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Critical infrastructure operation process related to operating environment threats and extreme weather hazards

Keywords

critical infrastructure, operation, prediction, environment threats, extreme weather hazards

Abstract

Considering a significant influence of the critical infrastructure operating environment threats on its operation process and safety, more precise and convergent to reality model of the critical infrastructure operation process related to critical infrastructure operating environment threats is built. The method of defining the parameters of this operation process is presented and a new procedure of their determining in the case when the critical infrastructure operating threats are not explicit separated in this process is proposed.

1. Introduction

Considering a significant influence of the critical infrastructure operating environment threats on its operation process and safety, more precise and convergent to reality model of the critical infrastructure operation process related to critical infrastructure operating environment threats is built. The method of defining the parameters of this operation process is presented and a new procedure of their determining in the case when the critical infrastructure operating threats are not explicit separated in this process is proposed.

The climate-weather change process for the critical infrastructure operating area is considered and its states are introduced. The semi-Markov process is used to construct a general probabilistic model of the climate-weather change process for the critical infrastructure operating area. To build this model the vector of probabilities of the climate-weather change process staying at the initials climate-weather states, the matrix of probabilities of the climate-weather change process transitions between the climate-weather states, the matrix of conditional distribution functions and the matrix of conditional density functions of the climate-weather change process conditional sojourn times at the climate-weather states are defined.

Further, these two precesses are joined into a general model of the critical infrastructure operation process including operating environmnt threats (OET) related to climate eather change process including extreme weather hazards (EWH).

The operation process of a critical infrastructure including operating environment threats often has significant influence on its safety. Also, a critical infrastructure operating environment area climate-weather conditions are essential in its safety analysis. Usually, the critical infrastructure operation process and the climate-weather conditions at its operating area interact and have either an explicit or an implicit strong joint influence on the critical infrastructure safety. Thus, considering together those two processes influence on the critical infrastructure safety is of grate practical value.

To construct a joint model of those two processes, first, the semi-Markov approaches to a critical infrastructure operation process including operating environment threats modeling and to climate-weather change process are separately developed. Next, those two separate models are linked into a general joint model of a critical infrastructure operation process including operating environment threats and related to the climate-weather change process including extreme hazards is build.

2. Critical infrastructure operation process including operating environment threats – modelling

2.1. Semi-markov model of critical infrastructure operation process including operating environment threats

We assume that the critical infrastructure [EU-CIRCLE Report D2.1-GMU4, 2016] operation process modelled in Section 3.2 can be affected by a number γ, $\gamma \in N$, of unnatural threats coming from the critical infrastructure operating environment and mark them by

$$ut_i,\ i = 1,2,...,\gamma.$$

We define new operation states considering the critical nfrastructure operating environment threats as follows:

- the operation states without including operating environment threats

$$z_i^{'} = z_i,\ i = 1,2,...,\nu,\ \nu \in N; \quad (1)$$

- the operation states including at least 1 and maximum w of operating environment threats

$$z_i^{'},\ i = \nu+1, \nu+2,...,\nu^{'},\ \nu^{'} \in N. \quad (2)$$

This way, we can have:

$$-\nu \cdot \binom{\gamma}{0} = \nu \quad (3)$$

operation states without including operating environment threats ut_i, $i = 1,2,...,\gamma$;

$$-\nu \cdot \binom{\gamma}{1} = \nu\gamma \quad (4)$$

operation states including 1 of the operating environment threats ut_i, $i = 1,2,...,\gamma$;

$$-\nu \cdot \binom{\gamma}{2} = \nu\gamma(\gamma-1)/2 \quad (5)$$

operation states including different 2 of the operating environment threats ut_i, $i = 1,2,...,\gamma$;

...;

$$-\nu \cdot \binom{\gamma}{\gamma} = \nu \quad (6)$$

operation states including all w operating environment threats ut_i, $i = 1,2,...,\gamma$.

Thus, considering (2.1)-(2.6) [EU-CIRCLE Report D2.1-GMU4, 2016], the maximum value of the number of new operation states is

$$\nu' = \nu \cdot [\binom{\gamma}{0} + \binom{\gamma}{1} + ... + \binom{\gamma}{\gamma}] = \nu \cdot 2^{\gamma}. \quad (7)$$

Practically most comfortable numeration of the operation states of the critical infrastructure operation process including its operating environment threats is as follows:

- the operation states without including operating environment threats by

$$z_i^{'} = z_1 \text{ for } i = 1,\ z_i^{'} = z_2$$
$$\text{for } i = 2^{\lambda} + 1,\ ...,\ z_i^{'} = z_{\nu}$$
$$\text{for } i = (\nu - 1)2^{\gamma} + 1; \quad (8)$$

- the operation states including state z_1 and successively 1, 2 until γ operating environment threats ut_i, $i = 1,2,...,\gamma$, by

$$z_i^{'},\ i = 2,\ ...,\ 2^{\gamma}, \quad (9)$$

- the operation states including state z_2 and successively 1, 2 until γ operating environment threats ut_i, $i = 1,2,...,\gamma$, by

$$z_i^{'},\ i = 2^{\gamma} + 2,\ ...,\ 2 \cdot 2^{\gamma}, \quad (10)$$

. . . ;

- the operation states including state z_{ν} and successively 1, 2 until w operating environment threats ut_i, $i = 1,2,...,\gamma$, by

$$z_i^{'},\ i = (\nu - 1)2^{\gamma} + 2,\ ...,\ \nu \cdot 2^{\gamma}. \quad (11)$$

In the case if operating environment threats are disjoint, the number of new operation states is

$$\nu' = \nu(\gamma + 1),$$

and their numeration is as follows:

- the operation states without including operating environment threats by

$$z_i^{'} = z_1 \text{ for } i = 1,\; z_i^{'} = z_2$$
$$\text{for } i = \gamma + 1, \; \ldots, \; z_i^{'} = z_\nu$$
$$\text{for } i = (\nu - 1)(\gamma + 1) + 1; \tag{12}$$

- the operation states including state z_1 and single successive operating environment threats ut_i, $i = 1,2,...,\gamma,$ by

$$z_i^{'}, \; i = 2, \; \ldots, \; \gamma + 1, \tag{13}$$

- the operation states including state z_2 and single successive operating environment threats ut_i, $i = 1,2,...,\gamma,$ by

$$z_i^{'}, \quad i = (\gamma + 1) + 2, \; \ldots, \; 2(\gamma + 1), \tag{14}$$

. . . ;

- the operation states including state z_ν and single successive operating environment threats ut_i, $i = 1,2,...,\gamma,$ by

$$z_i^{'}, \quad i = (\nu - 1)(\gamma + 1) + 2, \; \ldots, \; \nu(\gamma + 1). \tag{15}$$

In our further considerations, we assume that, the critical infrastructure during its operation process can take ν', $\nu' \in N$, defined above, by (8)-(11) or by (12)-(15) in a particular case of disjoint operating environment threats, different operation states

$$z_1^{'}, z_2^{'}, \ldots, z_\nu^{'}, z_{\nu+1}^{'}, \; \ldots, \; z_{\nu'}^{'}. \tag{16}$$

Further, we define the critical infrastructure new operation process $Z'(t)$, $t \in <0,+\infty)$, related to the critical infrastructure operating environment threats with discrete operation states from the set $\{z'_1, z'_2, \ldots, z'_{\nu'}\}$. Moreover, we assume that the critical infrastructure operation process $Z'(t)$ related to its operating environment threats is a semi-Markov process with the conditional sojourn times θ'_{bl} at the operation states z'_b when its next operation state is z'_l, $b, l = 1,2,...,\nu'$, $b \neq l$.

