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RECENT ADVANCES IN STUDIES OF THE STRENGTH OF ROCKS UNDER TRUE TRIAXIAL COMPRESSION CONDITIONS

POSTĘPY W BADANIACH NAD WYTRZYMAŁOŚCIĄ SKAŁ W WARUNKACH PRAWDZIWEGO TRÓJOSIOWEGO ŚCISKANIA

The results of true triaxial compression tests carried out by K. Mogi at the University of Tokyo, M. Takahashi at the Geological Survey of Japan and B. Haimson at the University of Wisconsin are summarized and the effect of the intermediate principal stress on the ultimate strength of rocks is discussed in the first part of the paper. Then, the Huber-Mises-Hencky failure theory, which was generalized by Nádai and further modified by Mogi to explain the stress-dependency of both the brittle fracturing and the ductile flow of rocks, is revisited. In the main part of the paper, the results of recent experimental studies carried out on samples of a Coal-Measure sandstone from the strata of the Upper Silesian Coal Basin under true triaxial compression conditions are presented. The studies focused on the effect of, independently, confining pressure, intermediate principal stress and minimum principal stress on the ultimate strength of this rock. The paper closes with a presentation and discussion of a general failure criterion that is capable of predicting the ultimate strength of rocks under both axisymmetric and true triaxial (asymmetric) compressive stress conditions.

Keywords: failure criterion, failure hypothesis, laboratory testing, rock mechanics, triaxial strength, true triaxial compression

W pierwszej części artykułu podsumowano wyniki badań eksperymentalnych na prawdziwe trójosiowe ściskanie przeprowadzonych przez K. Mogiego z Uniwersytetu Tokijskiego, M. Takahashiego z Japońskiego Instytutu Geologicznego i B. Haimsona z Uniwersytetu Wisconsin i omówiono wpływ pośredniego naprężenia głównego na wytrzymałość graniczną skał. Następnie, przypomniano uogólnioną przez Nádai'a hipotezę wytężeniową Hubera-Misesa-Hencky'ego, którą przed laty zmodyfikował Mogi żeby wyjaśnić zależność granicznego stycznego naprężenia oktaedrycznego od tzw. efektywnego naprężenia średniego dla, oddzielnie, skał zachowujących się krucho i skał doznających przy ściskaniu ciągliwego płynięcia.

W głównej części artykułu przedstawiono wyniki najnowszych badań eksperymentalnych przeprowadzonych na próbkach pewnego piaskowca z górotworu Górnośląskiego Zagłębia Węglowego (był

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to drobnoziarnisty arenit kwarcowy o stosunkowo wysokiej, równej około 146 MPa, wytrzymałości granicznej na jednoosiowe ściskanie). Celem tych badań było wykrycie wpływu, oddzielnie, ciśnienia okólnego ($p = \sigma_2 = \sigma_3$), pośredniego naprężenia głównego (σ_2) i najmniejszego naprężenia głównego (σ_3) na wytrzymałość graniczną tej skały.

W próbach na konwencjonalne trójosiowe ściskanie wytrzymałość badanego piaskowca silnie rosła ze wzrostem ciśnienia okólnego. Przy ciśnieniu równym 62,5 MPa, graniczne największe naprężenie główne (${}^F\sigma_1$) było, średnio, 2,8 razy większe od tego zanotowanego w próbach na jednoosiowe ściskanie. Wyniki prób na prawdziwie trójosiowe ściskanie pokazały, że wpływ pośredniego naprężenia głównego (σ_2) na wytrzymałość graniczną badanego piaskowca – chociaż ewidentny i znaczący – jest jednak wyraźnie słabszy niż wpływ ciśnienia okólnego (rys. 19). W przypadku próbek badanych przy najmniejszym naprężeniu głównym (σ_3) równym 25 MPa, największe naprężenie główne na granicy wytrzymałości odpowiadające naprężeniu pośredniemu σ_2 = 62,5 MPa było tylko o 19% większe niż to odpowiadające naprężeniu σ_2 = 25 MPa. Zaobserwowano przy tym, że tempo wzrostu wytrzymałości ze wzrostem pośredniego naprężenia głównego (w przedziale od 25 MPa do 62,5 MPa) nieznacznie maleje.

Badania nad wpływem najmniejszego naprężenia głównego na wytrzymałość graniczną pokazały, że naprężenie to odgrywa rolę podobną do roli naprężenia pośredniego (σ_2). Jednakże, tempo wzrostu wytrzymałości badanej skały ze wzrostem σ_3 było nieco większe od tego zaobserwowanego gdy zwiększano σ_2 przy stałym σ_3 (rys. 20). Przy pośrednim naprężeniu głównym równym 62,5 MPa, graniczne największe naprężenie główne (${}^F\sigma_1$) odpowiadające σ_3 = 62,5 MPa było wyższe o blisko 28% od tego odpowiadającego naprężeniu σ_3 równemu 25 MPa. Silniejszy wpływ najmniejszego naprężenia głównego na wytrzymałość graniczną badanej skały można wytłumaczyć w następujący sposób: Największy wzrost wytrzymałości granicznej skały na ściskanie w warunkach trójosiowego ściskania towarzyszy zwiększaniu ciśnienia okólnego, a więc właściwy jest osiowo-symetrycznemu stanowi naprężenia. Gdy naprężenie najmniejsze jest zwiększane przy stałym σ_2, trójosiowy stan naprężenia staje się coraz bardziej równomierny (coraz bliższy stanowi osiowo-symetrycznemu), co wpływa znacząco na wzrost wytrzymałości materiału. Z drugiej strony, gdy pośrednie naprężenie główne jest zwiększane względem σ_3, wytwarzany jest rosnąco asymetryczny (nierównomierny) stan naprężenia i wysokie naprężenie różnicowe (σ_2-σ_3) może wywoływać nawet spadek wytrzymałości materiału skalnego.

Na zakończenie artykułu przedstawiono i omówiono ogólny warunek wytrzymałościowy Mogiego, który dobrze nadaje się do oceny wytrzymałości granicznej skał w warunkach zarówno osiowo-symetrycznego, jak i asymetrycznego (prawdziwie trójosiowego) stanu naprężeń ściskających. Fizykalna interpretacja warunku zaproponowanego przez Mogiego dla skał kruchych jest następująca: W przeciwieństwie do hipotezy Hubera-Misesa-Hencky'ego, która obowiązuje dla materiałów ciągliwych, w przypadku skał wartość graniczna stycznego naprężenia oktaedrycznego nie jest stała lecz monotonicznie rośnie ze wzrostem tzw. efektywnego naprężenia średniego (σ_m). Ponieważ kruche pękanie ścięciowe zachodzi wzdłuż płaszczyzn zorientowanych równolegle do kierunku pośredniego naprężenia głównego, efektywne naprężenie średnie wydaje się być niezależne od naprężenia pośredniego, i w związku z tym $\sigma_m = \sigma_{m,2}=(\sigma_1 + \sigma_3)/2$.

Należy podkreślić, że potęgowy warunek wytrzymałościowy Mogiego (9) aproksymuje dane empiryczne z prób na konwencjonalne trójosiowe ściskanie i prób na prawdziwe trójosiowe ściskanie jednakowo dobrze (dotyczy to także liniowej wersji warunku Mogiego (11)). W związku z tym, żeby wyznaczyć wartości parametrów (stałych) występujących w tym warunku nie jest bezwzględnie konieczne wykonywanie – nawet gdy dysponuje się odpowiednią aparaturą – kłopotliwych, czasochłonnych i kosztownych testów na prawdziwe trójosiowe ściskanie. Wartości tych parametrów mogą być wyznaczone na podstawie wyników uzyskanych ze znacznie prostszych testów na konwencjonalne trójosiowe ściskanie. Potem, gdy wprowadzi się je do warunku Mogiego, mogą służyć do oceny wytrzymałości granicznej skał w warunkach zarówno osiowo-symetrycznego ($\sigma_1 > \sigma_2 = \sigma_3$), jak i asymetrycznego stanu naprężenia ($\sigma_1 > \sigma_2 > \sigma_3$).

Słowa kluczowe: badania laboratoryjne, hipoteza wytężeniowa, mechanika skał, prawdziwe trójosiowe ściskanie, warunek wytrzymałościowy, wytrzymałość trójosiowa

1. Introduction

The subject of the strength of rocks under triaxial compression conditions is addressed in this paper. Experimental data from conventional triaxial and true triaxial compression tests are used to reveal the features of the effect of, separately, confining pressure, intermediate principal stress (σ_2) and minimum principal stress (σ_3) on the ultimate strength of rocks. In addition, a failure hypothesis and a strength criterion which best allow these features to be taken into account are discussed.

