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MODELING BATHYMETRY CHANGES IN THE COASTAL ZONE – STATE OF KNOWLEDGE ANALYSIS

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Abstract

In this paper we briefly review the mathematical models used to describe the bathymetry changes in time and space. Overview of the models was carried out with particular emphasis on the problems encountered during the nonlinear equation solution, commonly used to describe the morphology of the bottom in the coastal zone of the sea. In contrast to the commonly used approach, it is proposed a linear relationship between volumetric flow of sediments transport rate and thickness of the layer of sediment grains, closely adjacent to each other and staying in motion. This linear relationship allows to precisely define the initial – boundary conditions and to apply the numerical scheme of finite difference method of „upwind" at the accuracy of the first order, not distorted by numerical errors. The author's method also allows to implement changes to the description of the bathymetry of simultaneous changes in the distributions of the sediment grain size.

Introduction

In the coastal zone of the sea there is a strong interaction of waves and currents on the bottom, which results in almost constant movement of sediment and the closely related variability of bottom shape. The importance of the sediment movement within the coastal areas underlines the need for mathematical description of sediment transport and bottom morphology. Cyclic erosion and accumulation processes, due to the volume of transported sediment cause that the local seabed is constantly in dynamic equilibrium. Thus, to describe the changes in the bottom morphology it is possible to use

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laws of conservation of sediment transport. In order to determine the prediction of changes in bottom morphology and the spatial variation of the transport flow (both alongshore and cross-shore) in the sea coastal zone, it is possible to use the numerical models. Morphological models of the coastal zone are essential means that allow for the prediction of changes in bathymetry within the coastal zone of the sea as well as the analysis of the bottom level changeability within the area of coastal structures at the stage of design, workmanship and usage.

Morphological models describing changes in the transverse profile in the coastal zone are formed by combining several models describing the cause-effect relationship: *waving and current → sediment transport → changes in bathymetry → changes in waving and currents*. At first, depending on the wave and wave induced current conditions from the sub-model of sediments transport it is required to determine the sediment transport rate. Then, on the basis of the sediment transport equation (conservation of sediment mass) it is required to perform bathymetric computations including the continuous redistribution of sediment in time.

Problem definition

Traditional Exner equation which is the basis for the common determination of bathymetry changes in time and space is based on the nonlinear relationship between sediment transport rates and elevation of the bottom level. This nonlinear relationship results that numerical schemes used to describe changes in bathymetry generate oscillations in the results of the modeling or they are unstable schemes. Oscillations are usually a sign of instability (dispersity) of numerical schemes themselves. In order to eliminate or control oscillations, it is usually necessary to use the solution of the numerical diffusion that causes smoothing the solution. The application of scheme generating a numerical diffusion is associated with reduced accuracy of the solution. Any attempts to „improve" the results have no relation to the actual physical processes occurring in the coastal zone. Thus, by selecting appropriate parameters you can only try to describe the bathymetry changes occurring in the past. Morphological models should serve the correct prediction of changes in bottom morphology instead of attempts to reflect the previously observed changes.

In particular, the open issue (crucial question) is the determination of bathymetry changes taking place under the influence of variations in the flow of sediment transport of non-uniform grain size, and the determination of the mutual impact of changes in grain size distribution of sediments and evolution of the bottom profile.

It should also be noted that when considering the sediment transport in the coastal zone of the sea, in case of assuming the asymmetric waving (2nd Stokes' approximation), customarily it is to operate the flow of the bottom sediments transport which is resultant in duration of wave period. This approach causes difficulties associated with the correct formulation of the boundary conditions in the calculation bottom morphology changes in case where the resultant stream of sediment transport changes the direction in space. In addition, handling the sediment transport rates resultant in wave period means, that in case of symmetric waving (sinusoidal) – where the sediment transport rate during the wave crest is equal to the absolute value of sediment transport rate during the wave through (opposite directions) – a resultant sediment transport rate is zero. As a result of bottom morphology calculations, conducted on the basis of the above assumption, no bathymetry changes are possible to be obtained which after all does not correspond to reality (SAWCZYŃSKI 2012).

