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METHODS OF THE SCHEDULE MATRIX FORMING BASED ON THE MODIFIED PERMANENT

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Summary. In this article a technique of schedule matrices analysis for scheduling task based on the certain modifications of permanent is proposed. The proposed method of schedule construction use various configurations and representatives of their formation and algorithm based on decompozition of the permanent by line.

Keywords: scheduling, task scheduling, configuration approach, permanent matrix

1. INTRODUCTION

Despite the fact that there is a number of approaches to solving scheduling tasks, particularly, tasks of lessons scheduling, they are relevant nowadays due to their multicriteria and computational complexity. After all, finding the optimal solutions in different classes of criteria requires significant computing resources in practice. Specificity of the problem of drawing up the studing schedule of an educational institution, obviously, is its multicriteria. According to R. Dechter [3], network constraints are graph representations used to search for problem-solving strategies.

Nowadays the theory of scheduling which is widely used both in organizing the work of enterprises and for constructing schedules in educational institutions and contains a number of specific approaches has already been formed. Also, this problem intersects with the dynamic programming section of the control theory and computer systems theory [1]. In this paper a new configuration approach to the scheduling tasks solution which allows taking into account a number of criteria that must be met by a schedule in the process of dynamically forming matrixes of schedules is proposed.

S.V. Davydov pointed to the complexity of the constructing schedules algorithms (NP-complete problems), proposing a solution by a method close to the exact one for a certain optimality criterion, since the realising scheduling task by the percolation method, as well as the search with a return, is not effective [2]. Therefore, modern scientific papers prefer heuristic approaches [5], because the real sizes of the tasks thus far do not allow them to be solved practically by their existing exact methods for quadratic programming tasks. G.A. Popov in his paper [6] considers the model of the constructing a curriculum task in the form of a 4-dimensional matrix of elements $x\_\rho g\alpha d = \{1,0\}$, where the lessons, subgroups, classrooms and disciplines are described in the indices, respectively. We can receive elements of the possibility of conducting a pair "1" or it's impossibility "0" by applying a set of priority requirements of different levels (criteria and restrictions), are obtained. Obviously, solving scheduling tasks within the above-mentioned approaches involves significant computational complexities therefore approaches that allow a significant reduction in the power of the permissible space (the number of choices) in solving such problems become actual.

The paper proposes a new configuration approach to the schedules matrices analysis based on combinatorial properties of algebraic structures, particularly, permanent.

Everybody knows criteria that must satisfy the classes schedule: students should not have "windows", teachers should not have "windows", the number of teacher's working days should be minimal, the teacher's wishes should be taken into account as much as possible [6]. Obviously the best in some meaning schedule is the best option for each day of the school week. So let's restrict ourselves to considering one working day.

If each teacher is assigned a certain natural number (number, weight, etc.), then the schedule will be a matrix of dimension 3 x n where 3 is the number of pairs, n-number of groups (subgroups) for which the schedule is composed. We call such matrix "decomposition matrix" and denote it R. Obviously a number of tasks related to the requirements for the classes schedule appears when analyzing schedule matrices. The task of this paper is to develop the criteria for the admissibility of the decomposition matrices particularly based on the specially introduced matrices of the configurations incidence and the theory of permanent and also to consider the algorithmic aspects of constructing admissible matrices of schedules on the basis of systems of different representatives of sets that form the "columns" of the original matrices.

The purpose of this work is to develop a new method of schedules matrixes analysis for the scheduling task based on the application of certain modifications of the permanence of incidence matrices.