Under these assumptions, the critical infrastructure operation process may be described by:

- the vector of the initial probabilities

$$p'_b(0) = P(Z'(0) = z'_b), \; b = 1,2,...,\nu', \tag{17}$$

of the critical infrastructure operation process $Z'(t)$ staying at particular operation states at the moment $t = 0$

$$[p'_b(0)]_{1\text{x}\nu'} = [p'_1(0), p'_2(0), ..., p'_{\nu'}(0)]; \tag{18}$$

- the matrix of probabilities

$$p'_{bl}, \; b, l = 1,2,...,\nu', \tag{19}$$

of the critical infrastructure operation process $Z'(t)$ transitions between the operation states z'_b and z'_l

$$[p'_{bl}]_{\nu'\text{x}\nu'} = \begin{bmatrix} p'_{11} \; p'_{12} \cdots p'_{1\nu'} \\ p'_{21} \; p'_{22} \cdots p'_{2\nu'} \\ \cdots \\ p'_{\nu'1} \; p'_{\nu'2} \cdots p_{\nu'\nu'} \end{bmatrix}, \tag{20}$$

where by formal agreement

$$p'_{bb} = 0 \text{ for } b = 1,2,...,\nu';$$

- the matrix of conditional distribution functions

$$H'_{bl}(t) = P(\theta'_{bl} < t), \; b, l = 1,2,...,\nu', \tag{21}$$

of the critical infrastructure operation process $Z'(t)$ conditional sojourn times θ'_{bl} at the operation states

$$[H'_{bl}(t)]_{\nu'\text{x}\nu'} = \begin{bmatrix} H'_{11}(t) \; H'_{12}(t) \ldots H'_{1\nu'}(t) \\ H'_{21}(t) \; H'_{22}(t) \ldots H'_{2\nu'}(t) \\ \cdots \\ H'_{\nu'1}(t) \; H'_{\nu'2}(t) \ldots H'_{\nu'\nu'}(t) \end{bmatrix}, \tag{22}$$

where by formal agreement

$$H'_{bb}(t) = 0 \text{ for } b = 1,2,...,\nu'.$$

We introduce the matrix of the conditional density functions

$$h'_{bl}(t), \quad b, l = 1,2,...,\nu',$$

of the critical infrastructure operation process $Z'(t)$ conditional sojourn times θ'_{bl} at the operation states corresponding to the conditional distribution functions $H'_{bl}(t)$

$$[h'_{bl}(t)]_{v' \mathrm{x} v'} = \begin{bmatrix} h'_{11}(t)\, h'_{12}(t) \ldots h'_{1v'}(t) \\ h'_{21}(t)\, h'_{22}(t) \ldots h'_{2v'}(t) \\ \ldots \\ h'_{v'1}(t)\, h'_{v'2}(t) \ldots h'_{v'v'}(t) \end{bmatrix}, \quad (23)$$

where

$$h'_{bl}(t) = \frac{d}{dt}[H'_{bl}(t)] \text{ for } b,l = 1,2,\ldots,v',$$

and by formal agreement

$$h'_{bb}(t) = 0 \text{ for } b = 1,2,\ldots,v'.$$

We assume that the suitable and typical distributions suitable to describe the critical infrastructure operation process *Z'*(*t*) conditional sojourn times θ'_{bl} , $b,l = 1,2,\ldots,v'$, $b \neq l$, in the particular operation states are of the same kind as that listed in Section 2.1 [EU-CIRCLE Report D3.3-GMU3-CIOP Model1, 2016] for the critical infrastructure operation process *Z*(*t*) conditional sojourn times θ_{bl}, eventually with different parameters they are dependent on.

3. Critical infrastructure operation process including operating environment threats - prediction

3.1. Prediction of critical infrastructure operation process characteristics including operating environment threats

Assuming that we have identified the unknown parameters of the critical infrastructure operation process including operating environment threats semi-Markov model:

– the initial probabilities $p'_b(0)$, $b = 1,2,\ldots,v'$, of the critical infrastructure operation process staying at the particular state z'_b at the moment $t = 0$;

– the probabilities p'_{bl}, $b,l = 1,2,\ldots,v'$, $b \neq l$, of the critical infrastructure operation process transitions from the operation state z'_b into the operation state state z'_l;

– the distributions of the critical infrastructure operation process conditional sojourn times θ'_{bl}, $b,l = 1,2,\ldots,v'$, $b \neq l$, at the particular operation states and their mean values $M'_{bl} = E[\theta'_{bl}]$, $b,l = 1,2,\ldots,v'$, $b \neq l$;

we can predict this process basic characteristics.

As the mean values of the conditional sojourn times θ'_{bl}, are given by

$$M'_{bl} = E[\theta'_{bl}] = \int_0^\infty t\,dH'_{bl}(t) = \int_0^\infty t h'_{bl}(t) \quad (24)$$
$$b,l = 1,2,\ldots,v',\ b \neq l,$$

then for the distinguished distributions (2.5)-(2.11) [Kolowrocki, Soszynska-Budny, 2011], the mean values of the system operation process $Z'(t)$ conditional sojourn times θ'_{bl}, $b,l = 1,2,\ldots,v'$, $b \neq l$, at the particular operation states can be found similarly as in Section …..

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times θ'_b, $b = 1,2,\ldots,v'$, of the system operation process $Z'(t)$ at the operation states z'_b, $b = 1,2,\ldots,v'$, are given by [3, 5, 6, 10, 12]

$$H'_b(t) = \sum_{l=1}^{v'} p'_{bl} H'_{bl}(t),\ b = 1,2,\ldots,v', \quad (25)$$

Hence, the mean values $E[\theta'_b]$ of the system operation process $Z'(t)$ unconditional sojourn times θ'_b, $b = 1,2,\ldots,v'$, at the operation states are given by

$$M'_b = E[\theta'_b] = \sum_{l=1}^{v'} p'_{bl} M'_{bl},\ b = 1,2,\ldots,v', \quad (26)$$

where M'_{bl} are defined by the formula (24) in a case of any distribution of sojourn times θ_{bl} and by the formulae (2.13)-(2.19) in the cases of particular defined respectively by (2.5)-(2.11) [Kolowrocki, Soszynska-Budny, 2011], distributions of these sojourn times.

The limit values of the system operation process $Z(t)$ transient probabilities at the particular operation states

$$p'_b(t) = P(Z'(t) = z'_b),\ t \in <0,+\infty), \quad (27)$$
$$b = 1,2,\ldots,v',$$

are given by [Kolowrocki, Soszynska-Budny, 2011],

$$p'_b = \lim_{t\to\infty} p'_b(t) = \frac{\pi_b M'_b}{\sum_{l=1}^{v'} \pi_l M'_l},\ b = 1,2,\ldots,v', \quad (28)$$

where M'_b, $b = 1,2,\ldots,v'$, are given by (26), while the steady probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases}[\pi_b]=[\pi_b][p_{bl}]\\ \sum_{l=1}^{v}\pi_l=1.\end{cases} \quad (29)$$

In the case of a periodic system operation process, the limit transient probabilities p'_b, $b=1,2,...,\nu'$, at the operation states defined by (28), are the long term proportions of the system operation process $Z'(t)$ sojourn times at the particular operation states z'_b, $b=1,2,...,\nu'$.

Other interesting characteristics of the system operation process $Z'(t)$ possible to obtain are its total sojourn times $\hat{\theta}'_b$ at the particular operation states z'_b, z_b, $b=1,2,...,\nu'$, during the fixed system opetation time. It is well known [Kolowrocki, Soszynska-Budny, 2011], that the system operation process total sojourn times $\hat{\theta}'_b$ at the particular operation states z'_b, for sufficiently large operation time θ, have approximately normal distributions with the expected value given by

$$\hat{M}'_b=E[\hat{\theta}'_b]=p'_b\,\theta,\ b=1,2,...,\nu', \quad (30)$$

where p'_b are given by (28).