Morphological models

In order to correctly predict the morphological changes at the bottom it is required to understand sufficiently the processes occurring in the coastal zone of the sea and describe them with the language of mathematics by making appropriate simplifications. Numerical simulations of long-term (years, decades) phenomena led at the significant area of the coastal zone should be carried out in a different way than the short-term (hours, days) or medium term (weeks, months, years). Pursuant to the purpose and mathematical simplification applied in the numeric modeling, the morphological models of the coastal zone can be divided into three main groups (CHIANG, HSIAO 2011): (i) models of the coastline (single or multiple lines), (ii) three-dimensional models and (iii) macromodels.

Models of coastline are forecasting models based on the equation of transport adapted to describe the evolution of the bottom in cross profile of the coast and possible in the equation describing the intensity of the alongshore transport. Two and multiple lines models are also helpful for the analysis of the specifically selected profiles. For the past twenty years, it has been made a significant growth of two-dimensional models, taking into account the spatial vertical dimension and the dimension which is perpendicular to the coast. They are often described as two-dimensional vertical models – 2DV (DE VRIEND et al. 1993a, HARRIS, WIBERG 2001, HSU et al. 2006, NICHOLSON et al. 1997, RAKHA et al. 1997, SATO et al. 1995, WANG 1992, ZHANG et al. 1999) with their application in a short (hours, days) and medium (weeks, months, years) period of time.

Three-dimensional models also called as coastal zone models are used to predict the level of the bottom and its variation in the horizontal plane. They take into account the calculations of sediment transport rates in the alongshore direction and cross- shore direction, arising from the interaction of waving and wave induced currents in the coastal zone (e.g. BLAAS et al. 2007, BROWN, DAVIES 2009, HARRIS et al. 2008, HU et al. 2009, LUMBORG 2005, SOUZA et al. 2007, ZANUTTIGH 2007). These models usually describe the short-term (hours or days) or medium-term (days or months) evolution of the bottom and usually have a modular structure. The core of the model is based on the different sub-models: wave, tidal currents, coastal currents, sediment transport, etc. closely coupled together. The core of the model, by solving the equation of mass conservation (sediment transport) determines the current elevations of the bottom level.

Macromodels make a more simplified, usually long-term empirical morphological analysis, based on evolution of trends, experience and gain of local data. These models can be effective in case of qualitative analysis, but they do not apply to the quantitative description.

Unfortunately, no morphological computational models, described in the literature provide valuable information on the spatial and time changes in bathymetry along with the simultaneously occurring changes in grain size distributions of the bottom sediment.

The problem of numerical solutions

Changes in the bottom level can be determined by the solutions of the commonly used equation of conservation of mass (Exner equation, e.g. YALIN, DA SILVA 2001) for the bottom sediment. For the two-dimensional case this can be written as follows:

$$\frac{\partial z_b}{\partial t} + \frac{1}{1 - n_p}\left(\frac{\partial q_{(x)}}{\partial x} + \frac{\partial q_{(y)}}{\partial y}\right) = 0 \tag{1}$$

where:

z_b – bottom level elevation [m],
x, y – horizontal space point coordinates [m],
t – time [s],
n_p – sediment porosity [–],
$q_{(x)}, q_{(y)}$ – volumetric flow of the sediment transport [m^2/s] towards x and y.

Sediment transport volumetric flow is expressed in the unit [m^2/s], which corresponds to the volume stream of the sediment, referred to the unitary width [$m^3/m \cdot s$] and it means the volume of the sediment, flowing in time unit through the rectangular cross-section $h \cdot 1$ m. In fact, the volumetric flows of sediment transport: $q_{(x)}$ and $q_{(y)}$ are very complex and complicated function of many parameters, including waving, wave induced currents, water depth, density, bed sediment material characteristics (the grain size distribution, porosity, etc.). Usually, it is assumed constant in time (at each time step) properties of the sediment and constant fill level. According to these assumptions, it is to determine (from various formulas based on experiments or theory) volumetric flow of sediment transport, caused by the interaction of waving and wave induced currents.