Knowledge of the mechanical behavior of rocks under triaxial compression conditions, and of strength criteria in particular, is of the utmost importance in all engineering activities in rock masses. An understanding of the stress conditions that lead to failure is also important in geophysical research such as the study of fault and earthquake mechanics and the determination of in-situ stresses using, for example, hydraulic fracturing or the borehole breakouts method.

At the beginning of this paper, the results of the true triaxial compression tests carried out by Professor Mogi at the Earthquake Research Institute of the University of Tokyo, Dr Takahashi at the Geological Survey of Japan and Professor Haimson and his doctoral students at the University of Wisconsin will be reviewed and the effect of the intermediate principal stress on the ultimate strength of rocks will be discussed.

Then, the features of two simple, and widely used, strength criteria, namely: the Hoek & Brown criterion and the Mohr-Coulomb criterion will be discussed, paying special attention to the ability of these criteria to express the limiting strength of rocks under various stress conditions. This will create the basis for the introduction of Mogi's failure hypothesis, which links the well-established Huber-Mises-Hencky failure theory with the experimental test results to explain the stress-dependency of both the brittle fracturing and ductile flow of rocks.

In the final part of the paper, the results of recent experimental studies on the mechanical properties of a Coal-Measure sandstone from the strata of the Upper Silesian Coal Basin under true triaxial compression conditions will be presented. The focus will be on the effect of confining pressure, intermediate principal stress and minimum principal stress on the ultimate strength of this rock.

The paper will close with a presentation and discussion of a general strength criterion that is capable of describing the ultimate strength of rocks under both axisymmetric and true triaxial (asymmetric) applied compressive stress conditions.

Throughout this paper, a differentiation between the axisymmetric and true triaxial states of compressive stresses will be made (see also Kwaśniewski, 2007). An axisymmetric state of stress is normally generated in solid cylindrical rock samples that are subjected to confining pressure and an axial load in a triaxial cell (Fig. 1a). Depending on the stress path in the stress space, a stress state may be generated where the intermediate and minimum principal stresses are equal to each other ($\sigma_1 > \sigma_2 = \sigma_3$); these are the so-called conventional triaxial compression (CTC) tests and reduced triaxial compression (RTC) tests, respectively. During the conventional triaxial extension (CTE) tests and the reduced triaxial extension (RTE) tests, a stress state is generated where the intermediate principal stress is equal to the maximum principal stress ($\sigma_1 = \sigma_2 > \sigma_3$) (Fig. 2).

A true triaxial or asymmetric triaxial stress state ($\sigma_1 \neq \sigma_2 \neq \sigma_3$) is normally generated in rectangular prismatic (cuboidal) rock samples loaded independently in three orthogonal directions (Fig. 1b). In this paper, the focus will mainly be on such a true triaxial compressive stress state.

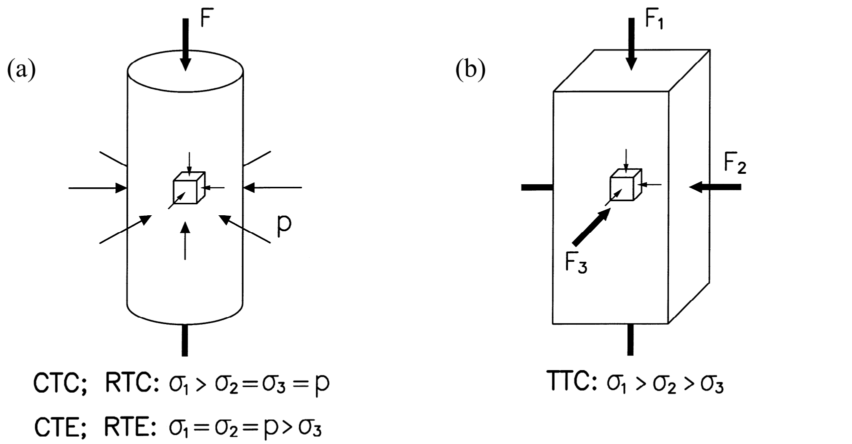


Fig. 1. Loading schemes to generate an axisymmetric (a) and true triaxial (b) compressive state of stress in rock samples

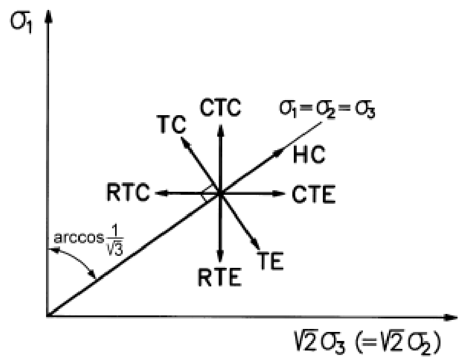
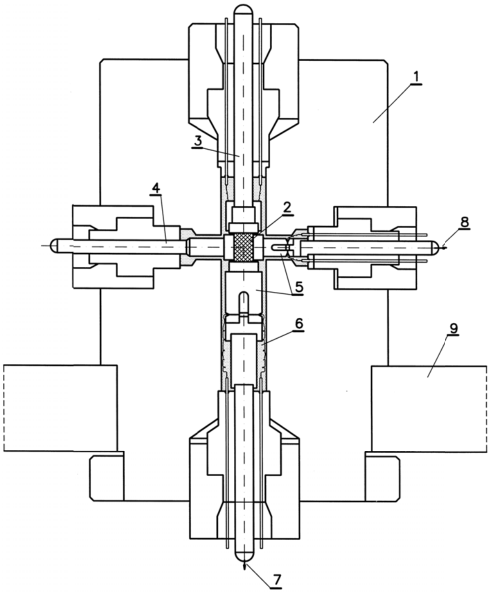


Fig. 2. Stress paths in the triaxial plane corresponding to different triaxial tests characterized by an axisymmetric state of stress (after Chen & Saleeb, 1982)

2. True triaxial compression test results; effect of the intermediate principal stress (σ_2) on the ultimate strength of rocks

Professor Kiyoo Mogi from the Earthquake Research Institute of the University of Tokyo, Japan, was the first to study the mechanical behavior and strength of rocks under true triaxial compression conditions in detail. In the late 1960s, he developed an ingenious, novel true triaxial compression apparatus. In the triaxial cell of that apparatus, three principal stresses could be generated independently in rock samples. The maximum and intermediate stresses were each induced by a pair of rigid pistons that were activated using manual hydraulic pumps, while the minimum principal stress was induced by oil pressure (Fig. 3). Using samples with the dimensions 15 × 15 × 30 mm, Mogi studied the effect of the intermediate principal stress on the deformational and



- sample dimensions: 15 × 15 × 30 mm
- max. p = 800 MPa
- manually driven pumps
- rocks tested: limestone, dolomite, marble, granite, monzonite, andesite, trachyte, crystalline schist

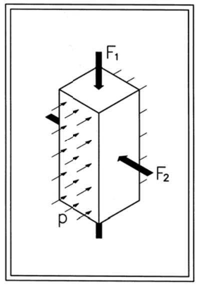


Fig. 3. Triaxial cell of Mogi's true triaxial compression apparatus; 1 – high pressure chamber, 2 – rock sample, 3 – axial piston, 4 – lateral piston, 5 – load cells, 6 – oil, 7 – to hydraulic jack (1), 8 – to hydraulic jack (2), 9 – fixed platen (Mogi, 1971a,b, 2006, 2012)

strength properties of carbonate and silicate rocks (Mogi, 1969-1974, 1977, 1979, 1981, 2006, 2012; Kwaśniewski & Mogi, 1990).

Dr Hitoshi Koide and Dr Manabu Takahashi from the Geological Survey of Japan, having built a slightly modified version of Mogi's apparatus (Fig. 4), tested large (50 × 50 × 100 mm) and small (35 × 35 × 70 mm) samples of three different sandstones, a shale, a marble, a granite and a limestone. In the triaxial cell of Koide and Takahashi's apparatus, the vertical load is applied to the sample by a servo-controlled hydraulic jack. Two servo-controlled hydraulic jacks apply the load to the sample in direction 2, that is the direction of the intermediate principal stress. The minimum principal stress is generated in the rock sample by the confining fluid (Takahashi 1984; Takahashi & Koide 1989).