4. Climate-weather change process including extreme weather hazards – modelling

4.1. Semi-Markov model of climate-weather change process including extreme weather hazards

To model the climate-weather change process for the critical infrastructure operating area we assume that the climate-weather in this area is taking w, $w \in N$, different climate-weather states $c_1, c_2,\dots, c_w$. Further, we define the climate-weather change process $C(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{c_1, c_2,\dots, c_w\}$. Assuming that the climate-weather change process $C(t)$ is a semi-Markov process it can be described by:

– the number of climate-weather states w, $w \in N$,

– the vector

$$[q_b(0)]_{1Xw}=[q_1(0), q_2(0),\dots, q_w(0)] \quad (31)$$

of the initial probabilities

$q_b(0) = P(C(0) = c_b)$, $b = 1,2,\dots, w$,

of the climate-weather change process $C(t)$ staying at particular climate-weather states c_b at the moment $t = 0$;

– the matrix

$$[q_{bl}]_{wxw}=\begin{bmatrix}q_{11}\,q_{12}\dots q_{1w}\\ q_{21}\,q_{22}\dots q_{2w}\\ \dots\\ q_{w1}\,q_{w2}\dots q_{ww}\end{bmatrix} \quad (32)$$

of the probabilities of transitions q_{bl}, b, $l = 1,2,\dots, w$, $b \neq l$, of the climate-weather change process $C(t)$ from the climate-weather states c_b to c_l, where by formal agreement

$q_{bb} = 0$ for $b = 1,2,\dots, w$;

– the matrix

$$[C_{bl}(t)]_{wxw}=\begin{bmatrix}C_{11}(t)\,C_{12}(t)\dots C_{1w}(t)\\ C_{21}(t)\,C_{22}(t)\dots C_{2w}(t)\\ \dots\\ C_{w1}(t)\,C_{w2}(t)\dots C_{ww}(t)\end{bmatrix}$$

of the conditional distribution functions

$C_{bl}(t) = P(C_{bl} < t)$, b, $l = 1,2,\dots, w$,

of the conditional sojourn times C_{bl} at the climate-weather states c_b when its next climate-weather state is c_l, b, $l = 1,2,\dots, w$, $b \neq l$, where by formal agreement

$C_{bb}(t) = 0$ for $b = 1,2,\dots, w$,

or equivalently the matrix

$$[c_{bl}(t)]_{wxw}=\begin{bmatrix}c_{11}(t)\,c_{12}(t)\dots c_{1w}(t)\\ c_{21}(t)\,c_{22}(t)\dots c_{2w}(t)\\ \dots\\ c_{w1}(t)\,c_{w2}(t)\dots c_{ww}(t)\end{bmatrix} \quad (34)$$

of the conditional density functions of the climate-weather change process $C(t)$ conditional sojourn times C_{bl} at the climate-weather states corresponding to the conditional distribution functions $C_{bl}(t)$, where

$$c_{bl}(t)=\frac{d}{dt}[c_{bl}(t)] \quad (35)$$

for b, $l = 1,2,\dots, w, b \neq l$,

and by formal agreement

$c_{bb}(t) = 0$ for $b = 1,2,\dots, w$.

We assume that the suitable and typical distributions suitable to describe the climate-weather change process $C(t)$ conditional sojourn times C_{bl}, b, l = 1,2,…, w, $b \neq l$, at the particular climate-weather states given by (4.5)-(4.12) [Kolowrocki, Soszynska-Budny, 2011].

5. Climate-weather change process including extreme weather hazards – prediction

5.1. Prediction of climate-weather process including extreme weather hazards characteristics

Assuming that we have identified the unknown parameters of the climate-weather change process semi-Markov model:
– the initial probabilities $q_b(0)$, b = 1,2,…,w, of the climate-weather change process staying at the particular state c_b at the moment $t = 0$;
– the probabilities q_{bl}, b, l = 1,2,…,w, $b \neq l$, of the climate-weather change process transitions from the climate-weather state c_b into the climate-weather state c_l;
– the distributions of the climate-weather change process conditional sojourn times C_{bl}, b, l = 1,2,…,w, $b \neq l$, at the particular climate-weather states and their mean values $M_{bl} = E[C_{bl}]$, b, l = 1,2,…,w, ;
we can predict this process basic characteristics.

As the mean values of the conditional sojourn times C_{bl} are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$N_{bl} = E[C_{bl}] = \int_0^\infty t dC_{bl}(t) = \int_0^\infty t c_{bl}(t) dt, \quad (36)$$
$$b,l = 1,2,...,w, \ b \neq l,$$

then for the distinguished distributions (4.5)-(4.12) [Kolowrocki, Soszynska-Budny, 2011], the mean values of the climate-weather change process $C(t)$ conditional sojourn times C_{bl}, $b,l = 1,2,...,v$, $b \neq l$, at the particular operation states are respectively given by (4.14)-(4.21) [Kolowrocki, Soszynska-Budny, 2011].
From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times C_b, $b = 1,2,...,w$, of the climate-weather change process $C(t)$ at the climate-weather states c_b, $b = 1,2,...,w$, are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$C_b(t) = \sum_{l=1}^{v} q_{bl} C_{bl}(t), \ b = 1,2,...,w. \quad (37)$$

Hence, the mean values $E[C_b]$ of the climate-weather change process $C(t)$ unconditional sojourn times C_b, $b = 1,2,...,w$, at the climate-weather states are given by

$$N_b = E[C_b] = \sum_{l=1}^{v} q_{bl} N_{bl}, \ b = 1,2,...,w, \quad (38)$$

where N_{bl} are defined by the formula (36) in a case of any distribution of sojourn times C_{bl} and by the formulae (37)-(38) in the cases of particular defined respectively by (4.14)-(4.21) distributions of these sojourn times.
The limit values of the climate-weather change process $C(t)$ transient probabilities at the particular operation states

$$q_b(t) = P(C(t) = c_b), \ t \in <0,+\infty), \quad (39)$$
$$b = 1,2,...,w,$$

are given by [Kołowrocki, Soszyńska-Budny, 2016]

$$q_b = \lim_{t\to\infty} q_b(t) = \frac{\pi_b N_b}{\sum_{l=1}^{w} \pi_l N_l}, \ b = 1,2,...,w, \quad (40)$$

where N_b, $b = 1,2,...,w$, are given by (5.3), while the steady probabilities π_b of the vector $[\pi_b]_{1xw}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][q_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \quad (41)$$

In the case of a periodic climate-weather change process, the limit transient probabilities q_b, $b = 1,2,...,w$, at the climate-weather states defined by (40), are the long term proportions of the climate-weather change process $C(t)$ sojourn times at the particular climate-weather states c_b, $b = 1,2,...,w$.
Other interesting characteristics of the system climate-weather change process $C(t)$ possible to obtain are its total sojourn times $\hat{C}_b$ at the particular climate-weather states c_b, $b = 1,2,...,w$, during the fixed time. It is well known [Kołowrocki, Soszyńska-Budny, 2011] that the climate-weather change process total sojourn times $\hat{C}_b$ at the particular climate-weather states c_b, for sufficiently

large time θ, have approximately normal distributions with the expected value given by

$$\hat{N}_b = E[\hat{C}_b] = q_b\theta, \ b = 1,2,...,w, \quad (42)$$

where q_b are given by (40).

6. Critical infrastructure operation process related to operating environment threats and extreme weather hazard - modelling

We assume, as in Section 3.2, that the critical infrastructure operation process including operating environment threats is taking $\nu', \nu' \in N$, different operation states $z_{,1}, z'_2, \ldots, z'_{\nu'}$. Further, we define the critical infrastructure operation process including operating environment threats $Z'(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{z'_1, z'_2, \ldots, z'_{\nu'}\}$. Moreover, we assume that the critical infrastructure operation process $Z(t)$ is a semi-Markov process that can be described by:

– the vector $[p'_b(0)]_{1\times\nu'}$ of the initial probabilities $p'_b(0)$, $b = 1,2,...,\nu'$, of the critical infrastructure operation process staying at the particular state z'_b at the moment $t = 0$;

– the matrix $[p'_{bl}]_{\nu'\times\nu'}$ of the probabilities p'_{bl}, $b,l = 1,2,...,\nu'$, $b \neq l$, of the critical infrastructure operation process transitions from the operation state z'_b into the operation state state z'_l;

– the matrix $[H'_{bl}(t)]_{\nu'\times\nu'}$ of the distributions of the critical infrastructure operation process conditional sojourn times θ'_{bl}, $b,l = 1,2,...,\nu'$, $b \neq l$, at the particular operation states and the matrix $[M'_{bl}]_{\nu'\times\nu'}$ of their mean values $M'_{bl} = E[\theta'_{bl}]$, $b,l = 1,2,...,\nu'$, $b \neq l$.