In the example shown by JOHNSON, ZYSERMAN (2002), bottom morphology simulation was performed using the model of MIKE 21 CAMS, the wave conditions simulation used MIKE 21 PMS model, description of flows was based on the model of MIKE 21 HD, and the calculations of sediments transport rates was made using MIKE 21 ST model. Models of MIKE 21 PMS and MIKE 21 HD are one of the most modern and well tested in many studies computational packages, used to assess the wave and wave induced current conditions. So what is the reason for unstable results?

According to JOHNSON, ZYSERMAN (2002), numerical spatial oscillations of morphological models occur as a result of the relationship between „bottom form velocity" (called bed celerity) and the elevation of the bottom level which is a non-linear relationship between sediment transport rate $q_{(x)}$ and bottom level elevation z_b. Thus, in morphological models the basic equations are nonlinear functions of the level bottom. Sediment transport, which occurs as a result of wave and wave induced current interactions is also determined on the basis of the nonlinear complex hydrodynamic relations. These nonlinear relations and numerical errors of the particular sub-models can generate instability and uncertainty of the results of calculation where the nature is still poorly understood. Despite the results of individual sub-models are accurate and stable, their connection with the equation of mass conservation (transport) also leads to instability and numerical oscillation (JENSEN et al. 1999). These models are therefore not able to accurately (physically) make a prediction of bathymetry in coastal areas in the vicinity of hydraulic structures, particularly in long-term simulations (JOHNSON, ZYSERMAN 2002).

Several techniques to improve the stability and accuracy of the solutions for modeling changes in bathymetry were developed in recent decades. As shown LONG et al. (2008), a number of cutting-edge computational models introduces numerical schemes, smoothing oscillations of bottom morphology evolution results. Delft 2D-MOR model (ROELVINK, VAN BANNING 1994, ROEL-

VINK et al. 1998) of a research institute of Delft Hydraulics uses an explicit numerical scheme with central differential quotient of FTCS (Forward Time Central Space), while at the same time it introduces the correction of the value of sediments transport rates in order to balance the diffusion generated by the numerical scheme. University of Liverpool model (O'CONNOR, NICHOLSON 1989, NICHOLSON et al. 1997) uses a two-step numerical scheme of Lax-Wendroff, taking into account the additional gravitational interactions on a sloping bottom when determining the sediment transport rate. VINCENT, CALTAGIRONE (1999) also use a modified Lax-Wendroff scheme along with LW-TVD scheme (Lax-Wendroff Total Variation Diminishing) with the limitation of the bottom slope. CAYOCCA (2001) uses a numerical „upwind" scheme with the correction of the sediments transport rate due to the nature of the bottom slope (DE VRIEND 1987a, DE VRIEND 1987b) and applies filtering techniques (DE VRIEND et al. 1993a, DE VRIEND et al. 1993b) in order to avoid oscillation. JOHNSON, ZYSERMAN (2002) shown that the nature of the bottom slope plays a major role in the instability of the numerical schemes, used to describe the changes in bathymetry. They expanded their discussion by the second term of Taylor series, concerning the changes of bottom level over time: the first term of the derivative with respect to time of the conservation of mass equation is calculated using the Lax-Wendroff scheme, and the second one is treated as a diffusion term of the advection equation. Calculation scheme used is further modified by introducing a LPF (Low Pass Filter) to dissipate random high frequency oscillation (suggested by JENSEN et al., 1999). In order to solve the equations of mass conservation, SAINT-CAST (2002) basing on the study of JIANG et al. (1998), used the NOC scheme (Non-Oscillating Centered) with a method of upgrading the bottom level (proposed by WATANABE 1988). He did not introduced into his model any filters or surges. CHIANG et al. (2010), LONG et al. (2008) and SHAO et al. (2004) in order to solve this equation for both one-dimensional (1D) and two-dimensional (2D) case, used WENO algorithm (Weighted Essentially Non – Oscillatory) derived from CFD scheme (Computational Fluid Dynamics), presented in papers of LIU et al. (1994), JIANG, WU (1999). This model, similarly to SAINT-CAST (2002) model is devoid of any filters or surges.