In the 1990s, Professor Bezalel Haimson and Dr Chandong Chang from the University of Wisconsin in Madison, USA, built another, highly sophisticated version of a Mogi-type apparatus. Haimson and Chang's true triaxial loading system consists of two main parts: a biaxial loading apparatus and a polyaxial pressure vessel (Fig. 5). The biaxial apparatus facilitates the application of two independent perpendicular loads: F_1 (axial) and F_2 (lateral). These two loads are transmitted to the rock sample via two orthogonal pairs of pistons located in the pressure vessel seated inside the biaxial apparatus. Loading on the third pair of sample faces (direction 3) is applied directly by hydraulic fluid inside the pressure vessel. Using rectangular prismatic samples 19 × 19 mm square and 38 mm high, Haimson and his fellow researchers studied the behavior of Westerly granite (Haimson & Chang, 2000, 2005; Chang, 2001), KTB amphibolite (Chang & Haimson, 2000; Chang, 2001; Haimson & Chang, 2005), Long Valley hornfels and

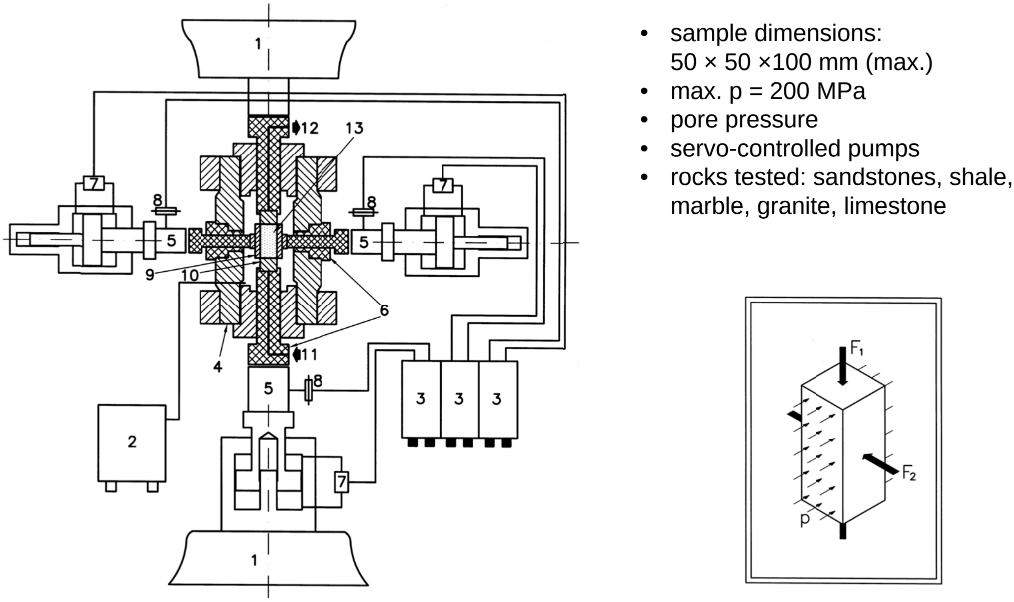
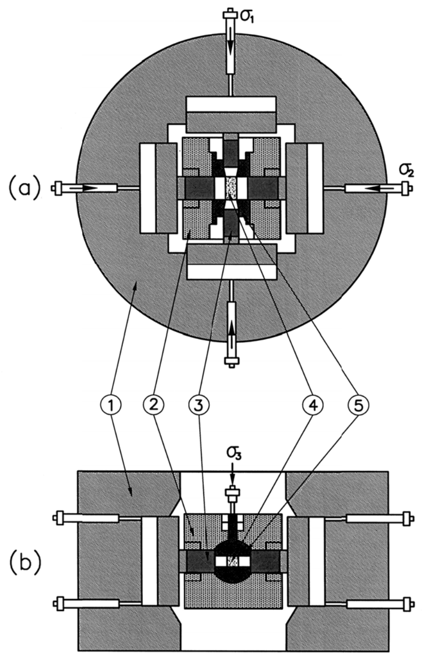


Fig. 4. Koide and Takahashi's true triaxial compression apparatus; 1 – fixed upper and lower platens, 2 – hydraulic power supply, 3 – control unit, 4 – pressure vessel, 5 – load cells for axial (σ_1) and lateral (σ_2) loads, 6 – pistons for axial (σ_1) and lateral (σ_2) loads, 7 – servovalve, 8 – displacement transducers for axial (1) and lateral (2) directions, 9 – lateral (2) steel end piece, 10 – axial (1) steel end piece, 11 – pore fluid inlet, 12 – pore fluid outlet, 13 – rock sample (after Takahashi & Koide, 1989)

metapelite (Chang & Haimson, 2005), Taiwan Chelungpu Fault siltstone (Oku et al., 2007) and San Andreas Fault granodiorite (Lee & Haimson, 2011) under high σ_2 and high σ_3 conditions. For a detailed review of true triaxial test results obtained at the University of Wisconsin see Haimson (2006, 2011, 2012).

Examples of test results obtained by Mogi, Takahashi and Chang in their studies of the effect of the intermediate principal stress on rock strength are shown in Figure 6. The data points represent the relationship between the major principal stress at failure (${}^F\sigma_1$) and the intermediate principal stress (σ_2) for different values of the minimum principal stress (σ_3). In all of the cases, the trend in the maximum principal stress at failure is to increase with the magnitude of σ_2, although the rate of this increase is somewhat lower than the rate at which the maximum principal stress at failure increases with an increase in confining pressure ($p = \sigma_2 = \sigma_3$).

Let's take a closer look at the triaxial test data for Dunham dolomite from north-western Vermont tested by Mogi (Fig. 6a). Note that as the magnitude of the intermediate principal stress is increased beyond that of the minimum stress, the strength of the dolomite is invariably higher than that under axisymmetric stress conditions ($\sigma_2 = \sigma_3 = p$). At each of the seven levels of applied minimum principal stress, rock strength first increases significantly with increasing σ_2, and reaches a peak at levels of σ_2 at least two or three times those of the applied σ_3. Beyond that the strength declines somewhat but is always significantly greater than under axisymmetric conditions for the given magnitude of σ_3.



- sample dimensions: 19 × 19 × 38 mm
- max. p = 400 MPa
- servo-controlled pumps
- rocks tested: granite, amphibolite, hornfels, metapelite, siltstone, granodiorite

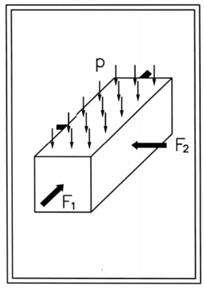


Fig. 5. Haimson and Chang's true triaxial compression apparatus: a – cross section, b – profile, 1 – biaxial loading apparatus, 2 – triaxial pressure cell, 3 – loading pistons, 4 – rock sample, 5 – confining fluid (after Chang & Haimson 2000, 2005)

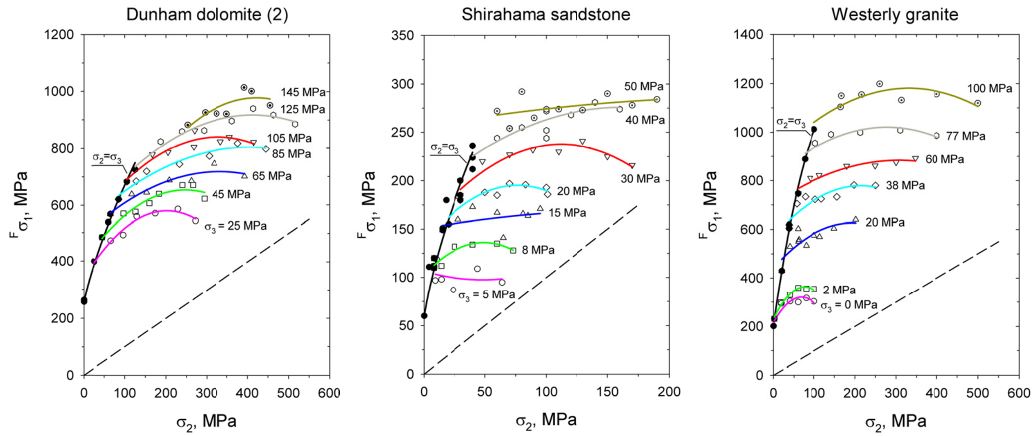


Fig. 6. Examples of the results of true triaxial compression tests conducted on samples of Dunham dolomite (2) (Mogi, 1971a, 2006), Shirahama sandstone (Takahashi, 1984) and Westerly granite (Chang, 2001)

The question now arises: Which of the known strength criteria can represent all of the data shown in this plot, that is which of the criteria takes into account the effect of σ_2 on rock strength?