Moreover, as in Section 4, we assume that the climate-weather change process $C(t)$, $t \in <0,+\infty)$, at the critical infrastructure operating area is taking w, $w \in N$, different climate-weather states $c_1, c_2, \ldots, c_w$. Further, we assume that the climate-weather change process $C(t)$ is a semi-Markov process and it can be described by:

– the vector $[q_b(0)]_{1\times w}$ of the initial probabilities $q_b(0)$, $b = 1,2,\ldots, w$, of the climate-weather change process $C(t)$ staying at particular climate-weather states c_b, $b = 1,2,\ldots, w$, at the moment $t = 0$;

– the matrix $[q_{bl}]_{w\times w}$ of the probabilities q_{bl}, $b, l = 1,2,...,w$, of transitions of the climate-weather change process $C(t)$ from the climate-weather states c_b to the climate-weather state c_l, $b, l = 1,2,\ldots, w$;

– the matrix $[C_{bl}(t)]_{w\times w}$ of the conditional distribution functions $C_{bl}(t)$, $b,l = 1,2,\ldots, w$, of the conditional sojourn times C_{bl} at the climate-weather states c_b when its next climate-weather state is c_l, b, $l = 1,2,\ldots, w$, the matrix $[N_{bl}]_{w\times w}$ of their mean values $N_{bl} = E[C_{bl}]$, $b,l = 1,2,...,w, b \neq l$.

6.1. Joint model of independent critical infrastructure operation process related to operating environment threats and extreme weather hazard

Under the assumption that the critical infrastructure operation process $Z'(t)$, $t \in <0,+\infty)$, and the climate-weather change process $C(t)$ are independent, we introduce the joint process of critical infrastructure operation process including operaring environment threats and climate-weather change process including extreme weather hazaerds called the critical infrastructure operation process related to operating environment threats and climate-weather hazards marked by

$$Z'C(t), t \in <0,+\infty), \quad (43)$$

and we assume that it can take $\nu' w, \nu, w \in N$, different operation states

$$z'c_{ij}, \ i = 1,2,...,\nu', \ j = 1,2,...,w. \quad (44)$$

We assume that the critical infrastructure operation process related to operating environment threats and climate-weather hazards $Z'C(t)$, at the moment $t \in <0,+\infty)$, is at the state $z'c_{ij}$, $i = 1,2,...,\nu$, $j = 1,2,...,w$, if and only if at that moment, the operation process $Z'(t)$ is at the operation states z'_i, $i = 1,2,...,\nu'$, and the climate-weather change process $C(t)$ is at the climate-weather state c'_j, $j = 1,2,...,w$, what we mark as follows:

$$(Z'C(t) = z'c_{ij}) \Leftrightarrow (Z'(t) = z'_i \cap C(t) = c_j), \quad (45)$$
$$t \in <0,+\infty), \ i = 1,2,...,\nu', \ j = 1,2,...,w.$$

Further, we define the initial probabilities

$$p'q_{ij}(0) = P(Z'C(0) = z'c_{ij}), \ i = 1,2,...,', \quad (46)$$
$$j = 1,2,...,w,$$

of the critical infrastructure operation process related to operating environment threats and climate-weather hazards $Z'C(t)$, at the initial moment $t = 0$ at the operation and climate-weather state $z'c_{ij}$,

$i = 1,2,...,\nu'$, $j = 1,2,...,w$, and this way we have the vector

$$[p'q_{ij}(0)]_{1x\nu'w}$$

$$= \begin{bmatrix} p'q_{11}(0), p'q_{12}(0), \ldots, p'q_{1w}(0); \\ p'q_{21}(0), p'q_{22}(0), \ldots, p'q_{2w}(0); \\ \ldots; p'q_{\nu'1}(0), p'q_{\nu'2}(0), \ldots p'q_{\nu'w}(0) \end{bmatrix} \quad (47)$$

of the initial probabilities the critical infrastructure operation process related to operating environment threats and climate-weather hazards $Z'C(t)$ staying at the particular operation and climate-weather state at the initial moment $t = 0$.
From the assumption that the critical infrastructure operation process $Z'(t)$ and climate-weather change process $C(t)$ are independent, it follows that

$$p'q_{ij}(0) = P(Z'C(0) = z'c_{ij})$$
$$= P(Z'(0) = z'_i \cap C(0) = c_j)$$
$$= P(Z'(0) = z'_i) \cdot P(C(0) = c_j)$$
$$= p'_i(0) \cdot q_j(0), \; i = 1,2,...,\nu', \; j = 1,2,...,w, \quad (48)$$

where $p'_i(0)$, $i = 1,2,...,\nu'$, and $q_j(0)$, $j = 1,2,...,w$, are respectively defined in Section 3 and Section 4.
Hence, the vector of the initial probabilities the critical infrastructure operation process related to operating environment threats and climate-weather hazards $Z'C(t)$ defined by (6.47) takes the following form

$$[p'q_{ij}(0)]_{1x\nu'w} = [p'_i(0)q_j(0)]_{1x\nu'w}$$

$$= \begin{bmatrix} p'_1(0)q_1(0), p'_1(0)q_2(0), \\ \ldots p'_1(0)q_w(0); \ldots; \\ p'_{\nu'}(0)q_1(0), p'_{\nu'}(0)q_2(0), \ldots, \\ p'_{\nu'}(0)q_w(0) \end{bmatrix}. \quad (49)$$

Further, we introduce the probabilities

$$p'q_{ij\,kl}, \; i = 1,2,...,\nu', \; j = 1,2,...,w, \; k = 1,2,...,\nu', \; l = 1,2,...,w, \quad (50)$$

of the transitions of the critical infrastructure operation process related to operating environment threats and climate-weather hazards $Z'C(t)$ between the operation states

$$z'c_{ij} \text{ and } z'c_{kl}, \; i = 1,2,...,\nu', \; j = 1,2,...,w, \; k = 1,2,...,\nu', \; l = 1,2,...,w, \quad (51)$$

and get their following matrix form

$$[p'q_{ij\,kl}]_{\nu'wx\nu'w}$$
$$= \begin{bmatrix} p'q_{1111} \; p'q_{1112} \ldots p'q_{111w}; p'q_{1121} p'q_{1122} \ldots p'q_{112w}; \ldots; p'q_{11\nu'1} p'q_{11\nu'2} \ldots p'q_{11\nu'w} \\ p'q_{1211} \; p'q_{1212} \ldots p'q_{121w}; p'q_{1221} p'q_{1222} \ldots p'q_{122w}; \ldots; p'q_{12\nu'1} p'q_{12\nu'2} \ldots p'q_{12\nu'w} \\ \ldots \\ p'q_{\nu'w11} \; p'q_{\nu'w12} \ldots p'q_{\nu'w1w}; p'q_{\nu'w21} p'q_{\nu'w22} \ldots p'q_{\nu'w2w}; \ldots; p'q_{\nu'w\nu'1} p'q_{\nu'w\nu'2} \ldots p'q_{\nu'w\nu'w} \end{bmatrix}. \quad (52)$$

From the assumption that the critical infrastructure operation process $Z(t)$ and climate-weather change process $C(t)$ are independent, it follows that

$$p'q_{ij\,kl} = p'_{ik} q_{jl}, \; i = 1,2,...,\nu', \; j = 1,2,...,w, \; k = 1,2,...,\nu', \; l = 1,2,...,w, \quad (53)$$

where

$$p'_{ik}, \; i = 1,2,...,\nu', \; k = 1,2,...,\nu', \text{ and } q_{jl}, \; j = 1,2,...,w, \; l = 1,2,...,w, \quad (54)$$

are respectively defined in Section 2.1 and Section 4.1 in [EU-CIRCLE Report D2.1-GMU4, 2016].

