 635	1389	9.657264	7.2363393	1 371	1.30	0.084834
5	21 532	1893	9.977284	7.5459182	1 869	1.29	0.083956
6	28 357	2469	10.252642	7.8115685	2 439	1.22	0.083207
7	35 125	3033	10.466678	8.0173075	3 000	1.09	0.082629
8	38 661	3324	10.562587	8.1089242	3 292	0.97	0.082371
9	43 606	3726	10.682941	8.2230906	3 698	0.75	0.082049
10	47 341	4026	10.765124	8.3005286	4 004	0.54	0.081830
11	52 789	4455	10.874056	8.4017823	4 449	0.13	0.081541
12	59 458	4974	10.993026	8.5119796	4 992	–0.36	0.081226
13	67 740	5616	11.123430	8.6333749	5 663	–0.84	0.080882
14	74 877	6165	11.223600	8.7266434	6 240	–1.21	0.080618
15	80 890	6624	11.300842	8.7984547	6 724	–1.50	0.080416
16	86 369	7041	11.366386	8.8595055	7 164	–1.74	0.080245

Tab. 2. Model coefficients of cumulative LPG consumption (Renault Traffic 3)

Coefficients	b_0	b_1	c	a
Values	–2.119680	0.967456	0.120070	–0.032544

Tab. 3. Parameters of the model prediction of the cumulative consumption of LPG powered Renault Traffic 3 car

Regression statistics				
Multiple of R	R-squared	Adjusted R-squared	Standard error SA	Observations
0.999920	0.999839	0.999828	0.013177	16

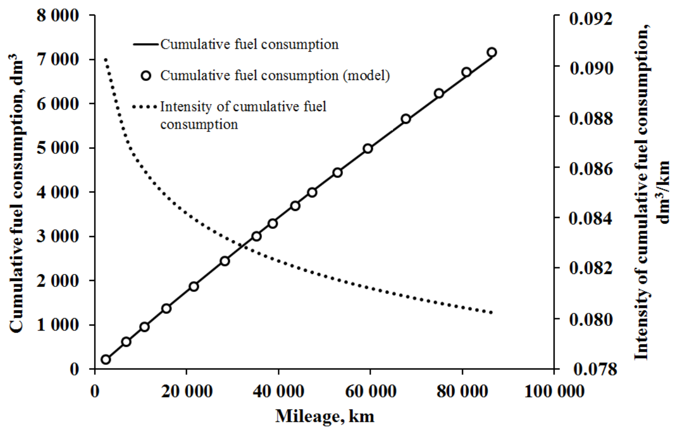

Fig. 1. Cumulative consumption of fuel by LPG powered Renault Traffic 3 car

2) The example of LPG car research results have shown the way of getting to mathematical model

of cumulative fuel consumption and the intensity of cumulative fuel consumption.

3) Data on outcomes of exploitation and operation fuel consumption acquired from the accounting documents of the company.

Tab. 4. The model coefficients of cumulative consumption LPG fuel of cars

	Tested cars	b_0	b_1	R-sq.	c	a
1	Fiat Ducato	–2.395192	0.974905	0.999898	0.091155	–0.025095
2	Renault Traffic 3	–2.119680	0.967456	0.999839	0.120070	–0.032544
3	Renault Kang. 4	–1.763006	0.944861	0.999816	0.171529	–0.055139
4	Renault Kang. 3	–1.819454	0.943761	0.999808	0.162114	–0.056239
5	Renault Kang. 2	–1.805672	0.950142	0.999802	0.164364	–0.049858
6	Renault Traffic 2	–2.149325	0.969470	0.999785	0.116563	–0.030530
7	Fiat Doblo 2	–2.022880	0.976935	0.999760	0.132274	–0.023065
8	Fiat Doblo 1	–1.958163	0.969716	0.999735	0.141117	–0.030284
9	Opel Astra	–2.020415	0.956704	0.999724	0.132600	–0.043296
10	Renault Traffic 1	–2.075968	0.959196	0.999710	0.125435	–0.040804
11	Renault Kang. 1	–1.850900	0.953435	0.999646	0.157096	–0.046565
12	Skoda Fabia 3	–1.466967	0.923717	0.999411	0.230624	–0.076283
13	Skoda Fabia 2	–1.564869	0.931084	0.999336	0.209115	–0.068916
14	Skoda Fabia 1	–1.737393	0.951020	0.999315	0.175979	–0.048980

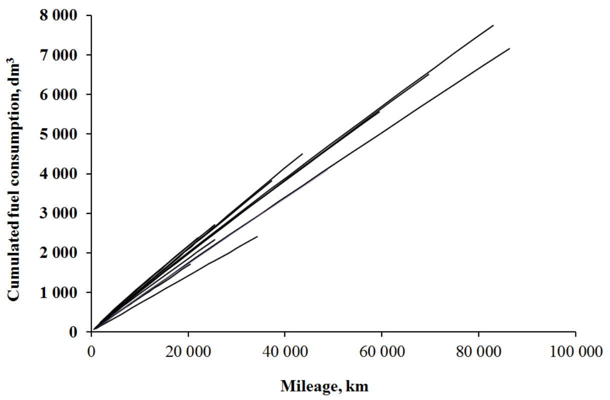

Fig. 2. Cumulative fuel consumption all investigated cars

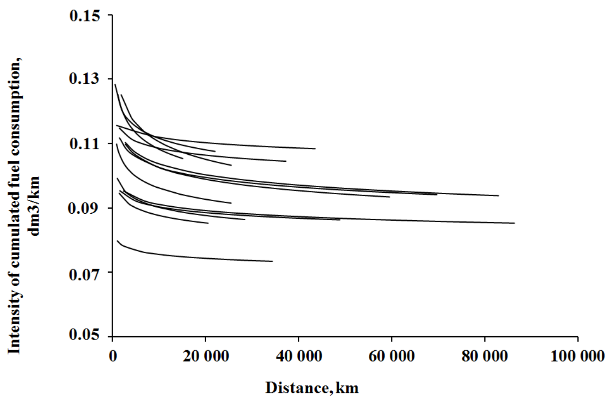

Fig. 3. Intensity of cumulative fuel consumption all investigated cars

4) Very good prediction results in mathematical model of operational data are obtained. The high value of prescience quotients (in this case R-sq > 9.999) are similar to values which were obtained in various other cases.
5) Conversance of mathematical model of cumulative fuel consumption allows to carry out comprehensive analysis of this significant exploitative parameter.
6) Presented theory must not only be regarded as a theory of cumulative fuel consumption but also can be seen more broadly as a theory of cumulative energy consumption.

Research is being conducted on the application of theory to assess the energy consumption of hybrid vehicles – the results can be very interesting.

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