3. Features of the selected strength criteria

At present, one of the most popular and widely used strength or failure criteria is the Hoek & Brown criterion:

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_C\sigma_3 + \sigma_C^2}\,, \quad \text{where} \quad \sigma_3 = \sigma_2 = p \tag{1}$$

It was originally developed in the late 1970s and was first published in 1980. It is purely empirical, having been found in the process of trial and error when Hoek and Brown were trying to fit the available conventional triaxial compression test results with a possibly simple function that represents the nonlinear dependency of rock strength on confining pressure well (Fig. 7). The criterion enjoys its wide popularity due to the fact that although it is simple, it accounts for the nonlinear dependency of rock strength on pressure and fits the empirical data from the conventional triaxial compression tests (where $\sigma_3 = \sigma_2 = p$) fairly well. Only two material parameters occur in the criterion: the uniaxial compressive strength of intact rock material (σ_C) and

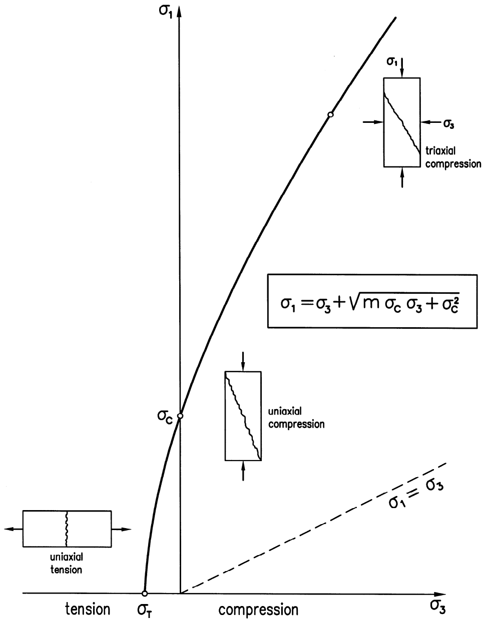


Fig. 7. Basic features of Hoek & Brown's failure criterion

a constant m that depends on rock type and characterizes the rate of increase of the maximum principal stress at failure with an increase in confining pressure. An important limitation of this criterion is that it is valid only for the axisymmetric stress state; it cannot be employed for true triaxial or asymmetric stress states, where $\sigma_1 \neq \sigma_2 \neq \sigma_3$. This shortcoming of the Hoek & Brown criterion is well illustrated by the ${}^{F}\sigma_1 = f(\sigma_2)$ plots shown in Figure 6.

The simplest, and still widely used, failure criterion is that of C.A. Coulomb (1773-1776) (see Timoshenko 1953; Jaeger & Cook 1969). Based on his extensive experimental investigations into friction, Coulomb assumed that failure in a rock or soil takes place along a plane due to the shear stress τ acting in that plane, where motion is resisted by a frictional force whose magnitude is a product of the normal stress σ_n acting on this plane and some constant factor μ. In the case of the initially intact failure plane, motion is also resisted by an internal cohesive force of the material. According to these considerations, failure occurs along a plane if the following condition is satisfied:

$$\tau = c + \mu\sigma_n \quad \text{or} \quad \tau = c + \tan\varphi\sigma_n \tag{2}$$

The parameter c is known as the cohesion, μ is the coefficient of internal friction and φ is the angle of internal friction (φ = arctanμ).

When expressed in terms of the applied principal stresses, the Coulomb criterion assumes the following form:

$$\sigma_1 = \sigma_C + B\sigma_3 \tag{3a}$$

where

$$B = \tan^2\left(45° + \frac{\varphi}{2}\right) \equiv \frac{1+\sin\varphi}{1-\sin\varphi} \tag{3b}$$

The criterion is valid for $\sigma_1 > {}^1/_2\,\sigma_C$ (where $\sigma_C = 2c\sqrt{B}$), which corresponds to the condition that the normal stress acting on the failure plane is compressive, i.e. positive ($\sigma_n > 0$) (Fig. 8).

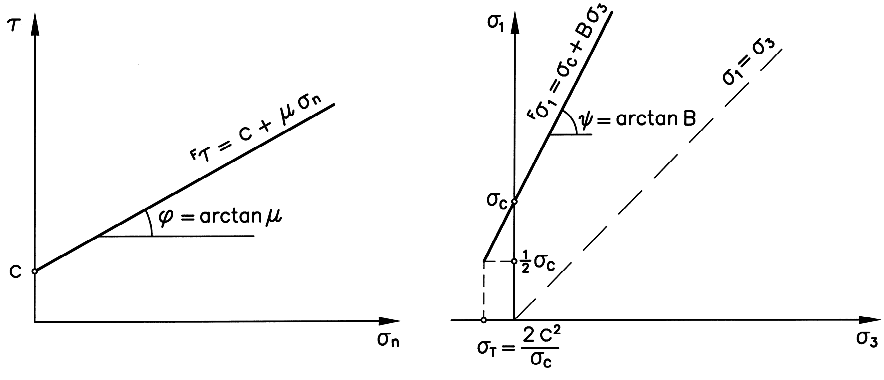


Fig. 8. Graphical representation of Coulomb's strength criterion expressed in terms of stresses acting on the plane of failure (a) and in terms of principal (applied) stresses (b)

The linear Coulomb criterion is linked by many to a limiting shear strength theory that was formulated in 1882-1900 by O. Mohr (Fig. 9). According to this theory, under complex stress conditions material undergoes failure through shearing along a plane where shear stress reaches its limiting value which is a certain increasing function of the normal stress acting on this plane. Mohr did not proposed any specific form of the function ${}^{F}\tau = f(\sigma_n)$. In general, it can be nonlinear as, for example, the following one, which was proposed by Fairhurst (1964):

$$\tau_2 = (\kappa - 1)^2 \sigma_T (\sigma_T - \sigma_n) \tag{4}$$

where $\kappa = \sqrt{\sigma_C / \sigma_T + 1}$, σ_C is the uniaxial compressive strength, and σ_T is the uniaxial tensile strength.

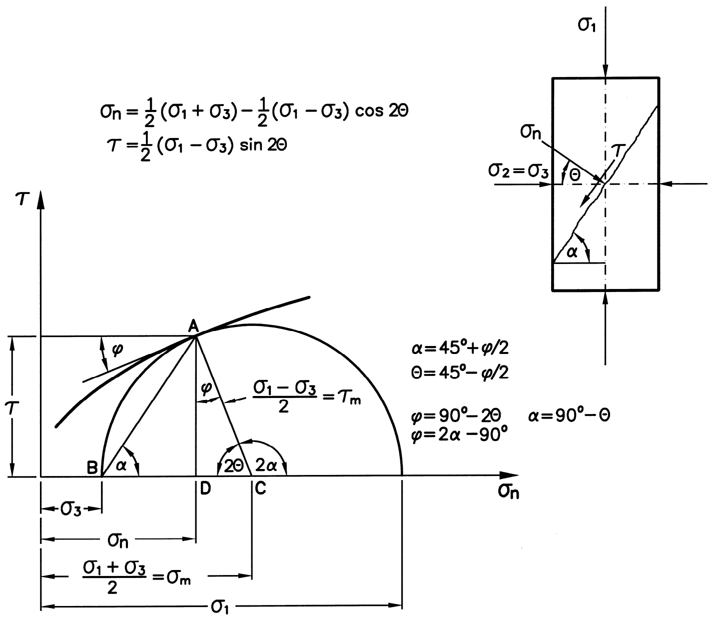


Fig. 9. Graphical representation of Mohr's theory

Of course, the Coulomb criterion is equivalent to the assumption of a linear envelope of the Mohr stress circles.

The major limitation of the Mohr theory is that it does not take into account the effect of the intermediate principal stress on rock strength. Therefore, the Mohr envelopes established for a given rock material are not unique in the case of true triaxial or asymmetric stress state. As was first shown by Mogi (1972b), different strength envelopes correspond to a given value of normal stress (σ_n) that depends on the orientation of the plane of failure (angle θ) and is related to the applied principal stresses by the following equation:

$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta, \tag{5}$$

that is a family of strength functions $\tau = f(\sigma_n, \sigma_2)$ exists instead of a single strength function $\tau = f(\sigma_n)$ (Fig. 10).

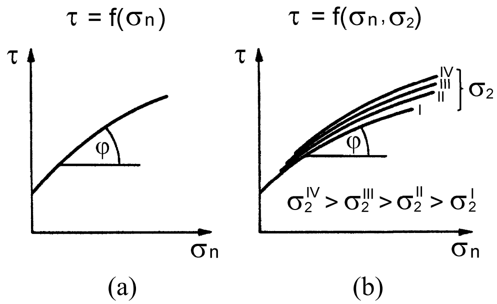


Fig. 10. The limiting shear stress as a function of stress normal to the failure plane: a – according to the Mohr theory, b – according to the results of true triaxial compression tests (Mogi, 1972b)

An intriguing question for some time was how to generalize the true triaxial test results like those presented in Figure 11 for Inada granite. Is there any all-inclusive strength criterion that can represent all the data shown in such a plot? Criteria which relate the maximum principal stress at failure to confining pressure, as the Hoek & Brown criterion does, cannot be used as they are suitable only for the axisymmetric stress conditions; in Figure 11 the ${}^{F}\sigma_1 = f(p)$ relationship is represented by the steep black line that fits the data points obtained from the conventional triaxial compression tests. The Mohr theory-related strength criteria also are not a solution as this theory ignores the effect of the intermediate principal stress on rock strength and different Mohr envelopes correspond to different values of σ_2.