Hence, the matrix of the probabilities of transitions between the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ defined by (6.52) takes the following form

$$[p'q_{ij\,kl}]_{v'wxv'w} = [p'_{ik}\, q_{jl}]_{v'wxv'w}$$

$$= \begin{bmatrix} p'_{11} q_{11}\ p'_{11} q_{12} \cdots p'_{11} q_{1w}; p'_{12} q_{11}\ p'_{12} q_{12} \cdots p'_{12} q_{1w}; \ldots; p'_{1v'} q_{11} p'_{1v'} q_{12} \cdots p'_{1v'} q_{1w} \\ p'_{11} q_{21}\ p'_{11} q_{22} \cdots p'_{11} q_{2w}; p'_{12} q_{21} p'_{12} q_{22} \cdots p'_{12} q_{2w}; \ldots; p'_{1v'} q_{21} p'_{1v'} q_{22} \cdots p'_{1v'} q_{2w} \\ \cdots \\ p'_{v'1} q_{w1}\ p'_{v'1} q_{w2} \cdots p'_{v'1} q_{ww}; p'_{v'2} q_{w1} p'_{v'2} q_{w2} \cdots p'_{v'2} q_{ww}; \ldots; p'_{v'v'} q_{w1} p'_{v'v'} q_{w2} \cdots p'_{v'v'} q_{ww} \end{bmatrix}. \tag{55}$$

The matrix of conditional distribution functions

$$H'C_{ij\,kl}(t) = P(\theta'C_{ij\,kl} < t),\ t \in <0,+\infty),\ i = 1,2,\ldots,v',\ \ j = 1,2,\ldots,w,\ \ k = 1,2,\ldots,v',\ \ l = 1,2,\ldots,w, \tag{56}$$

of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ conditional sojourn times $\theta'C_{ij\,kl}$, $i = 1,2,\ldots,v'$, $j = 1,2,\ldots,w$, $k = 1,2,\ldots,v'$, $l = 1,2,\ldots,w$, at the operation state $z'c_{ij}$, $i = 1,2,\ldots,v'$, $j = 1,2,\ldots,w$, when the next operation state is $z'c_{kl}$, $k = 1,2,\ldots,v'$, $l = 1,2,\ldots,w$, takes the following form

$$[H'C_{ij\,kl}(t)]_{v'wxv'w} = \begin{bmatrix} H'C_{1111}(t) \ldots HC_{111w}(t); H'C_{1121}(t) \ldots H'C_{112w}(t); \ldots; H'C_{11v'1}(t) \ldots H'C_{11v'w}(t) \\ H'C_{1211}(t) \ldots H'C_{121w}(t); H'C_{1221}(t) \ldots H'C_{122w}(t); \ldots; H'C_{12v'1}(t) \ldots H'C_{12v'w}(t) \\ \cdots \\ H'C_{v'w11}(t) \ldots H'C_{v'w1w}(t); H'C_{v'w21}(t) \ldots H'C_{v'w2w}(t); \ldots; H'C_{v'wv'1}(t) \ldots H'C_{v'wv'w}(t) \end{bmatrix} \tag{57}$$

and the matrix of their corresponding conditional density functions

$$h'c_{ij\,kl}(t) = \frac{d}{dt}[H'C_{ij\,kl}(t)] \text{ for } t \in <0,+\infty),\ i = 1,2,\ldots,v',\ \ j = 1,2,\ldots,w,\ \ k = 1,2,\ldots,v',\ \ l = 1,2,\ldots,w, \tag{58}$$

the form

$$[h'c_{ij\,kl}(t)]_{v'wxv'w} = \begin{bmatrix} h'c_{1111}(t) \ldots hc_{111w}(t); h'c_{1121}(t) \ldots h'c_{112w}(t); \ldots; h'c_{11v'1}(t) \ldots h'c_{11v'w}(t) \\ h'c_{1211}(t) \ldots h'c_{121w}(t); h'c_{1221}(t) \ldots h'c_{122w}(t); \ldots; h'c_{12v'1}(t) \ldots h'c_{12v'w}(t) \\ \cdots \\ h'c_{v'w11}(t) \ldots h'c_{v'w1w}(t); h'c_{v'w21}(t) \ldots h'c_{v'w2w}(t); \ldots; h'c_{v'wv'1}(t) \ldots h'c_{v'wv'w}(t) \end{bmatrix}. \tag{59}$$

From the assumption that the critical infrastructure operation process $Z(t)$ and climate-weather change process $C(t)$ are independent, it follows that

$$H'C_{ij\,kl}(t) = P(\theta'C_{ij\,kl} < t) = P(\theta'_{ik} < t \cap C_{jl} < t) = H'_{ik}(t) C_{jl}(t),\ t \in <0,+\infty),$$
$$i = 1,2,\ldots,v',\ \ j = 1,2,\ldots,w,\ \ k = 1,2,\ldots,v',\ \ l = 1,2,\ldots,w, \tag{60}$$

and

$$h'c_{ij\,kl}(t) = \frac{d}{dt}[H'C_{ij\,kl}(t)] = \frac{d}{dt}[H'_{ik}(t) C_{jl}(t)] = h'_{ik}(t) C_{jl}(t) + H'_{ik}(t) c_{jl}(t),\ t \in <0,+\infty),$$

$$i = 1,2,\ldots,\nu', \quad j = 1,2,\ldots,w, \quad k = 1,2,\ldots,\nu', \quad l = 1,2,\ldots,w, \tag{61}$$

where

$$H'_{ik}(t),\ i = 1,2,\ldots,\nu',\ k = 1,2,\ldots,\nu', \text{ and } C_{jl}(t),\ \ j = 1,2,\ldots,w,\ \ l = 1,2,\ldots,w, \tag{62}$$

and

$$h'_{ik}(t),\ \ i = 1,2,\ldots,\nu',\ \ k = 1,2,\ldots,\nu', \text{ and } c_{jl}(t),\ \ j = 1,2,\ldots,w,\ \ l = 1,2,\ldots,w, \tag{63}$$

are respectively defined in Chapter 3, and Chapter 4 [Kolowrocki, Soszynska-Budny, 2011],.
Hence, the matrix of the conditional distribution functions and the matrix of the conditional density functions of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $ZC(t)$ conditional sojourn times defined by (57) and (59) respectively take the following forms

$$[H'C_{ij\,kl}(t)]_{\nu'wx\nu'w} = [H'_{ik}(t)C_{jl}(t)]_{\nu'wx\nu'w}$$

$$= \begin{bmatrix} H'_{11}(t)C_{11}(t)\ H'_{11}(t)C_{12}(t)\ldots H'_{11}(t)C_{1w}(t);\ldots;H'_{1\nu'}(t)C_{11}(t)H'_{1\nu'}(t)C_{12}(t)..H'_{1\nu'}(t)C_{1w}(t) \\ H'_{11}(t)C_{21}(t)H'_{11}(t)C_{22}(t)\ldots H'_{11}(t)C_{2w}(t);\ldots;H'_{1\nu'}(t)C_{21}(t)H'_{1\nu'}(t)C_{22}(t)..H'_{1\nu'}(t)C_{1w}(t) \\ \ldots \\ H'_{\nu'1}(t)C_{w1}(t)\ H'_{\nu'1}(t)C_{w2}(t)\ldots H'_{\nu'1}(t)C_{ww}(t);\ldots;H'_{\nu'\nu'}(t)C_{w1}(t)H'_{\nu'\nu'}(t)C_{w2}(t)..H'_{\nu'\nu'}(t)C_{ww}(t) \end{bmatrix} \tag{64}$$

and

$$[h'c_{ij\,kl}(t)]_{\nu'wx\nu'w} = [h'_{ik}(t)C_{jl}(t) + H'_{ik}(t)c_{jl}(t)]_{\nu'wx\nu'w}$$

$$= \begin{bmatrix} h'_{11}(t)C_{11}(t) + H'_{11}(t)c_{11}(t)\ldots h'_{11}(t)C_{1w}(t) + H'_{11}(t)c_{1w}(t);\ldots; \\ h'_{11}(t)C_{21}(t) + H'_{11}(t)c_{21}(t)\ldots h'_{11}(t)C_{2w}(t) + H'_{11}(t)c_{2w}(t);\ldots; \\ \ldots \\ h'_{\nu'1}(t)C_{w1}(t) + H'_{\nu'1}(t)c_{w1}(t)\ldots h'_{\nu'1}(t)C_{ww}(t) + H'_{\nu'1}(t)c_{ww}(t);\ldots; \end{bmatrix}$$

$$\begin{bmatrix} h'_{1\nu'}(t)C_{11}(t) + H'_{1\nu'}(t)c_{11}(t)..h'_{1\nu'}(t)C_{1w}(t) + H'_{1\nu'}(t)c_{1w}(t) \\ h'_{1\nu'}(t)C_{21}(t) + H'_{1\nu'}(t)c_{21}(t)..h'_{1\nu'}(t)C_{1w}(t) + H'_{1\nu'}(t)c_{1w}(t) \\ \ldots \\ h'_{\nu'\nu'}(t)C_{w1}(t) + H'_{\nu'\nu'}(t)c_{w1}(t)..h'_{\nu'\nu'}(t)C_{ww}(t) + H'_{\nu'\nu'}(t)c_{ww}(t) \end{bmatrix} \tag{65}$$