CALLAGHAN et al. (2006) have subjected to verify the advantages and disadvantages of morphological in terms of oscillation control. They reviewed a number of numerical schemes, including among others: „upwind", Lax-Wendroff (JOHNSON, ZYSERMAN 2002, VINCENT, CALTAGIRONE 1999), NOC (SAINT-CAST 2002). LONG et al. (2008) discussed the use of two numerical schemes of Lax-Wendroff (based on the diagram of MacCormack and Richtmyer) and three schemes of WENO (TVD – RKWENO, used among others by SHAO et al. 2004, Euler-Weno by LONG et al. 2008 and the two-step, three-

-level scheme of WENO, also discussed by CHIANG et al. 2010). On the basis of results numerical schemes review, CALLAGHAN et al. (2006) found that Lax-Wendroff schemes an any modifications to these schemes are not stable in case of long-term simulation of the bathymetry changes. Therefore, it is necessary to supplement these schemes with additional information in the form of filters, limiters or artificial viscosity to prevent numerical oscillations, generated by these schemes. Moreover, the authors of models review shown, that it is difficult to describe the phase velocity of propagation of bottom forms which is the most important parameter responsible for the stability of these numerical schemes. The requirement of numerical schemes stability is usually determined by the condition concerning Courant number, namely: $C_r = C_a \dfrac{\Delta t}{\Delta x} \leq 1$ (C_a is the speed of propagation of bottom forms). Values greater than unity suggest that a reduction in simulation time can help to improve the stability of the numerical schemes. However, it is related to the increase in demand for computing capacity. If a diffusive term of the transport equation is properly removed, the limit value of Courant number may be exceeded (CHIANG, HSIAO 2011). This can be done by introducing a diffusion constants which are adopted depending on the actual environmental conditions (CAYOCCA 2001, CHIANG et al. 2010, KUROIWA et al. 2003, STRUIKSMA et al. 1985, WATANABE 1988).

It should be emphasized that in order to make the correct long-term simulation of the bathymetry changes occurring as a result of the interaction of waves and wave induced currents in the coastal zone, the computational model should be able to control the spatial oscillations but also to ensure the accuracy of results, providing at the same time the physics of phenomena. Numerical schemes used to describe changes in bathymetry generate oscillations in the results of the modeling or they are unstable schemes, and all attempts to „improve" the results have no relation to the actual physical processes taking place in the coastal zone. So, it seems doubtful to find ways to describe changes in bathymetry with these methods, taking into account simultaneous changes in the distribution of sediment grain that builds the bottom.

New concept

If we assume that z_m means the thickness of cell $z_m \cdot \mathrm{d}x \cdot 1$, which is eroded over time of dt from the cross-shore profile (Fig. 1), as a result of sediment transport rates of q_x with the porosity of n_p, it is possible to write this down as follows:

$$z_m = \frac{1}{(1 - n_p)} \frac{q_x \mathrm{d}t}{\mathrm{d}x} \tag{2}$$

Furthermore, it is assumed that due to the shear stress influence on the bottom, the sediment is pulled off directly from the bottom, which is understood as an immediate bottom's „response" to the particular hydrodynamic conditions. As a result of this response – in hydrodynamic equilibrium circumstances – the stream of sediment pulled from the bottom equals (at each level) the stream of sediments falling onto the bottom. The entire sediment moving over the bottom originates exclusively from the bottom. Therefore, the movement of the sediment in the layer above the bottom with the average speed of implies a kind of sediment movement which takes place at the bottom

$$\bar{U} = \frac{\int_0^H UC\mathrm{d}z}{\int_0^H C\mathrm{d}z} \tag{3}$$

of the layer z_m (Fig. 1) described by the relationship (1) with the advance velocity of U_{L_1}:

$$U_{L_1} = \frac{\mathrm{d}x}{\mathrm{d}t} \tag{4}$$

while

$$\frac{\bar{U}}{U_{L_1}} = \kappa \tag{5}$$

In the above equations, H determines the water depth, C is the volumetric concentration of the transported sediment, U is the velocity of sediment and q_x is the sediment transport rate in hydrodynamic equilibrium conditions.