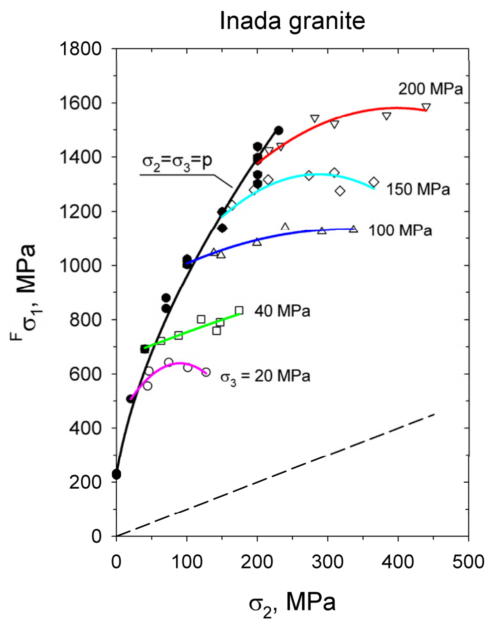


Fig. 11. Results of true triaxial compression tests for Inada granite (data taken from Mogi, 2006)

The inclusion of σ_2 as a quantity that affects rock strength seemed to be possible by using the concept of Nádai (1950), who generalized the original Huber-Mises-Hencky yield criterion for ductile metals and suggested that the limiting strength of brittle materials is reached when the octahedral shear stress τ_{oct} and octahedral normal stress σ_{oct} (the mean normal stress) are related by a monotonically increasing function F, i.e. $\tau_{oct} = F(\sigma_{oct})$. It was Mogi who in the early 1970s confirmed to what extent the H-M-H failure criterion generalized by Nádai is suitable to describe the experimentally observed failure of rocks under true triaxial compression conditions.

It has been well established that under true triaxial stress conditions brittle fracture, either extensional or shear fracture, occurs along planes striking in the σ_2 direction, that is the fracture planes are oriented parallel to the direction of σ_2 (Fig. 12). This is why a hypothesis that the mean normal stress that opposes the creation of the fracture plane is $\sigma_{m,2}$, as defined by the following equation, was put forward by Mogi (1971a, b):

$$\sigma_{m,2} = \frac{1}{2}(\sigma_1 + \sigma_3) \tag{6}$$

rather than the octahedral normal stress σ_{oct} or $\sigma_{m,3}$, which is given by

$$\sigma_{oct} \equiv \sigma_{m,3} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \tag{7}$$

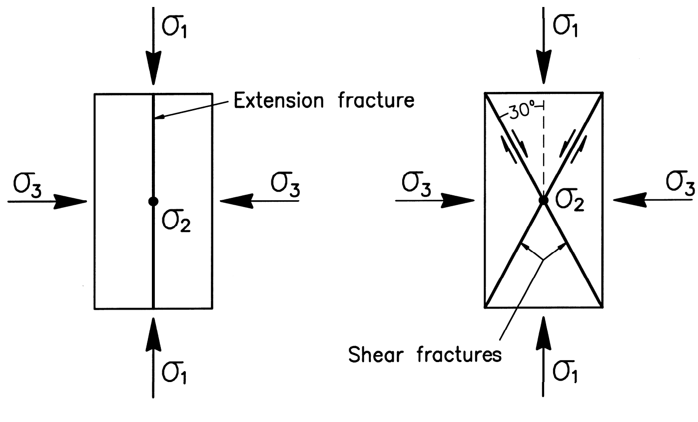


Fig. 12. The brittle fracture pattern in rocks

In other words, Mogi suggested that the so-called effective normal stress is independent of σ_2, and proposed a new failure criterion expressed as

$$\tau_{oct} = f(\sigma_{m,2}) \tag{8}$$

where f is a certain monotonically increasing function.

Mogi verified his new strength criterion for several rock types and found that it is a power function given by

$$\tau_{oct} = A\sigma_{m,2}^{n}, \quad \text{where} \quad n \leq 1 \tag{9}$$

that fits the experimental data from true triaxial compression tests extremely well.

As an example, results of Mogi's (2006) studies of the triaxial strength of Inada granite (cf. Fig. 11) are presented in Figure 13. Samples of this rock were tested under a minimum principal stress and an intermediate principal stress reaching 200 MPa and 450 MPa, respectively. Note how perfectly the power strength function (9) fits the empirical data yielding a value of the constant *A* equal to 2.666 and a value of the exponent *n* equal to 0.8.

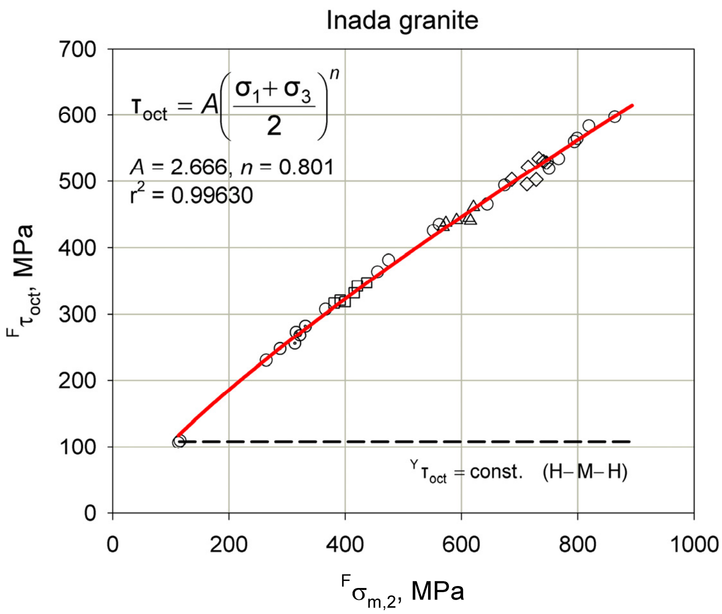


Fig. 13. Results of true triaxial compression tests on Inada granite fitted using the Mogi power-law failure criterion (9)

Note also the horizontal (dashed) line on the plot, which corresponds to the original H-M-H failure criterion.

According to Mogi's triaxial test results, the failure mode of carbonate rocks, like limestone, marble and dolomite, was completely different from that of brittle silicate rocks. Samples of carbonate rocks did not fracture under triaxial compression conditions. On the contrary, they behaved in a ductile manner undergoing yielding and plastic flow. It is very interesting to note that the experimental test data corresponding to the yield point do not follow unique, single curves when plotted on the τ_{oct}–$\sigma_{m,2}$ plane (Fig. 14). This means that Mogi's failure criterion, although quite applicable for brittle silicate rocks, is not suitable for ductile carbonate rocks.

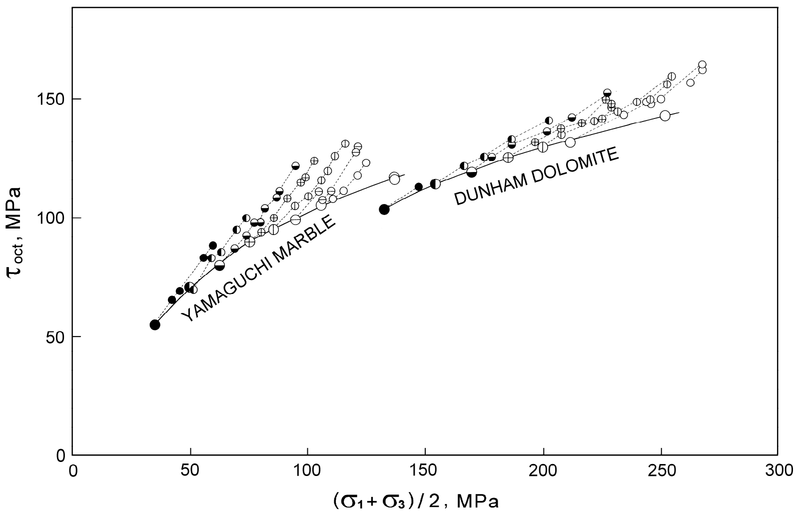


Fig. 14. Results of true triaxial compression tests on two carbonate rocks plotted on the $\tau_{oct}-\sigma_{m,2}$ plane (after Mogi, 2006)

The reason is that, as explained by Mogi, the yielding of rock material is not localized along a single plane or shear band but occurs over the entire volume of a rock when subjected to triaxial loading (Fig. 15). Consequently, the Nádai failure function should be employed in such a case as the octahedral normal stress σ_{oct} or the mean normal stress $\sigma_{m,3}$ includes not only the maximum and the minimum principal stresses but also the intermediate principal stress.

