Model predictive control of the two-mass drive system with mechanical backlash

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In the paper an application of the model predictive controller for the drive system with elasticity and backlash is presented. In the introduction the control problem of the drive system with mechanical backlash is introduced. Next, the mathematical model of the drive is presented. Then the predictive algorithm is described briefly. The performance of the predictive controller is examined in the simulation study. The influence of the changing of the matrix Q is tested.

1. Introduction

The nonlinearities of the mechanical part of the drive, such as friction or backlash decrease the performance the system [1]-[4]. The mechanical backlash evident in gearboxes shortens the life-time of the whole drive system [2]-[4]. This is especially evident in multi-mass systems in which the load (machine) is connected to a driving motor through one or multiple flexible shafts. Excessive shaft twists and poorly damped torsional vibrations are detrimental to the drive's performance, greatly decreasing product quality and system reliability, and in some cases leading to instability and failure of the entire drive system. This problem commonly occurs in rolling-mill drives, belt-conveyors, paper machines, robotic-arm drives including space manipulators, servo-drives and throttle systems [1]-[12].

The control methodologies developed in this type of the drive may be based on linear PI controllers with an additional feedback [1, 3, 5, 6], state controllers [6], or robust controllers using the H^{∞} control law [7].

In recent years, model predictive control (MPC) has been widely investigated for its potential in controlling modern electrical drives and power electronics circuits [15]–[18]. Predictive control presents several advantages that make it suitable for the control of power converters and drives. The central feature of MPC is that it enables the process operating and physical constraints (due to e.g. resource limitations, operational or safety concerns as well as limits arising from various economic objectives) to be taken directly into consideration in the control problem formulation so that any potential constraint violations are anticipated and prevented. Additionally, as the control input is obtained by solving an optimization problem at each sampling time, it can ensure truly optimal performance of the closed-loop control system [8]-[12].

The main contribution of this paper is the design and validation of an explicit model predictive controller for a two-mass elastic drive system with mechanical backlash. The explicit version of the MPC algorithm presented here does not involve complex optimization to be performed in a control unit but requires only a piecewise linear function evaluation, which can be realized through a simple look-up table approach. The effect of the additional input of the system, on the drive dynamic not considered in the literature is examined. Also the robustness of the control structure to the changes of the mechanical gap is considered.

2. The mathematical model of the drive

Many industrial drive systems can be modeled as two-mass systems, where the first mass represents the moment of inertia of the motor and the second mass refers to the moment of inertia of the load machine. In this paper, the commonly used inertia-free-shaft dual-mass system model will be employed, which is described by the following normalized differential equations

$$\frac{d}{dt}\begin{bmatrix}\omega_1\\ \omega_2\\ \alpha_1\\ \alpha_2\\ m_s\end{bmatrix}=\begin{bmatrix}0 & 0 & 0 & 0 & \frac{-1}{T_1T_c}\\ 0 & 0 & 0 & 0 & \frac{1}{T_2T_c}\\ 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ \varphi(\Delta\theta) & -\varphi(\Delta\theta) & 0 & 0 & 0\end{bmatrix}\cdot\begin{bmatrix}\omega_1\\ \omega_2\\ \alpha_1\\ \alpha_2\\ m_s\end{bmatrix}+\begin{bmatrix}\frac{1}{T_1}\\ 0\\ 0\\ 0\\ 0\end{bmatrix}\cdot[m_e]+\begin{bmatrix}0\\ \frac{-1}{T_2}\\ 0\\ 0\\ 0\end{bmatrix}\cdot[m_L] \tag{1}$$

where: ω_1 ,ω_2 – motor and load speeds, m_e, m_s, m_L – electromagnetic, shaft ad load torques, α_2 – shaft position (on the load side), T_1, T_2 – mechanical time constant of the motor and the load machine, T_c – stiffness time constant, α_1, α_2 – mechanical angle of the motor and the load machine, $\varphi(\Delta\theta)$ – the function of the mechanical couplings.