We assume that the suitable and typical distributions suitable to describe the critical infrastructure operation process $Z'(t)$ conditional sojourn times θ'_{bl}, $b,l = 1,2,\ldots,\nu'$, $b \neq l$, in the particular operation states are that defined in [Kołowrocki, Soszyńska-Budny, 2011], [EU-CIRCLE Report D2.1-GMU4-Part1, 2016] and [EU-CIRCLE Report D2.1-GMU4-Part2, 2016]. The suitable and typical distributions suitable to describe the climate-weather change process $C(t)$ conditional sojourn times C_{bl}, b, l = 1,2,…, w, $b \neq l$, at the particular climate-weather states are given by (4.5)-(4.12) [EU-CIRCLE Report D2.1-GMU4-Part2, 2016].

6.2. Joint model of dependent critical infrastructure operation process related to operating environment threats and extreme weather hazards

Under the assumption that the critical infrastructure operation process including operating environment threats $Z'(t)$, $t \in <0,+\infty)$, and the climate-weather change process $C(t)$ including extreme weather hazards are dependent, we introduce the joint process of critical infrastructure operation process and climate-weather change process called the critical infrastructure operation process related to operating environment threats and extreme weather hazards marked by

$$Z'C(t), \ t \in <0,+\infty), \tag{66}$$

and we assume that it can take $\nu' w, \nu, w \in N$, different operation states

$$z'c_{ij}, \quad i = 1,2,\ldots,\nu', \quad j = 1,2,\ldots,w. \tag{67}$$

We assume that the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$, at the moment $t \in <0,+\infty)$, is at the state $z'c_{ij}$, $i = 1,2,\ldots,\nu'$, $j = 1,2,\ldots,w$, if and only if at that moment, the operation process $Z'(t)$ is at the operation states z'_i, $i = 1,2,\ldots,\nu'$, and the climate-weather change process $C(t)$ is at the climate-weather state c_j, $j = 1,2,\ldots,w$, what we mark as follows:

$$= P(C(0) = C_j) \cdot P(Z'(0) = z'_i \mid C(0) = c_j)$$

$$(Z'C(t) = z'c_{ij}) \Leftrightarrow (Z'(t) = z_i \cap C(t) = c_j), \tag{68}$$
$$t \in <0,+\infty), \ i = 1,2,\ldots,\nu', \ j = 1,2,\ldots,w.$$

Further, we define the initial probabilities

$$p'q_{ij}(0) = P(Z'C(0) = z'c_{ij}), \ i = 1,2,\ldots,\nu', \tag{69}$$
$$j = 1,2,\ldots,w,$$

of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$, at the initial moment $t = 0$ at the operation and climate-weather state $z'c_{ij}$, $i = 1,2,\ldots,\nu' \in N$, $j = 1,2,\ldots,w$, and this way we have the vector

$$[p'q_{ij}(0)]_{1x\nu' w}$$

$$= \begin{bmatrix} p'q_{11}(0), p'q_{12}(0), \ldots, p'q_{1w}(0); \\ p'q_{21}(0), p'q_{22}(0), \ldots, p'q_{2w}(0); \\ \ldots; p'q_{\nu'1}(0), p'q_{\nu'2}(0), \ldots, p'q_{\nu'w}(0) \end{bmatrix} \tag{70}$$

of the initial probabilities the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ staying at the particular operation and climate-weather state at the initial moment $t = 0$.

In the case when the processess $Z'(t)$ and $C(t)$ are dependent the initial probabilities existing in (70) can be expresed either by

$$p'q_{ij}(0) = P(Z'C(0) = z'c_{ij}) = P(Z(0) = z'_i \cap C(0) = c_j)$$

where

$$p'_i(0) \ = P(Z'(0) = z'_i),$$
$$= p'_i(0) \cdot q_{j/i}(0), \tag{72}$$
$$i = 1,2,\ldots,\nu',$$

are the initial probabilities of the operation process $Z'(t)$ defined in Chapter 3 and

$$q_{j/i}(0) = P(C(0) = c_j \mid Z'(0) = z'_i), \tag{73}$$
$$j = 1,2,\ldots,w, \qquad i = 1,2,\ldots,\nu',$$

are conditional initial probabilities of the climate-weather change process $C(t)$ defined in Chapter 4 in case they are not conditional or by

$$p'q_{ij}(0) = P(Z'C(0) = z'c_{ij}) = P(Z'(0) = z'_i \cap C(0) = c_j)$$

$$= q_j(0) \cdot p'_{i/j}(0), \quad i = 1,2,\ldots,\nu', \quad j = 1,2,\ldots,w, \tag{74}$$

where

$$q_j(0) \quad = P(C(0) = c_j), \quad j = 1,2,\ldots,w, \tag{75}$$

are initial probabilities of the climate-weather change process $C(t)$ defined in Chapter 4 and

$$p'_{i/j}(0) \ = P(Z'(0) = z'_i \mid C(0) = c_j), \tag{76}$$
$$j = 1,2,\ldots,w, \ i = 1,2,\ldots,\nu',$$

are conditional initial probabilities of the operation process $Z'(t)$ defined in Chapter 3 in case they are not conditional.
Further, we introduce the probabilities

$$p'q_{ij\,kl}, \quad i = 1,2,\ldots,\nu', \quad j = 1,2,\ldots,w, \quad l = 1,2,\ldots,w, \quad k = 1,2,\ldots,\nu', \tag{77}$$

of the transitions of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ between the operation states

$$z'c_{ij} \quad \text{and} \quad z'c_{kl}, \quad i = 1,2,\ldots,\nu', \quad j = 1,2,\ldots,w, \quad k = 1,2,\ldots,\nu', \quad l = 1,2,\ldots,w, \tag{78}$$

and get their following matrix form

$$[p'q_{ij\,kl}]_{\nu'w x \nu'w} = \begin{bmatrix} p'q_{11\,11}\ p'q_{11\,12}\ \ldots\ p'q_{11\,1w};\ p'q_{11\,21}p'q_{11\,22}\ldots p'q_{11\,2w};\ldots;p'q_{11\,\nu'1}p'q_{11\,\nu'2}\ldots p'q_{11\,\nu'w} \\ p'q_{12\,11}\ p'q_{12\,12}\ \ldots\ p'q_{12\,1w};\ p'q_{12\,21}p'q_{12\,22}\ldots p'q_{12\,2w};\ldots;p'q_{12\,\nu'1}p'q_{12\,\nu'2}\ldots p'q_{12\,\nu'w} \\ \ldots \\ p'q_{\nu'w\,11}\ p'q_{\nu'w\,12}\ \ldots\ p'q_{\nu'w\,1w};\ p'q_{\nu'w\,21}p'q_{\nu'w\,22}\ldots p'q_{\nu'w\,2w};\ldots;p'q_{\nu'w\,\nu'1}p'q_{\nu'w\,\nu'2}\ldots p'q_{\nu'w\,\nu'w} \end{bmatrix}. \tag{79}$$