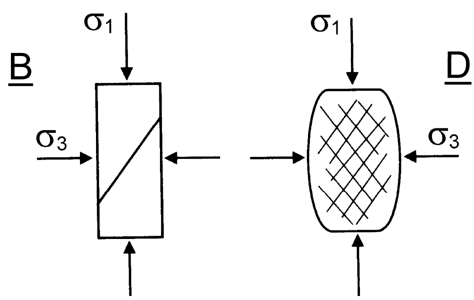


Fig. 15. Brittle (B) and ductile (D) faulting in rocks

Note that the Nádai yield function fits the experimental data for Yamaguchi marble much better than the Mogi strength failure function did (Fig. 16).

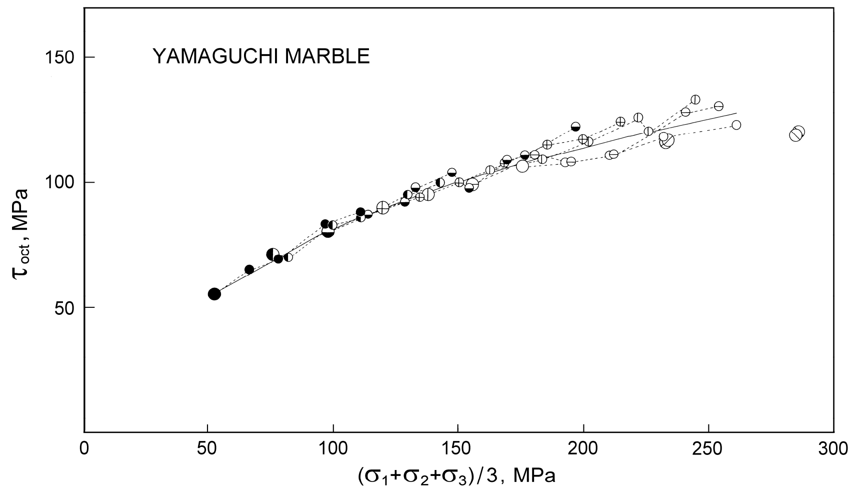


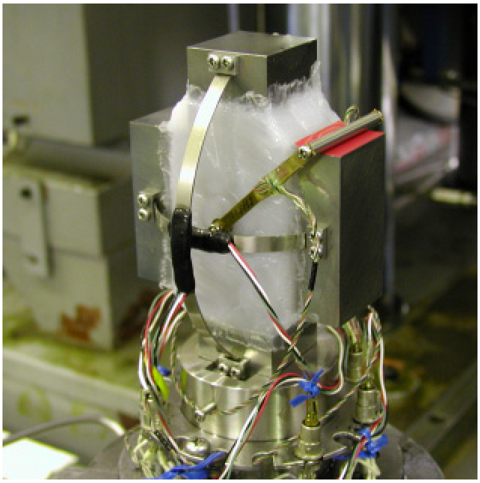
Fig. 16. Results of true triaxial compression tests on Yamaguchi marble plotted on the $\tau_{oct}-\sigma_{m,3}$ plane (after Mogi, 2006)

4. True triaxial compression test results; effect of the intermediate (σ₂) and the minimum (σ₃) principal stresses on the ultimate strength of rocks

To date, the researchers engaged in the testing of rock samples under true triaxial compression conditions have focused only on the effect of confining pressure and on the effect of the intermediate principal stress on the mechanical behavior of rocks. No attempts have been made to study the effect of the minimum principal stress. In order to fill this gap, I devised a testing program that included loading and unloading tests to and from different levels of differential stress in the pre- and post-failure stages under conditions of: (i) uniaxial compression, (ii) conventional triaxial compression and (iii) true triaxial compression conditions. Four different varieties of fairly homogeneous and isotropic Coal-Measure sandstones with different grain sizes from the strata of the Upper Silesian Coal Basin were selected for the studies. To reveal the effect of confining pressure, compression tests were carried out at confining pressures reaching 62.5 MPa. The effect of the intermediate principal stress was studied under conditions where σ_3 was equal to 25 MPa, and σ_2 was 1.5, 2 and 2.5 times higher than σ_3. To reveal the effect of the minimum principal stress, the rock samples were tested under conditions where σ_2 was equal to 62.5 MPa and σ_3 was equal to $0.4\sigma_2$, $0.6\sigma_2$ and $0.8\sigma_2$.

Triaxial tests were carried out at Dr Manabu Takahashi's Rock Deformation Laboratory at the National Institute of Advanced Industrial Science and Technology (AIST) in Tsukuba using the servo-controlled true triaxial apparatus that was described in the first part of this paper (see Figure 4).

The rock sample assembly consisted of a rectangular prismatic rock sample of the dimensions 35 mm × 35 mm × 70 mm, with steel end pieces attached to the top and to the bottom of the sample and steel end pieces attached to these side walls of the sample that were to be loaded in direction 2. The sample was covered with a silicone rubber jacket and equipped with strain-gauged displacement transducers to measure the axial strain and the lateral strains in two directions (Fig. 17).



- Rectangular prismatic sample of the dimensions:
  35 mm × 35 mm × 70 mm
- Steel end pieces (directions 1 & 2)
- Silicone rubber jacket (direction 3)
- Strain-gauged displacement transducers (directions 1, 2 & 3)

Fig. 17. Rock sample assembly ready for testing

In this paper, the results obtained using samples of a fine-grained Rozbark sandstone, a rock with a uniaxial compressive strength of 146 MPa, will be presented.

In the conventional triaxial compression tests, the ultimate strength of the sandstone tested increased markedly with an increase in confining pressure. At a confining pressure equal to 62.5 MPa, the maximum principal stress at failure was, on average, 2.8 times higher than the uniaxial compressive strength. The relationship between the maximum principal stress at failure and the confining pressure is represented very well by the Hoek & Brown strength criterion with parameter m equal to 10.5 (Fig. 18).

The results of the true triaxial compression tests show that the effect of intermediate principal stress (σ_2) on the ultimate strength of the sandstone tested – although evident and significant – is, however, considerably weaker than the effect of confining pressure (Fig. 19). In the case of samples tested at a minimum principal stress (σ_3) equal to 25 MPa, the maximum principal stress at failure (${}^F\sigma_1$) corresponding to $\sigma_2 = 62.5$ MPa was only about 19% greater than that corresponding to $\sigma_2 = 25$ MPa. It can be observed at the same time that the rate of strength increase with an increase in the intermediate principal stress (within a range from 25 MPa to 62.5 MPa) decreased slightly.

Studies of the effect of minimum principal stress on the strength properties of the Rozbark sandstone showed that σ_3 plays a role that is similar to that of σ_2. However, the rate of strength increase with an increase of σ_3 was somewhat higher than that observed when σ_2 was increased under constant σ_3 conditions (Fig. 20). At an intermediate principal stress equal to 62.5 MPa, the maximum principal stress at failure (${}^F\sigma_1$) corresponding to $\sigma_3 = 62.5$ MPa was nearly 28% higher than that corresponding to $\sigma_3 = 25$ MPa.

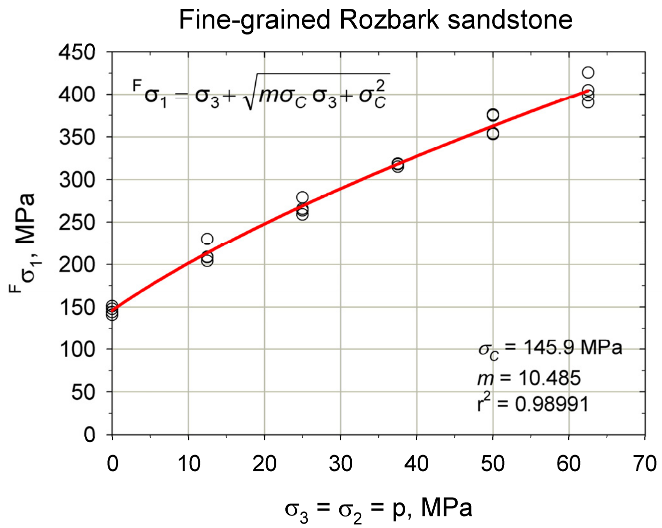


Fig. 18. Effect of confining pressure on the ultimate strength of the Rozbark sandstone; empirical data fitted using the Hoek & Brown failure criterion