The backlash is described by the following function:

$$\varphi(\Delta\theta)=\begin{cases}\Delta\theta-\varepsilon & dla \quad \Delta\theta>\varepsilon\\ 0 & dla \quad \Delta\theta<\varepsilon\\ \Delta\theta+\varepsilon & dla \quad \Delta\theta<\varepsilon\end{cases} \tag{2}$$

$$\Delta\theta=\alpha_1-\alpha_1 \tag{3}$$

where: ε is the backlash width. The block diagram of the considered system is presented in Fig.1. The main parameters of the considered system are as follows: $T_1=T_2=203ms$ oraz $T_c=1,2ms$.

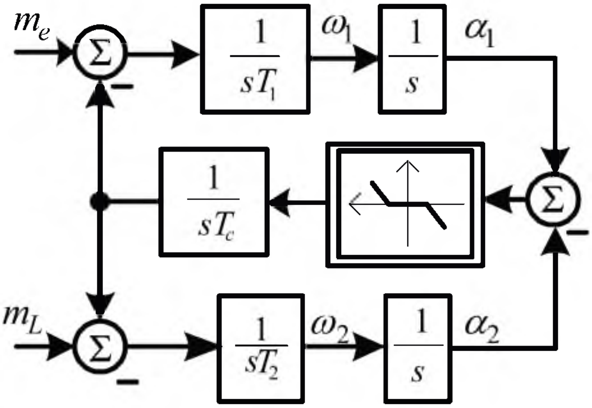

Fig. 1. The block diagram of the considered system

3. The predictive control structure

The control structure for the considered electrical drive system is shown in Fig. 2. The inner torque loop (here for simplicity represented as a single block) is composed of the power converter, electromagnetic part of the motor, current sensor and respective current or torque controller. As this control loop is designed to provide sufficiently fast torque control, it can be approximated by an equivalent first order term with a small time constant. For a well-tuned torque controller, the drive machines could be AC or DC motors without any impact on the outer speed control loop. The outer loop consists of the mechanical part of the motor, speed sensor and speed controller with reference ω_r. For the estimation of signals required in the MPC control structure, the Luenberger observer is used in this paper.

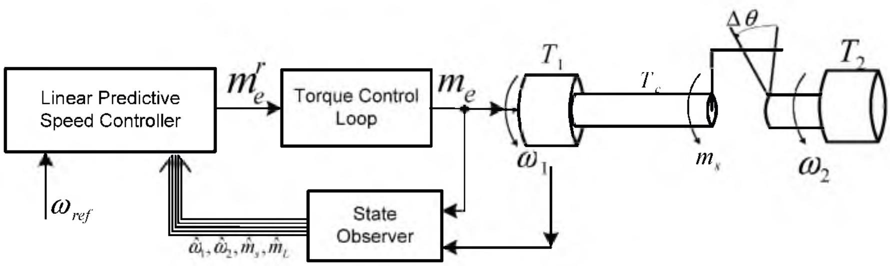

Fig. 2. The schematic diagram of the considered control structure

In its core, Model Predictive Control employs an explicit process model that is used to predict the effect of future actions of the manipulated variables on the process output. The model choice is open but typically the following linear discrete-time state-space form is considered [13]-[14]:

$$\begin{aligned}\mathbf{x}(t+1) &= \mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t)\end{aligned} \tag{4}$$

where $\mathbf{x}(t)\in\Re^{n}$, $\mathbf{u}(t)\in\Re^{m}$, $\mathbf{y}(t)\in\Re^{p}$ are the state, input and output variables, respectively, and $\mathbf{A}\in\Re^{n\times n}$, $\mathbf{B}\in\Re^{n\times m}$ $\mathbf{C}\in\Re^{p\times n}$ are known time-invariant system matrices.

Let $\mathbf{y}_k$ denote the prediction of the output variable at a future time k, given the input sequence $\mathbf{u}_k$, an initial state $\mathbf{x}_0$ and the model (2). At each time step k, an MPC algorithm attempts to optimize future plant behavior while respecting the system input/output constraints by solving the following optimization problem [13]-[14]

$$\min_{u_0^T,\ldots,u_{N_{c-1}}^T} \sum_{k=0}^{N_p}\mathbf{y}_k^T\mathbf{Q}\mathbf{y}_k+\sum_{k=0}^{N_c-1}\mathbf{u}_k^T\mathbf{R}\mathbf{u}_k \tag{5}$$