In the case when the processess $Z'(t)$ and $C(t)$ are dependent the probabilities of transitions between the operation states existing in (6.79) can be expresed either by

$$p'q_{ij\,kl} = p'_{ik} \cdot q_{jl/ik}, \quad i = 1,2,\ldots,\nu', \quad j = 1,2,\ldots,w, \quad k = 1,2,\ldots,\nu', \quad l = 1,2,\ldots,w, \tag{80}$$

where

$$p'_{ik}, \quad i = 1,2,\ldots,\nu', \quad k = 1,2,\ldots,\nu', \tag{81}$$

are transient probabilities of the operation process $Z'(t)$ defined in Chapter 3 and

$$q_{jl/ik}, \quad i = 1,2,\ldots,\nu', \quad j = 1,2,\ldots,w, \quad k = 1,2,\ldots,\nu', \quad l = 1,2,\ldots,w, \tag{82}$$

are conditional transient probabilities of the climate-weather change process $C(t)$ defined in Chapter 4 in case they are not conditional or by

$$p'q_{ij\,kl} = q_{jl} \cdot p'_{ik/jl}, \quad i = 1,2,\ldots,\nu', \quad k = 1,2,\ldots,\nu', \quad j = 1,2,\ldots,w, \quad l = 1,2,\ldots,w, \tag{83}$$

where

$$q_{jl}, \quad j = 1,2,\ldots,w, \quad l = 1,2,\ldots,w, \tag{84}$$

are transient probabilities of the climate-weather change process $C(t)$ defined in Chapter 4 and

$$p'_{ik/jl}, \quad i = 1,2,\ldots,\nu', \quad k = 1,2,\ldots,\nu', \quad j = 1,2,\ldots,w, \quad l = 1,2,\ldots,w, \tag{85}$$

are conditional transient probabilities of the operation process $Z'(t)$ defined in Chapter 3 in case they are not conditional.
The matrix of conditional distribution functions

$$H'C_{ij\,kl}(t) = P(\theta'C_{ij\,kl} < t) \quad i = 1,2,\ldots,\nu', \quad t \in <0,+\infty), \quad (86) \quad j = 1,2,\ldots,w, \quad k = 1,2,\ldots,\nu', \quad l = 1,2,\ldots,w,$$

of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ conditional sojourn times $\theta'C_{ij\,kl}$, $i = 1,2,\ldots,\nu'$, $j = 1,2,\ldots,w$, $k = 1,2,\ldots,\nu'$, $l = 1,2,\ldots,w$, at the operation state $z'c_{ik}$, $i = 1,2,\ldots,\nu'$, $k = 1,2,\ldots,\nu'$, when the next operation state is zc_{jl}, $j = 1,2,\ldots,w$, $l = 1,2,\ldots,w$, takes the following form

$$[H'C_{ij\,kl}(t)]_{\nu'wx\nu'w}$$

$$= \begin{bmatrix} H'C_{1111}(t)\ldots H'C_{111w}(t); H'C_{1121}(t)..H'C_{112w}(t);\ldots; H'C_{11\nu'1}(t)..H'C_{11\nu'w}(t) \\ H'C_{1211}(t)\ldots H'C_{121w}(t); H'C_{1221}(t)..H'C_{122w}(t);\ldots; H'C_{12\nu 1}(t)..H'C_{12\nu w}(t) \\ \ldots \\ H'C_{\nu'w11}(t)\ldots H'C_{\nu'w1w}(t); H'C_{\nu'w21}(t)..H'C_{\nu'w2w}(t);\ldots; H'C_{\nu'w\nu'1}(t)..H'C_{\nu'w\nu'w}(t) \end{bmatrix} \quad (87)$$

and the matrix of their corresponding conditional density functions

$$h'c_{ij\,kl}(t) = \frac{d}{dt}[H'C_{ij\,kl}(t)] \text{ for } t \in <0,+\infty), \; i = 1,2,\ldots,\nu', \; j = 1,2,\ldots,w, \; k = 1,2,\ldots,\nu', \; l = 1,2,\ldots,w, \quad (88)$$

the form

$$[h'c_{ij\,kl}(t)]_{\nu'wx\nu'w}$$

$$= \begin{bmatrix} h'c_{1111}(t)\ldots h'c_{111w}(t); h'c_{1121}(t)..h'c_{112w}(t);\ldots; h'c_{11\nu'1}(t)..h'c_{11\nu'w}(t) \\ h'c_{1211}(t)\ldots h'c_{121w}(t); h'c_{1221}(t)..h'c_{122w}(t);\ldots; h'c_{12\nu'1}(t)..h'c_{12\nu'w}(t) \\ \ldots \\ h'c_{\nu'w11}(t)\ldots h'c_{\nu'w1w}(t); h'c_{\nu'w21}(t)..h'c_{\nu'w2w}(t);\ldots; h'c_{\nu'w\nu'1}(t)..h'c_{\nu'w\nu'w}(t) \end{bmatrix} \quad (89)$$

In the case when the critical infrastructure operation process $Z'(t)$ and climate-weather change process $C(t)$ are dependent, the distribution functions existing in (6.88) can be expressed either by

$$H'C_{ij\,kl}(t) = P(\theta'C_{ij\,kl} < t) = P(\theta'_{ik} < t \cap C_{jl} < t)$$
$$= H'_{ik}(t)C_{jl/ik}(t), \quad t \in <0,+\infty), \quad (90)$$
$$i = 1,2,\ldots,\nu', \quad j = 1,2,\ldots,w, \quad k = 1,2,\ldots,\nu', \quad l = 1,2,\ldots,w,$$

where

$$H'_{ik}(t), \; i = 1,2,\ldots,\nu', \; k = 1,2,\ldots,\nu', \quad (91)$$

are distribution functions defined of the sojourn lifetimes of the operation process $Z'(t)$ defined in Chapter 3 and

$$C_{jl/ik}(t) = P(C_{jl} < t \mid \theta'_{ik} < t), \; i = 1,2,\ldots,\nu', \quad (92)$$
$$j = 1,2,\ldots,w, \; k = 1,2,\ldots,\nu', \; l = 1,2,\ldots,w,$$

are conditional distributions of the sojourn lifetimes at the climate-weather states of the climate-weather change process $C(t)$ defined in Chapter 4 in case they are not conditional or by

$$H'C_{ij\,kl}(t) = P(\theta' C_{ij\,kl} < t) = P(\theta'_{ik} < t \cap C_{jl} < t)$$
$$= C_{jl}(t) H'_{ik/jl}(t), \quad t \in <0,+\infty), \tag{93}$$
$i = 1,2,\dots,\nu'$, $j = 1,2,\dots,w$, $k = 1,2,\dots,\nu'$, $l = 1,2,\dots,w$,

where

$$C_{jl}(t), \quad j = 1,2,\dots,w, \quad l = 1,2,\dots,w, \tag{94}$$

are distribution functions defined of the sojourn lifetimes at the climate-weather states of the climate-weather change process $C(t)$ defined in the Chapter 4 and

$$H'_{ik/jl}(t) = P(\theta'_{ik} < t \mid C_{jl} < t), \; i = 1,2,\dots,\nu', \tag{95}$$
$j = 1,2,\dots,w$, $k = 1,2,\dots,\nu'$, $l = 1,2,\dots,w$,

are conditional distributions of the sojourn lifetimes at the operation states of the critical infrastructure operation process $Z'(t)$ defined in Chapter32 in case they are not conditional.
Hence, the density functions existing in (6.89) can be expressed either by

$$h'c_{ij\,kl}(t) = \frac{d}{dt}[H'C_{ij\,kl}(t)] = \frac{d}{dt}[H'_{ik}(t) C_{jl/ik}(t)]$$
$$= h'_{ik}(t) C_{jl/ik}(t) + H'_{ik}(t) c_{jl/ik}(t),$$
(96)
$t \in <0,+\infty)$,
$i = 1,2,\dots,\nu'$, $j = 1,2,\dots,w$, $k = 1,2,\dots,\nu'$, $l = 1,2,\dots,w$,