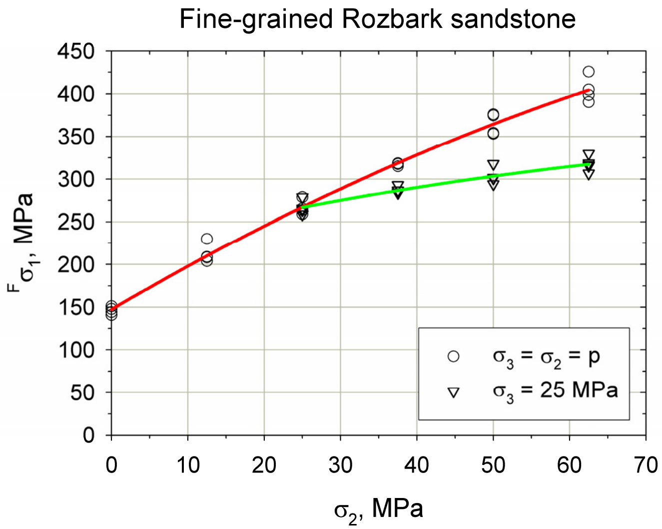


Fig. 19. Effect of confining pressure and of the intermediate principal stress on the maximum principal stress at strength failure of the Rozbark sandstone (after Kwaśniewski & Takahashi, 2007)

The stronger effect of the minimum principal stress on the ultimate strength of the rock tested in comparison to the effect of the intermediate principal stress may be explained in the following way: When the minimum stress is increased under constant σ_2, the general triaxial stress state becomes more and more uniform (axisymmetric), which enhances the strengthening

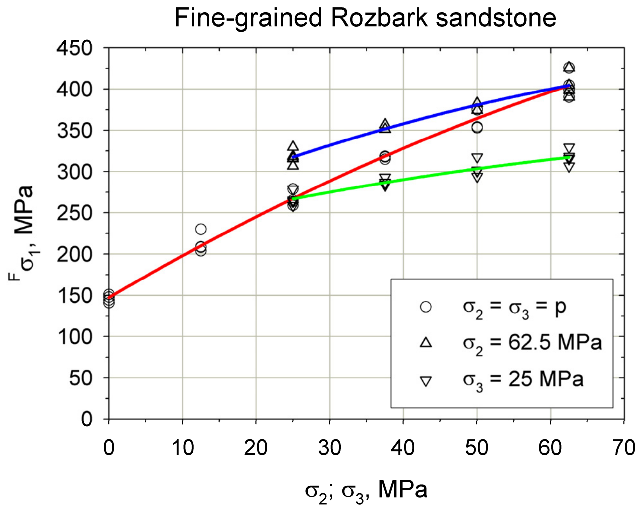


Fig. 20. Effect of confining pressure, of the intermediate principal stress and of the minimum principal stress on the maximum principal stress at strength failure of the Rozbark sandstone (after Kwaśniewski & Takahashi, 2007)

of the material (Fig. 21a). On the other hand, when the intermediate principal stress is increased relative to σ_3, an increasingly asymmetric or non-uniform state of stress is produced, which is disadvantageous to strength and at high stress differences ($\sigma_2 - \sigma_3$) may even lead to a decrease in strength (Fig. 21b).

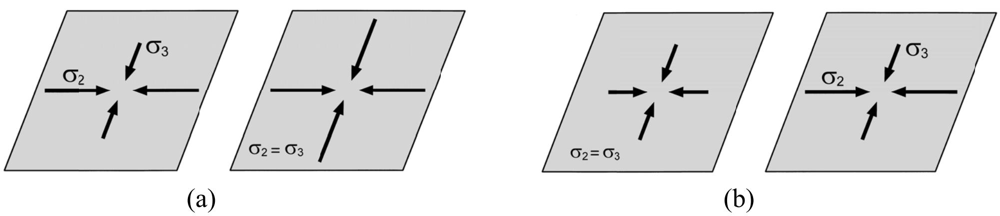


Fig. 21. Schemes illustrating how the minimum principal stress increases to form an axisymmetric stress state (a) and how the intermediate principal stress increases to form an asymmetric stress state (b) (Kwaśniewski 2012)

The results of all of the experimental studies on the ultimate strength of the Rozbark sandstone under uniaxial and triaxial compressive stress conditions were fitted by Mogi's power-law strength criterion very well. The fitting procedure employing the method of least squares yielded values of parameters A and n equal to 2.879 and 0.740, respectively (Fig. 22).

It should be noted, that Mogi's criterion fits the results of axisymmetric triaxial tests (note the circles on the plot) and asymmetric triaxial tests (triangles on the plot) equally well. In the domain of true triaxial tests, it covers results obtained under conditions in which at a given mini-

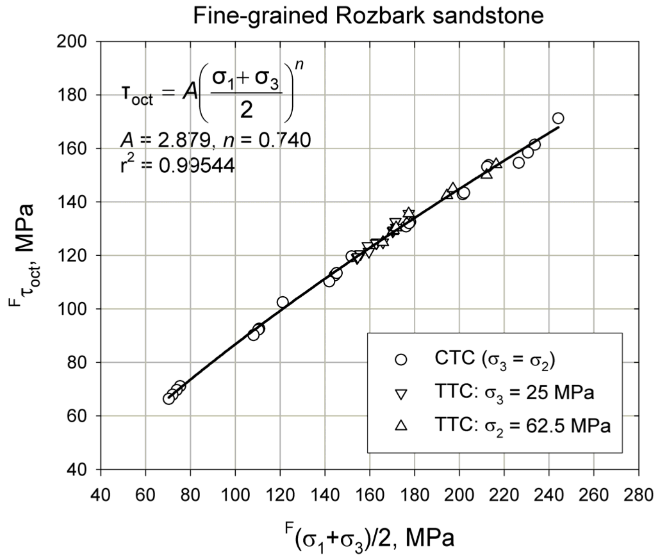


Fig. 22. Results of empirical studies on the true triaxial strength of the Rozbark sandstone plotted as the octahedral shear stress vs mean normal stress at failure $\sigma_{m,2}$ and fitted using Mogi's power-law failure criterion (Kwaśniewski & Takahashi, 2006, 2007)

mum principal stress, the values of the intermediate principal stress were different (downward triangles on the plot), as well as those obtained under conditions where the minimum principal stress values were different at a given intermediate principal stress (upward triangles).

5. Summary and final remarks

The following statements briefly summarize the results of the experimental studies presented in this paper:

(i) Although the effect of the intermediate principal stress and the minimum principal stress on the ultimate strength of rocks is weaker than the effect of confining pressure, it is considerable and must not be neglected.

(ii) The minimum principal stress affects the ultimate strength of rocks slightly more than the intermediate principal stress.

(iii) The Huber-Mises-Hencky failure criterion generalized by Nádai and modified by Mogi fits the triaxial test results very well.

The physical interpretation of the criterion proposed by Mogi for brittle rocks is as follows: Contrary to the stipulations of the Huber-Mises-Hencky theory which are valid for ductile metals, in the case of rocks the limiting value of the octahedral shearing stress is not constant but monotonically increases with an increase in the effective mean stress (σ_m). Since brittle shear fracturing occurs along the planes oriented parallel to the direction of the intermediate principal stress, the effective mean stress seems to be independent of this stress, and therefore $\sigma_m = \sigma_{m,2} = (\sigma_1 + \sigma_3)/2$.

Plots shown in Figure 23 confirm that Mogi's interpretation of the mechanism of failure under true triaxial compression conditions is much more suitable for brittle rocks than Nádai's approach, which was proven by Mogi to be valid for rocks which behave in a ductile manner.