Conditions of the hydrodynamic equilibrium assume that q_x sediment transport rate that results from Eq. (3) in traditional (e.g. VAN RIJN (1984) form:

$$q_x = (1 - n_p)\, U_{L_1} z_m \tag{6}$$

in the bottom's layer of the thickness of z_m equals the sediments transport rate above the bottom (Fig. 1) described by the following relation:

$$q_x = \bar{U} \int_0^H C\mathrm{d}z \tag{7}$$

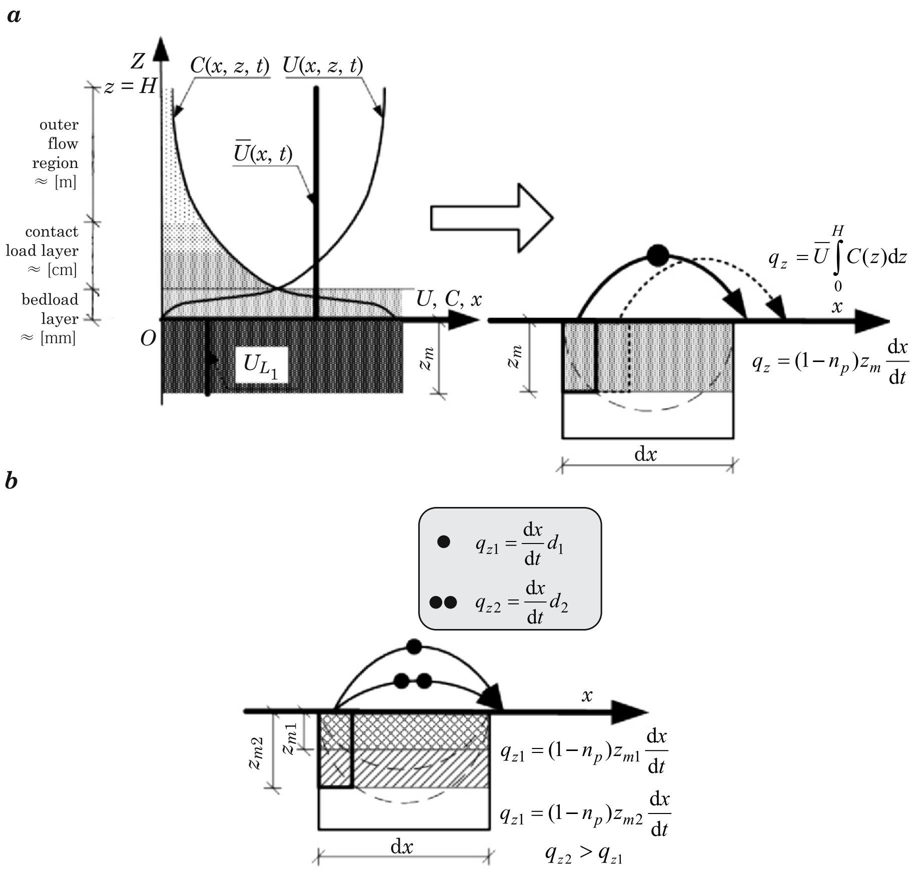

Fig. 1. Concept of the hydrodynamic equilibrium

Essential role in the initiation of sediment movement is played by grains saltations. Figure 1a shows, that saltation movement of the sediment grains results in a shift of stroke at the dx length of the total amount of grains with the equivalent diameter of $d = (1 - n_p)z_m$. Thus, the saltation movement (Fig. 1b) of sediment grains with the diameter of $d_2 > d_1$, under the influence of sediments transport rates $q_{x_2} > q_{x_1}$ results in dt time the bottom erosion with the thickness of $z_{m_2} > z_{m_1}$ in the cell with the length of dx. This means, that under conditions of hydrodynamic equilibrium, the thickness of z_m is the function of sediments transport rates and the advanced velocity of U_{L_1}, described with the relation of (4) does not depend on it. The above considerations are compatible with Einstein's conclusions. Based on experimental observations, EINSTEIN (1950) assumed that the averaged distance travelled by sand particle between erosion and subsequent deposition, is simply proportional to the grain diameter and independent of the hydraulic conditions and the amount of sediment in motion.