2. METHODS OF DECOMPOSITION MATRIXES RESEARCH BASED ON MODIFIED PERMANENTS THEORY

2.1. SCHEDULES CONFIGURATION

The elements set of the matrix R such that $\alpha_{i,j} = \alpha_{k,l} = \alpha_{m,p} = d$, $i, k, m \in \{1, 2, 3\}$, $1 \le j$, $p \le n$ will be called the ternary configuration of the decomposition formed by the element of the decomposition matrix d. The concept of binary and unary configurations can be introduced similarly (the binary configuration can't be a ternary's element). A natural condition for ternary configuration is a condition $i \ne k \ne m$ (the teacher can't conduct several pairs simulnateously). A similar condition holds for binary configurations. Thus, the schedules matrix is a collection of ternary, binary, unary configurations and zeros (zero means that there are no pairs). Obviously, the set of configurations should be such that the decomposition matrix could be formed in general (there should be no more than 3 pairs per day in one group). The schedule matrix is called admissible if there are no two identical elements in the line.

Obviously in the case of flows the decomposition matrix may be inadmissible. Consider for example the such matrix:

$$\begin{pmatrix} 1 & 1 & 1 & 2 & 3 & 5 \\ 2 & 3 & 1 & 4 & 4 & 4 \\ 3 & 1 & 3 & 4 & 2 & 4 \end{pmatrix} \qquad (1)$$

Let's assume that the "one" in the first row and the "fourth" in the second correspond to the flows. This matrix is inadmissible. Indeed this matrix has four ternary configurations formed by elements 1, 2, 3, 4 and one binary one formed by element 5. There must necessarily be one representative of each ternary configuration in each line additionally two lines have one representative of the ternary and binary configuration in the admissible form. Totally we have 6 columns. But it's impossible to create a line that would contain elements 1, 2, 3, 4 and 5 in this example. Indeed, if "one" or "four" is current, then three of the six elements are "busy." And you need to write four more different elements. If we choose all non-current representatives (such a situation is possible for this matrix, we can form a string for example 3, 1, 4, 5, 2), then we can't avoid the coincidence since there is no unary element in the matrix .

Let's first consider a general approach based on building systems of different column representatives. Under the system of different column representatives (SDCR) we mean the set formed by column elements, in which all the columns elements are different except for the current elements. Selecting of a current element from a column will automatically select it from all other columns where the corresponding flow is located.

2.2. ACCEPTABLE FORM OF THE DECOMPOSITION MATRIX

Let's formulate the criterion for the existence of a matrix acceptable form of day-to-day decomposition.

Obvious is the following simple statement:

Statement 0. *The day-by-day schedule matrix R has a valid form if and only if there is a system of different columns representatives* α *that* R/α *has some SDCR* β *and* $R/\alpha/\beta$ *is SDCR (the multiplicative theorem operation is implemented "in columns").*

Let matrix A have a permissible form. Then there is no coincidence of any two elements in the matrix strings. But this means that all lines are the SDCR and the statement condition is executed. Let the statement condition be fulfilled. But then we can create a matrix, whose rows are α, β and $A/\alpha/\beta$. Obviously this matrix coincides with the output for the sets of column elements. We need to modify the incidence matrix and also introduce the concept of a modified permanent later. We introduce the concept of the configurations incidence matrix to represent a decomposition matrix containing streams. Suppose we have an arbitrary matrix of daily schedule, dimensionality 3xn. Consider the columns of the decomposition matrix R_1, R_2, ...R_n. The modified incidence matrix is constructed in the following way. The teachers numbers are displayed horizontally, groups, in which classes are held are displayed vertically. Each teacher is associated with matrix column in which zeros and singles are written, depending on whether the teacher has pairs in the respective groups. If a teacher x has a current pair we allocate him a separate column of the incidence matrix, denoting it x^p (you can use an index that is the number of stream elements). Thus the matrix of the form is formed:

$$A = \begin{array}{c} \\ R_1 \\ R_2 \\ \cdots \\ R_m \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \end{array}, \tag{2}$$

here $\alpha_{i,j} = \begin{cases} 1, \text{if } x_i \in R_j , \\ 0 \text{ in other cases.} \end{cases}$

For example, for a schedules matrix of the form:

$$\begin{pmatrix} 1 & 1 & 1 & 2 & 3 & 5 \\ 2 & 3 & 1 & 4 & 4 & 4 \\ 3 & 1 & 4 & 5 & 2 & 4 \end{pmatrix} \tag{3}$$

we get the following incidence matrix:

$$\begin{pmatrix} & 1 & 1^p & 2 & 3 & 4 & 5^p & 5 \\ R_1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ R_2 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ R_3 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ R_4 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ R_5 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ R_6 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \tag{4}$$

where 1^p and 4^p are the indices of the corresponding teachers having "current" pairs.