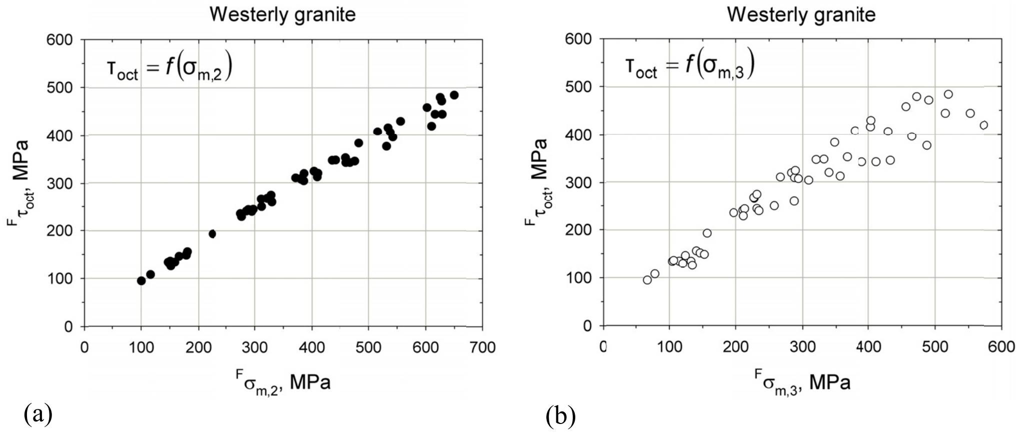


Fig. 23. Comparison of the suitability of Mogi's failure criterion $\tau_{oct} = f(\sigma_{m,2})$ (a) and Nádai's failure criterion $\tau_{oct} = f(\sigma_{m,3})$ (b) to fit the true triaxial strength data for the Westerly granite tested by Chang (2001)

It is important to note that since Mogi's power-law failure criterion fits empirical data from conventional triaxial tests and true triaxial tests equally well (see Figure 22), it is not absolutely necessary to conduct true triaxial tests (which are troublesome, time-consuming and expensive, even if a suitable apparatus is at hand, which normally is not the case) in order to estimate the values of parameters A and n that occur in the criterion. Values of these parameters can be determined based on the results of much simpler conventional triaxial compression tests. Then, after being incorporated into Mogi's criterion they may serve to assess the ultimate strength of rocks under both axisymmetric and asymmetric stress conditions.

An example of such an approach using empirical data from CTC tests carried out on samples of a fine-grained Bogdanka sandstone from the Lublin Coal Basin in south-east Poland is shown in Figure 24. Note that these data were fitted very well using Hoek & Brown's strength criterion and even better (with a coefficient of determination equal to 0.99663) using Mogi's criterion. The Hoek & Brown criterion (with the material constants σ_C and m equal to 110.3 MPa and 12.25, respectively) can be used to assess the ultimate strength of the Bogdanka sandstone under axisymmetric stress conditions only, while the Mogi power-law criterion (with the material constants $A = 2.276$ and $n = 0.776$) is suitable for any configuration of the magnitudes of the principal compressive stresses applied.

It is worth noting that parameter A in Mogi's brittle failure criterion must not necessarily be considered as an entirely independent constant. It can easily be shown that it is related to the uniaxial compressive strength and the exponent n by the following equation:

$$A = \frac{2^{n+0.5}\sigma_C^{1-n}}{3} \tag{10}$$

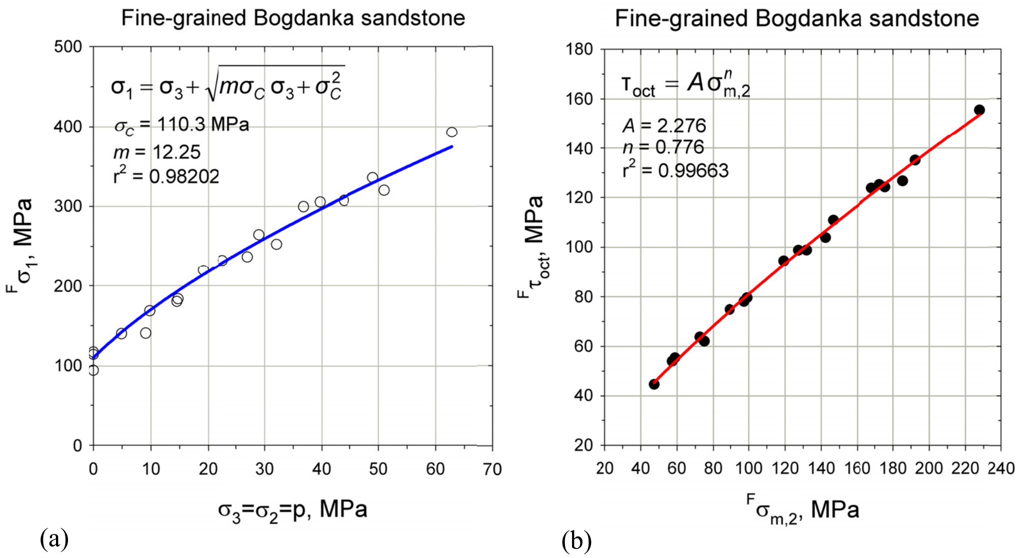


Fig. 24. Results of conventional triaxial compression tests on samples of the Bogdanka sandstone fitted using the original Hoek & Brown failure criterion (a) and Mogi's power-law failure criterion (b)

Finally, it should be added that Mogi's failure criterion (9) may be considered somewhat inconvenient in certain applications because of its power-law form. In such a case it can be replaced with the linear equation

$$\tau_{oct} = a + b\sigma_{m,2} \tag{11}$$

which, in many cases, gives an almost equally good fit to the experimental data. Note that in the case of the fine-grained Rozbark sandstone, samples of which were tested under different p, different σ_2 and different σ_3 conditions, all of the data lie in the $\tau_{oct} - \sigma_{m,2}$ plane satisfactorily close to the straight line given by the linear-law model (Fig. 25).

Moreover, Al-Ajmi and Zimmerman (2005) showed that for an axisymmetric state of stresses ($\sigma_2 = \sigma_3$), the linear Mogi criterion (11) reduces to the Coulomb criterion (2) or (3) with parameters a and b equal, respectively, to:

$$a = \frac{2\sqrt{2}}{3} c \cos\varphi, \quad b = \frac{2\sqrt{2}}{3} \sin\varphi \tag{12}$$

or

$$a = \frac{2\sqrt{2}}{3} \frac{\sigma_C}{B+1}, \quad b = \frac{2\sqrt{2}}{3} \frac{B-1}{B+1} \tag{13}$$

They even proposed that the linear version of Mogi's failure criterion could be named the "Mogi-Coulomb" criterion.

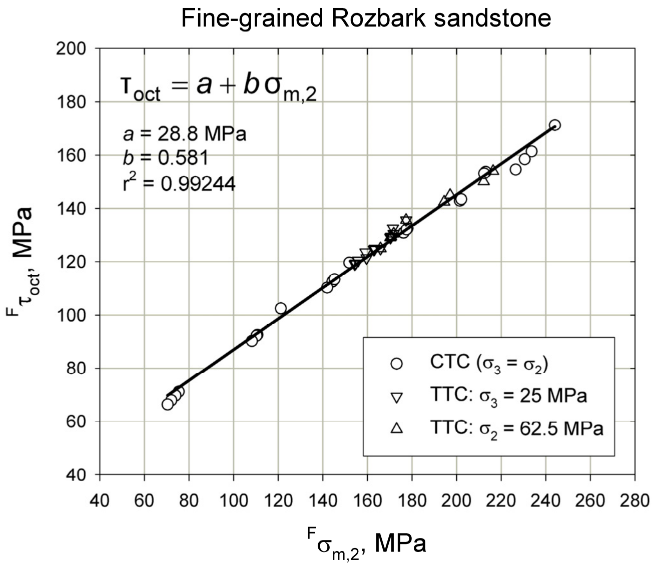


Fig. 25. Results of CTC and TTC tests on samples of the Rozbark sandstone fitted using the linear form of Mogi's failure criterion

In recent years, extended and generalized forms of the Hoek & Brown failure criterion have been developed by Zhang & Zhou (2007) and Zhang (2008), respectively. In the extended form:

$$\frac{9}{2\sigma_C}\tau_{oct}^2+\frac{3}{2\sqrt{2}}m\tau_{oct}-m\sigma_{m,2}=\sigma_C \tag{14}$$

the criterion can be applied not only for axisymmetric stress conditions but also for conditions where $\sigma_2 \neq \sigma_3$. Generalization of the criterion to the following form:

$$\frac{1}{\sigma_C^{(1/a-1)}}\left(\frac{3}{\sqrt{2}}\tau_{oct}\right)^{1/a}+\frac{m}{2}\left(\frac{3}{\sqrt{2}}\tau_{oct}\right)-m\sigma_{m,2}=\sigma_C \tag{15}$$

has made it possible to account for different rates of increase of the octahedral shear stress (τ_{oct}) at strength failure with the effective mean normal stress ($\sigma_{m,2}$). Note, that for a = 0.5 Equation (15) reduces to Equation (14).

In a review of the experimental studies on the mechanical behavior of rocks under general triaxial compression conditions to-date, that was compiled for the International Workshop on True Triaxial Testing of Rocks (Beijing, October 17, 2011), the linear version of Mogi's failure criterion (Equ. (11)), the power-law version of Mogi's criterion (Equ. (9)) and Zhang & Zhu failure criterion, i.e. an extended version of the original Hoek & Brown criterion (Equ. (14)) were used to fit twenty-nine data sets $[\tau_{oct}=f(\sigma_{m,2})]_F$ collected for a variety of magmatic, sedimentary and metamorphic rocks. For details see Kwaśniewski (2012).

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