A good experimental illustration of the sediment movement under conditions of hydrodynamic equilibrium is the water flow over the box filled with sediment (Fig. 1). If the bottom on the outside of the box is concrete, it is not difficult to imagine that the entire sediment moving over the bottom comes exclusively from the bottom. With sufficient approximation it can be assumed, that under steady water flow conditions, erosion of sediment from the box (Fig. 1a) is caused by the simultaneous (along the entire length dx) grains strokes, with the total size of z_m. In such a case, the bottom sediment erosion is constant at the entire dx length and it amounts z_m (Fig. 1).

It should be at the same time noted, that the saltation movement, occurring as a form of grains strokes with the simultaneous mass exchange with bottom may result in change of grain size distribution in the control volume. It can be assumed that the sediment movement occurs in as short as possible grains strokes – at the distance of $\mathrm{d}x/2$ allowing the transition from the first to the second area, as shown in Fig. 2. Then, the thickness of the mixing layer h_{mj}, defined at j point, according to the formula:

$$q_{xj} = (1 - n_p)\,\frac{\mathrm{d}x}{2\mathrm{d}t}\,2z_{mj} = (1 - n_p)\,\frac{U_{L_1}}{2}\,h_{mj} \tag{8}$$

must be equal to:

$$h_{mj} = 2z_{mj} \tag{9}$$

It is assumed, that over time of dt, the half of material that residues at the bottom, in the cell with a length of dx (Fig. 2) flows out from the area of *II* and the other half is moved from the first to the second area and it is mixed with the sediment, flowing into the area *I* and resulting in the change of grain size distribution in the layer of thickness mixing, described with (9) relation.

It should also be noted that for the purpose of considerations concerning the transport of sediments in the wave motion, it is necessary to introduce the decomposition of movement into movement associated with the phase of wave crest (which takes place during the wave crest) and movement associated with the phase of the wave through (which takes place during the wave through), see KACZMAREK et al. (2011), SAWCZYŃSKI (2012).

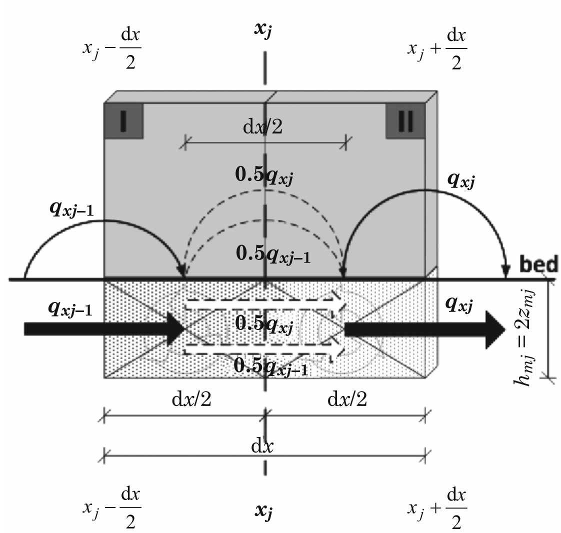

Fig. 2. Mixed vertical sediments

The advantages of the new concept

The linear dependence (6), valid under the hydrodynamic equilibrium conditions was confirmed by the results of laboratory experimental studies (KACZMAREK et al. 2011, SAWCZYŃSKI 2012). This linear relationship makes it possible to use the numerical scheme of the first order „upwind" the finite difference method to solve the proposed equation describing the changes of the bathymetry in time and space (SAWCZYŃSKI 2012):

$$\frac{\partial z_b}{\partial t} + \frac{1}{1 - n_p} \frac{\partial q_x}{\partial x} = 0 \tag{10}$$

– ensuring an accurate solution. At the same time, the following relation applies:

$$z_b(x, t + dt) = z_b(x, t) + \frac{\partial z_m}{\partial t} dt \tag{11}$$

It should be noted now, that generally valid relationship between the resultant (during the wave period) sediment transport rate $q_{(x)}$ and the elevation of the bottom level z_b is a non-linear relationship. In case of non-linear, the use of the numerical scheme of the first order „upwind" finite difference method leads to inaccurate solution of equation (1), as this diagram is responsible for introducing maximal numerical diffusion to the solution (SAWCZYŃSKI 2012). Moreover, handling the resultant (within the wave period duration) sediment transport rate means that in case of symmetrical waving (sinusoidal waving) it leads to the situation where as a result of bottom morphology calculations, no bathymetry changes are possible to be obtained which after all does not correspond to reality (SAWCZYŃSKI 2012). In the case, where the resultant stream of the sediments transport flow changes the direction in the space, then there are difficulties with the proper formulation of issue, associated with the appropriate preset of boundary conditions.