$$\begin{aligned}\text{subject to: } & \mathbf{u}_{\min}\le\mathbf{u}\le\mathbf{u}_{\max} \quad k=0,1,\cdots,N_{c-1}\\ & \mathbf{x}_{\min}\le\mathbf{x}\le\mathbf{x}_{\max} \quad k=0,1,\cdots,N_p\\ & \mathbf{x}_{k+1}=\mathbf{A}\mathbf{x}_k+\mathbf{B}\mathbf{u}_k,\ k\ge 0,\\ & \mathbf{y}_k=\mathbf{C}\mathbf{x}_k \quad k\ge 0,\\ & \mathbf{x}_0=\mathbf{x}(0),\end{aligned} \tag{6}$$

where: $\mathbf{Q}\ge$ and $\mathbf{R}>0$ denote the weighting matrices and N_p and N_c are the prediction and control horizons, respectively. Constraints u_{min} and u_{max} arise naturally from physical restrictions of control actuators whereas restrictions due to safety, quality, environmental and economic targets may be incorporated in x_{min} and x_{max}.

The implementation of the MPC controller amounts to solving problem (3) on-line for a given $\mathbf{x}_0$ in a receding horizon fashion. This means that, at time k, only the first element $\mathbf{u}_0^*$ of the optimal input sequence is applied to the plant and the remaining future control actions $u_1^*,\ldots,u^*_{Nc-1}$ are discarded. At the next time step the whole procedure is repeated for the new measured or estimated output y(k + 1) [13]-[14]. This strategy can be computationally intensive for systems with fast sampling requirements thus greatly limiting the scope of applicability to systems with relatively slow dynamics. Alternatively, rather than using the initial state x_0 to "update" the optimization problem (3) at each time k, the idea is to treat the state vector as a parameter vector and then solve problem (3) *off-line* for all realizations of x_0 within a predefined set of states using multi-parametric programming [15]-[20]. In this strategy, the parameter space is subdivided into characteristic regions where the optimizer is given as an explicit piecewise affine (PWA) function of the parameters:

$$u_0^*(x_0)=K_r x_0+g_{r,} \qquad \forall x\in P_r \tag{6}$$

where P_r are polyhedral sets defined as

$$P_r=\{x\in\Re^n \mid H_r x\le d_r\}, \qquad r=1,\ldots N_r \tag{7}$$

Here $N_r > 0$ represents the total number of polyhedral regions in the partition. The main advantage of this approach is that the optimal input u_0^* for a given initial state x_0 can be obtained by evaluating a PWA function in the control unit thus greatly simplifying the controller implementation process as numerical optimization is no longer required [15]-[20].

As an internal model of the plant used for the optimization problem, a linear model of the two-mass system is used (7). Consequently, the mechanical backlash is omitted for the prediction of the future response of the object.

$$\frac{d}{dt}\begin{bmatrix}\omega_1\\ \omega_2\\ \alpha_1\\ \alpha_2\\ m_s\\ m_L\end{bmatrix}=\begin{bmatrix}0 & 0 & 0 & 0 & \frac{-1}{T_1} & 0\\ 0 & 0 & 0 & 0 & \frac{1}{T_2} & \frac{-1}{T_2}\\ 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ \frac{1}{T_C} & \frac{-1}{T_C} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\cdot\begin{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \alpha_1\\ \alpha_2\\ m_s\\ m_L\end{bmatrix}\end{bmatrix}+\begin{bmatrix}\frac{1}{T_1}\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}\cdot[m_e] \tag{8}$$

The considered model includes two additional variables, namely the reference speed and the load torque. The main design parameters of the MPC controller are the following: prediction horizon, the selection of the inputs, and the values of the cost matrices Q and R.

3. The study

In this section, the proposed single-loop explicit MPC control strategy for the drive system with an elastic coupling will be evaluated through simulations. A primary design objective for the MPC controller is to ensure that the load speed response follows the set-point with the desired dynamics. This needs to be achieved without generating excessive shaft torque responses and without violating the input and output constraints of the drive. The first two requirements can be addressed by defining the following auxiliary output variables:

$$\begin{cases} y_1 = \left(\omega_1 - \omega^{ref}\right)\\ y_2 = \left(\omega_2 - \omega^{ref}\right)\\ y_3 = \left(m_s - m_L\right)\\ y_4 = \left(\omega_1 - \omega_2\right)\end{cases} \tag{9}$$

where: y_1, y_2 account for tracking performance, y_3 relates to load-shaft torque imbalance. The factor y_4 reduces the difference of the speed which reduces the maximal shaft torque. Due to (9) the reference speed variable and the disturbance torque need to be directly incorporated into the drive system model.