Definition: *the modified permanent of the incidence matrix is the sum of all possible items of the matrix elements, each containing one element per row and from different columns, with the item of the stream column (the column corresponding to the stream element) can't be in the product along with elements of other lines, corresponding to the same stream.*

Note that in the absence of flow elements, the modified permanent is a usual permanent.

We can use the schedule by line to find the modified permanent of the incidence matrix. Based on the definition the scheduling procedure will be as follows: the non-zero element of the line is multiplied by the modified permanent of the matrix formed by the following rules: if the element of the string belongs to the thread column, then the matrix is formed with the original stroke of the column where the element is located, and all the rows that correspond to all elements of this thread. If the line item does not belong to the thread column, then the matrix is formed by removing the row and column where this item is located, as well as all the flow columns at the intersection of which this line contains non-zero elements. Let's also note that the removal of all available items will ultimately result in a "one".

Consider the procedure for calculating a modified permanent for a schedules matrix (3). We have a matrix of the form:

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}. \tag{5}$$

When calculating a permanent operator, we will construct a schedule for the first row (stream columns 2nd and 4th):

$$per\,mod\,A = 1 * per\,mod\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} + 1 * per\,mod\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} +$$
$$1 * per\,mod\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}. \tag{6}$$

Next the procedure is scheduled similarly.

Statement 1. *The schedules matrix has an SDCR if and only if the modified permanence of the incidence matrix is non-zero.*

Proving

Let's have a schedules matrix of 3xn dimensional. Let there is a matrix of time schedules for the SDCR $x^1, x^2, ..., x_n$. Then, constructing the incidence matrix we see that the permanent of the matrix is non-zero since the permanence of the matrix formed by the columns corresponding to the elements $x^1, x^2, ..., x_n$ is "one":

$$\begin{pmatrix} x_1 & x_2 & \cdots & \cdots & x_n \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix} \tag{7}$$

The presence of stream elements does not worsen the situation. For example if $x^1 = x^2$ and the other elements are different, then we have an incidence matrix:

$$\begin{pmatrix} x_1^p & x_2 & \cdots & \cdots & x_n \\ 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix} \tag{8}$$

The modified permanent is obviously a "one".

Let the modified permanent of the incidence matrix is non-zero. Then there is a single element, in its schedule (the permanent of a matrix formed by the subsystem of the columns is equal to 1). But if we consider the elements that correspond to the columns of this subsystem, then it is easy to see that they form the SDRC. The statement is proven.

2.3. THE PROCEDURE OF FORMING A SYSTEM OF DIFFERENT COLUMN REPRESENTATIVES

Proof of the previous assertion theorem suggests that it is easy to construct all possible SCRS in the process of the expansion of the modified permanent of the incidence matrix of column configurations. For each SCRS corresponds the submatrix of incidence matrix with a single permanent. In addition, the SPRS can be obtained if you enter the process of "memorizing" the column item corresponding to the unit used in the construction of the product at the moment and the line in which this unit is in the original matrix of incidence. In this regard, in the process of calculating the permanent, we will write two indice for each "one": the upper element corresponding to the column where this unit is, the lower line of the number in which this unit is in the original matrix.