Instances of the above problems are possible to be avoided in case of the proposed approach – the use of the decomposition sediment transport direction towards the direction associated with the phase of the wave crest and through, as well as through the implementation of the hydrodynamic equilibrium concept. The proposed method allows for a clear definition of the initial-boundary conditions and makes it possible to apply the numerical scheme not distorted by numerical errors. When making calculation based on an authors concept, as in reality – it is possible to obtain changes in bathymetry even when the resultant (within the wave period duration) stream of sediment transport flow is zero and the wave propagates over the sloped bottom (SAWCZYŃSKI 2012).

Postulate of the linear relationship (6) between the sediment transport rate and the thickness of densely packed grains of sediment in motion also allows for the introduction of the mathematic description of sediments grain size changeability in time and space into the calculations of bottom morphology changes. As it has been shown by SAWCZYŃSKI (2012), the impact of change of grain size of bottom sediment is crucial for the description of changes in bathymetry in the coastal zone of the sea. Moreover, the knowledge of grain size may be crucial, e.g. in planning and conducting silting works associated with artificial edges supply. Resilting material drawn from the water lanes is often used to reinforce the edge in the vicinity of ports. In this case, the knowledge of the grain size distributions of resilting material is necessary, as for example the use of very fine sand can make the operation of strengthening the bank useless and unprofitable.

Comparison of the results of calculations performed with the use of the authors model and results of laboratory experiments is presented in studies: KACZMAREK et al. (2011), SAWCZYŃSKI (2012), SAWCZYŃSKI et al. (2013). These comparisons became to be very impressive, including both changes in bottom morphology as well as changes in bottom sediment grain size distributions.

It should be emphasized that the postulate of the linear relationship (6) between sediment transport rate q_x and the thickness of sediment grains z_m in motion and closely adjacent to each other does not correspond to postulate of a linear relationship between the sediment transport rate q_x and the elevation of the bottom level z_b. The last relationship is still postulated as non-linear.

Conclusions

This paper presents the problems of modeling bathymetry changes. It indicated the difficulties associated with the solution of the nonlinear transport equation, commonly used to describe the changes in bathymetry in the coastal zone of the sea. Numerical schemes, used to resolve the non-linear equation generate oscillations in the results of the calculations or they are unstable schemes. In order to eliminate or control the oscillations, it is usually necessary to use the solution of the numerical diffusion that causes smoothing the solution what is in turn related with the decrease of solution accuracy. Computational models, based on non-linear relationship between the sediment transport rate and the elevation of the bottom level are not able to describe the physical processes occurring in the coastal zone.

In this paper, it is proposed a linear relationship between sediments transport rate and thickness of the layer of sediment grains, closely adjacent to each other and staying in motion. The method proposed by authors allows to precisely define the initial – boundary conditions and to apply the numerical scheme of finite difference method of „upwind" at the accuracy of the I-st order, not distorted by numerical errors. This scheme is used to solve the equations describing the bottom bathymetry changes in time and space. The possibility to resolve the transport equation concerning the above numerical scheme, resulting from the use of authors approach, make it possible to implement (to the description of bottom morphological changes) the simultaneous changes, occurring in distributions of bottom sediment grain size, which is so far unique matter in the literature. It should be emphasized that the authors concept of the linear relationship between sediment transport rate and the thickness of sediment grains in motion and closely adjacent to each other does not correspond to postulate of a linear relationship between the sediment transport rate and the elevation of the bottom level. The last relationship is still postulated as non-linear in modelling of bathymetry changes.

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