The task of the MPC controller is to bring the output variables (9) to zero by manipulating m_{er} while respecting the safety and physical limitations of the drive system:

$$-\overline{m}_{er} \leq m_{er} \leq \overline{m}_{er} \tag{10}$$

$$-\overline{m}_{s} \leq m_{s} \leq \overline{m}_{s} \tag{11}$$

The selection of the prediction and control horizons is a compromise between the drive performance and computational complexity. In practice, $N_c \leq N_p$ to avoid large computational burden for the standard MPC and large number of regions for the explicit MPC. In this paper, the state variables will be predicted using a 10ms prediction window, whereas the control input will be calculated over 1ms intervals. This translates to N_p = 10, N_c = 2. The R is set to the 0,0001. The limit of the electromagentic torque is set to ±3[p.u], and the shaft torque to ±1.5[p.u].

In the first step the influence of the fourth input and the changing of the value q_4 on the value of the cost function and the location of the MPC controller regions is presented. The investigated characteristic is depicted in Fig. 3. As can be concluded from the presented characteristic, the variation of the q_4 value influences the cost function significantly (Fig. 3a). Also the controller regions are changing according to the value q_4.

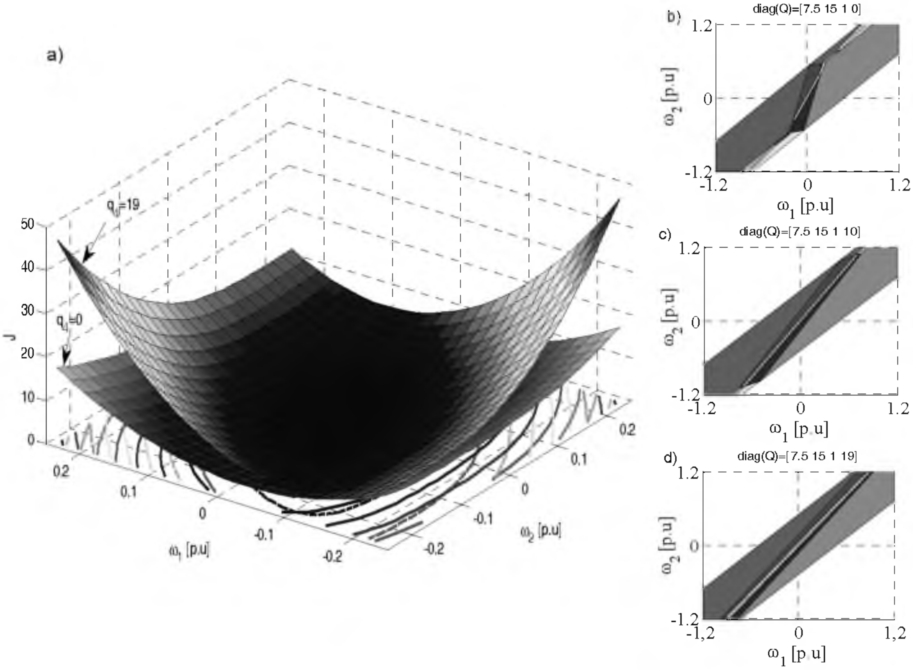

Fig. 3. The relationship between the cost function value and the value of the speed for the two value of the q_4 (a), and controller regions (b,c,d) for diag(Q)=[7.5 15 1 0] (b), diag(Q)=[7.5 15 1 10] (c), diag(Q)=[7.5 15 1 29] (d)

The transients of the system state variables for different q_4 value are presented in Fig. 4. The enlarged fragments of those transients are displayed in Fig. 5.