Let we have for example a schedule matrix of the form:

$$R = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 3 & 1 & 4 \end{pmatrix} \tag{9}$$

Then the incidence matrix has the form:

$$\begin{pmatrix} & 1 & 1^n & 2 & 3 & 4 \\ R_1 & 0 & 1 & 1 & 1 & 0 \\ R_2 & 1 & 1 & 0 & 1 & 0 \\ R_3 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \tag{10}$$

We construct the process of a modified permanent decomposition with "remembering" on the first line:

$$per\,mod\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix} = 1_1^{1^p} * 1 + 1_1^2 * per\,mod\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + \cdots \tag{11}$$

$$+1_1^3 * per\,mod\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} = 1_1^{1^n} * 1 + 1_1^2 * \left(1_2^1\, per\,mod\begin{pmatrix} 0 & 1 \end{pmatrix} + 1_2^3\, per\,mod\begin{pmatrix} 1 & 1 \end{pmatrix}\right) + \cdots$$

$$+1_1^3 1_2^1\, per\,mod\begin{pmatrix} 0 & 1 \end{pmatrix} = 1_1^{1^n} + 1_1^2 1_2^1 1_3^4 + 1_1^2 1_2^3 1_3^1 + 1_1^2 1_2^3 1_3^4 + 1_1^3 1_2^1 1_3^4.$$

We see that the modified permanent is equal to 5 (and this is the number of all possible SPRCs). In addition the CPPS themselves are known. They are written as upper "singles" indices. Moreover, if certain lower indices are absent, then the upper element is repeated the corresponding number of times (the situation of the stream element): *111, 214, 231, 234, 314*.

In the presence of all possible SRPCs, it is a permissible solution to the tasks of different classes. For example, if we are interested in the admissible form of the decomposition matrix then it exists only if in the set of all SRPC there are three SRPCs that do not contain identical elements in identical places, except for the elements of flows. There is only one set of SRPCs for this example: *111, 314, 231*.

3. FORMATION OF AN ADMISSIBLE DECOMPOSITION MATRIX BASED ON SRPC

Consequently, we will form a permissible matrix in stages, forming the first, second, third lines (etc.), by permuting the elements of the columns. In order for the first line of the allowable schedule matrix to be formed, it is necessary that the of the column sets $\{R_1, R_2, ..., R_n\}$ configuration contains a different representatives system (DRS). But the DRS exists if and only if the permanence matrix of the incident configuration is different from zero. Consequently, we will consider the permanence of the matrix A. If it is different from zero, then there is a DRS: $a11$, $a12$, ..., $a1n$. This DRS forms the first line of the allowed decomposition matrix. In order to form the second line of an admissible matrix, it is necessary that there exist a DRS of sets configurations $\{R_1 / a11, R_2 / a12, ..., R_n / a1n\}$ where $a11$, $a12$, ..., $a1n$. is the first line of the allowed decomposition matrix. Appropriate considerations continue.

Note that the initial process of forming configurations can be either automatic, for reasons of minimum number of teacher's working days or it is the result of agreement with the teacher (for example, it is difficult for some teachers to spend three pairs in one day, then you can create the appropriate binary or even unary configurations, if all of them are satisfied). With this approach, the problem of minimizing the "windows" of teachers is already at the initial stage of the problem solution: the schedule matrix is a set of already optimized (not having windows) configurations.

4. CONCLUSION

In this paper a new approach to solving scheduling problems based on concepts of configurations and schedules matrixes is proposed. A criterion for the existence of a different representatives system of schedules matrices is obtained based on the introduction of certain modifications of the incidence matrices permanence. A special procedure for constructing a different representatives of sets system that form columns of the schedules matrix is proposedThe obtained theoretical results can be the basis for the development of an automated system for drawing up the classes schedule, as well as solving other problems of the schedules theory.

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SPOSOBY TWORZENIA MATRYC ROZKŁADU W OPARCIU O TRWAŁE LINIOWE MODYFIKOWANIE

Streszczenie

W artykule zaproponowano technikę analizy matryc harmonogramu dla zadania szeregowania w oparciu o pewne modyfikacje trwałe. Zaproponowana metoda budowy rozkładu wykorzystuje różne konfiguracje oraz przedstawicieli ich tworzenia i algorytmów opartych na liniowym trwałym rozkładzie.

Słowa kluczowe: planowanie, planowanie zadań, podejście konfiguracyjne, macierz rozkładu