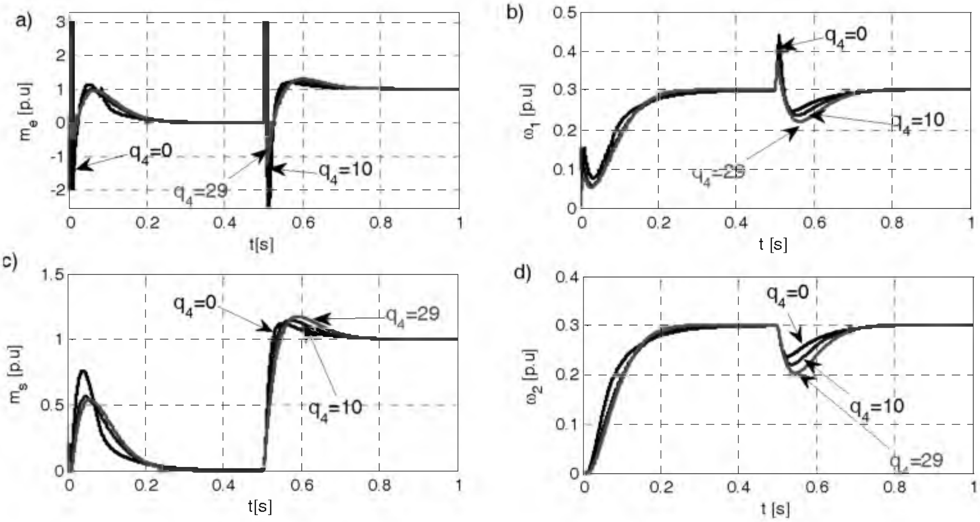

Fig. 4. Transients of the system state variable: electromagnetic torque (a), motor speed (b), shaft torque (c) and load speed (d) for different value of q_4

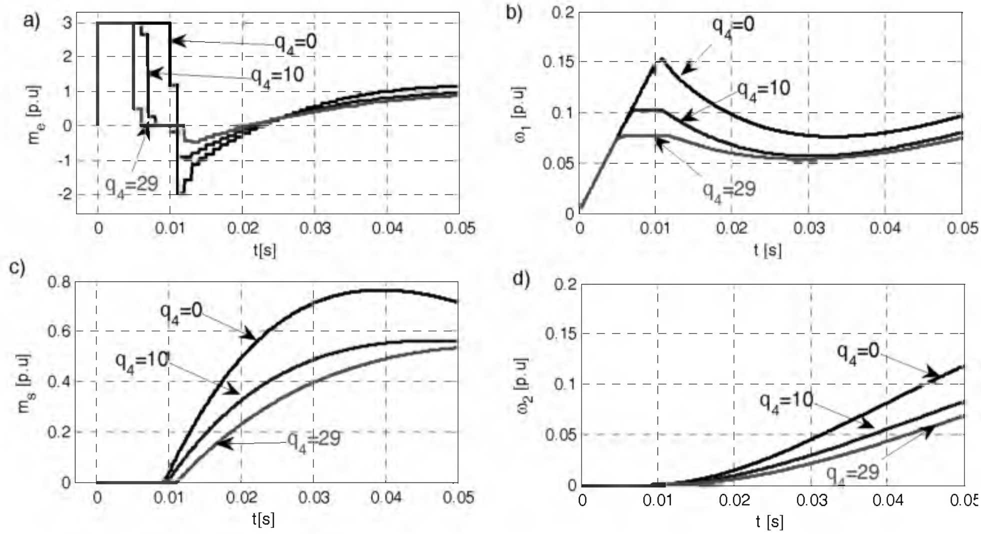

Fig. 5. Enlarged transients of the system state variable: electromagnetic torque (a), motor speed (b), shaft torque (c) and load speed (d) for different value of q_4

As can be deduced from the presented figures, the increase of the value q_4 makes the torques decline when the gap of the backlash is closing (Fig. 5a,c). At the same time, the increase of the q_4 value slowest the system responses to the changing of the load torque. The dynamics of the control structure is reduced and the system needs more time to eliminate the static error. It stems from the fact that the bigger value of the q_4 counteracts the large difference between the motor and the load speeds. Thus, that the motor cannot accelerate as dynamically as in the case when the input q_4 is neglected.

Then, the influence of the backlash width on the dynamics of the system has been investigated. At this point the values of the matrix Q are constant and equal (*diag(Q)=[7.5 15 1 29]*). The transients of the system are presented in Fig. 6. Similarly as in the previous case the fragments of the transients are shown in Fig. 7.

It is obvious from the presented results that the considered system is robust to changes of the backlash gap within the selected range without visible performance degradation. The two backlash parts hit each other softly. Even the system with a relatively big gap equal to 11,4 degree works correctly. The only noticeable difference between the tested systems exists during the start-up. SO, the bigger the gap of the backlash, the slower response time of the plant. This effect can be reduced by decreasing in the value q_4. In that case the speed of the motor will starts more dynamical, which results in the shortening the settling time of the plant. The reaction of the system to the changing of the load torque is the same in every tested structure.

Then the system was tested for the nominal value of the speed. In this case the shaft torque was limited at the maximal set value by the MPC algorithm. The obtained results for two values of q_4 are demonstrated in Fig. 8.

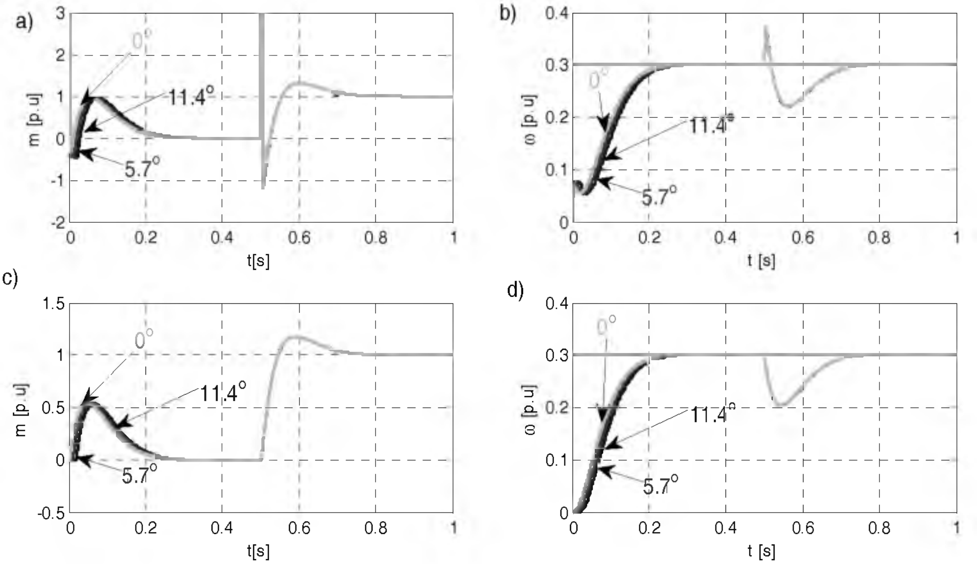

Fig. 6. Transients of the system state variables: electromagnetic torque (a), motor speed (b), shaft torque (c) and load speed (d) for different value of the backlash gap

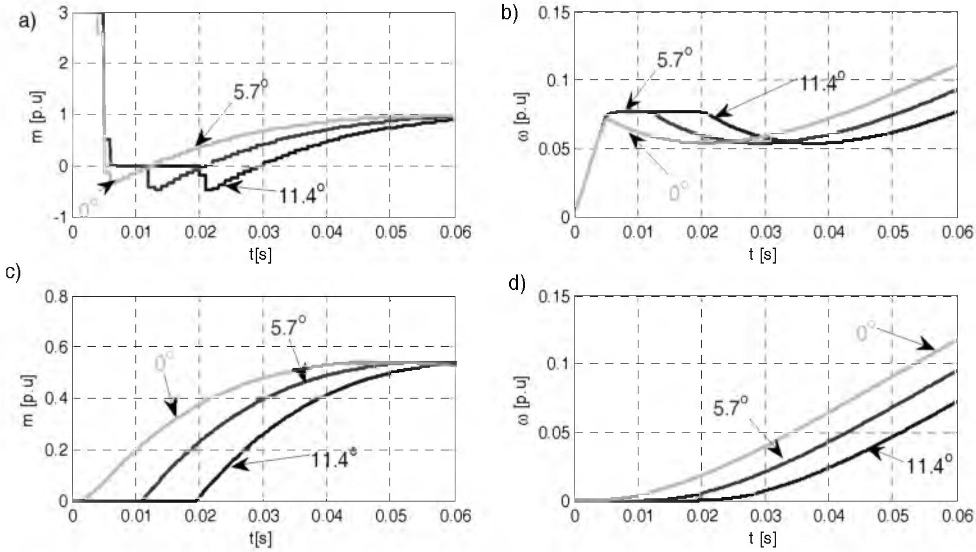

Fig. 7. Enlarged transients of the system state variable: electromagnetic torque (a), motor speed (b), shaft torque (c) and load speed (d) for different value of the backlash gap

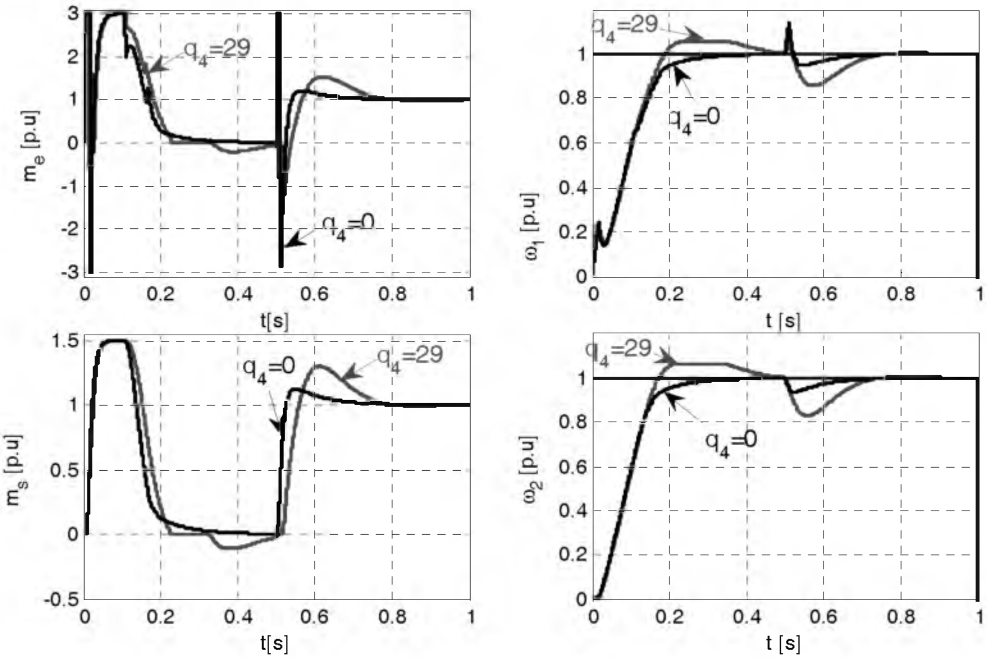

Fig. 8. The transients of the system state variable: electromagnetic torque (a), motor speed (b), shaft torque (c) and load speed (d) for different value of q_4 and nominal value of the system speed

The drive system works properly and the system speeds follow the reference value with the desired dynamics (results from the values of the matrices **Q** and **R**) taking into account the system constraints. At the times t_1=35ms and t_2=100ms the electromagnetic torque decreased rapidly in order to avoid the validation of the shaft torque constrains. In the electromagnetic torque transients some overshoots (over the limit) are visible resulting from the neglected dynamic of the motor torque control loop. The application of the load torque causes a quickly eliminated speed fall.

4. Conclusions

The paper is devoted to issues related to the application of the MPC control structure to the two-mass system with backlash are presented. In order to damp torsional vibrations an MPC controller based on the linear model is applied. From the presented study the following conclusions can be formulated:
- The application of the MPC control structure to the drive system with mechanical backlash allows to damp the torsional vibrations effectively.
- The proposed MPC strategy allows to close the mechanical gap of the backlash softly.
- The bigger value q_4, the softer the mechanical gap closes, however, at the same time the dynamics of the drive are affected.
- Despite of a linear model of the two-mass system is used in the optimization algorithm, the set limits of the system state variable are not validated.
- The drive system works correctly for small as well as for big values of the reference speed.
- For the system with a relatively big value of q_4 the variation of the backlash width has an insignificant effect on the performance of the system.

The future work will be devoted to experimental validation of the proposed MPC control structure. Also a system with a nonlinear model of the mechanical part of the drive in the MPC algorithm will be considered.

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