where

$H'_{ik}(t)$, $i = 1,2,\dots,\nu'$, $k = 1,2,\dots,\nu'$,
(97)
and $C_{jl/ik}(t)$, $j = 1,2,\dots,w$, $l = 1,2,\dots,w$,

and

$$h'_{ik}(t), \quad i = 1,2,\dots,\nu', \quad k = 1,2,\dots,\nu', \tag{98}$$
and $c_{jl/ik}(t)$, $j = 1,2,\dots,w$, $l = 1,2,\dots,w$,

are respectively defined in Chapter 3 and Chapter 4 or by

$$h'c_{ij\,kl}(t) = \frac{d}{dt}[H'C_{ij\,kl}(t)] = \frac{d}{dt}[C_{jl}(t) H'_{ik/jl}(t)]$$
$$= c_{jl}(t) H'_{ik/jl}(t) + C_{jl}(t) h'_{ik/jl}(t), \quad t \in <0,+\infty),$$
(99)
$i = 1,2,\dots,\nu'$, $j = 1,2,\dots,w$, $k = 1,2,\dots,\nu'$, $l = 1,2,\dots,w$,

where

$$C_{jl}(t), \quad j = 1,2,\dots,w, \quad l = 1,2,\dots,w, \tag{100}$$
and $H'_{ik/jl}(t)$, $i = 1,2,\dots,\nu'$, $k = 1,2,\dots,\nu'$,

and

$$c_{jl}(t), \quad j = 1,2,\dots,w, \quad l = 1,2,\dots,w, \tag{101}$$
and $h'_{ik/jl}(t)$, $i = 1,2,\dots,\nu'$, $k = 1,2,\dots,\nu'$,

are respectively defined in Chapter 4 and Chapter 3.
We assume that the suitable and typical distributions suitable to describe the critical infrastructure operation process $Z'(t)$ conditional sojourn times θ'_{bl}, $b,l = 1,2,\dots,\nu'$, $b \neq l$, in the particular operation states are that defined in [Kołowrocki, Soszyńska-Budny, 2011], [EU-CIRCLE Report D2.1-GMU4-Part1, 2016] and [EU-CIRCLE Report D2.1-GMU4-Part2, 2016]. The suitable and typical distributions suitable to describe the climate-weather change process $C(t)$ conditional sojourn times C_{bl}, b, l = 1,2,…, w, $b \neq l$, at the particular climate-weather states are given by (4.5)-(4.12).

7. Critical infrastructure operation process related to operating environment threats and extreme weather hazards – prediction

Assuming that we have identified the unknown parameters of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$, $t \in <0,+\infty)$, that can take $\nu' w, \nu', w \in N$, different operation states $z'c_{ij}$, $i = 1,2,\dots,\nu'$, $j = 1,2,\dots,w$, defined in Chapter 6 and described by :

- the vector $[p'q_{ij}(0)]_{1x\nu' w}$ of initial probabilities of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ staying at the initial moment $t = 0$ at the operation and climate-weather states $z'c_{ij}$, $i=1,2,\ldots,\nu'$, $j=1,2,\ldots,w$;

- the matrix $[p'q_{ij\,kl}]_{\nu'w x\nu'w}$ of the probabilities of transitions of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ between the operation states $z'c_{ij}$ and $z'c_{kl}$, $i=1,2,\ldots,\nu'$, $j=1,2,\ldots,w$, $k=1,2,\ldots,\nu'$, $l=1,2,\ldots,w$;
- the matrix $[H'C_{ij\,kl}(t)]_{\nu'w x\nu'w}$ of the matrix of conditional distribution functions of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ conditional sojourn times $\theta' C_{ij\,kl}$, $i=1,2,\ldots,\nu'$, $j=1,2,\ldots,w$, $k=1,2,\ldots,\nu'$, $l=1,2,\ldots,w$, at the operation state $z'c_{ij}$, $i=1,2,\ldots,\nu'$, $j=1,2,\ldots,w$, when the next operation state is $z'c_{kl}$, $k=1,2,\ldots,\nu'$, $l=1,2,\ldots,w$,

we can predict this process basic characteristics.

7.1. Critical infrastructure operation process related to operating environment threats and extreme weather hazards – independent critical infrastructure operation process and climate-weather change process

The mean values of the conditional sojourn times $\theta' C_{ij\,kl}$, $i=1,2,\ldots,\nu$, $j=1,2,\ldots,w$, $k=1,2,\ldots,\nu'$, $l=1,2,\ldots,w$, at the operation state $z'c_{ij}$, $i=1,2,\ldots,\nu'$, $j=1,2,\ldots,w$, when the next operation state is $z'c_{kl}$, $j=1,2,\ldots,w$, $k=1,2,\ldots,\nu'$, are defined by [Kołowrocki, Soszyńska-Budny, 2011]

$$M'N_{ij\,kl}=E[\theta' C_{ij\,kl}]=\int_0^\infty t dH'C_{ij\,kl}(t)dt$$

$$=\int_0^\infty t h'c_{ij\,kl}(t)dt, \qquad (102)$$

$i=1,2,\ldots,\nu'$, $j=1,2,\ldots,w$, $k=1,2,\ldots,\nu'$, $l=1,2,\ldots,w$.

In the case when the processess $Z'(t)$ and $C(t)$ are independent, according to (65) the expessions (102) tasks the form

$$M'N_{ij\,kl}=E[\theta' C_{ij\,kl}]$$

$$=\int_0^\infty t[h'_{ik}(t)C_{jl}(t)+H'_{ik}(t)c_{jl}(t)]dt, \qquad (103)$$

$i=1,2,\ldots,\nu'$, $j=1,2,\ldots,w$, $k=1,2,\ldots,\nu'$, $l=1,2,\ldots,w$.

Since from the formula for total probability, it follows that the unconditional distribution functions of the conditional sojourn times $\theta' C_{ij}$, of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the operation states state $z'c_{ij}$, $i=1,2,\ldots,\nu'$, $j=1,2,\ldots,w$, are given by

$$H'C_{ij}(t)=\sum_{k=1}^{\nu'}\sum_{l=1}^{w} p'_{ij\,kl}H'C_{ij\,kl}(t), \qquad (104)$$

$t\in<0,+\infty)$, $i=1,2,\ldots,\nu'$, $j=1,2,\ldots,w$,

In the case when the processess $Z'(t)$ and $C(t)$ are independent, according to (53) and (60) the expessions (104) takes the form

$$H'C_{ij}(t)=\sum_{k=1}^{\nu'}\sum_{l=1}^{w} p'_{ik}q_{jl}H'_{ik}(t)C_{jl}(t), \qquad (105)$$

$t\in<0,+\infty)$, $i=1,2,\ldots,\nu'$, $j=1,2,\ldots,w$,

From (104) it follows that the mean values $E[\theta' C_{ij}]$ of the unconditional distribution functions of the conditional sojourn times $\theta' C_{ij}$, of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the operation states $z'c_{ij}$, $i=1,2,\ldots,\nu'$, $j=1,2,\ldots,w$, are given by

$$M'N_{ij}=E[\theta' C_{ij}]=\sum_{k=1}^{\nu'}\sum_{l=1}^{w} p'_{ij\,kl}M'N_{ij\,kl}, \qquad (106)$$

$i=1,2,\ldots,\nu'$, $j=1,2,\ldots,w$,

where $M'N_{ij\,kl}$ are given by the formula (102).

In the case when the processess $Z(t)$ and $C(t)$ are independent, considering (105) and (60) the expession (106) takes the form

$$M'N_{ij}=E[\theta' C_{ij}]=\sum_{k=1}^{\nu'}\sum_{l=1}^{w} p'_{ik}q_{jl}M'N_{ij\,kl}, \qquad (107)$$

$i=1,2,\ldots,\nu'$, $j=1,2,\ldots,w$,

where $M'N_{ij\,kl}$ are given by the formula (103).

The transient probabilities of the critical infrastructure operation process related to operating environment threats and extreme weather hazards

$Z'C(t)$ at the operation states $z'c_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, can be defined by

$$p'q_{ij}(t) = P(Z'C(t) = zc_{ij}), \quad t \in <0,+\infty), \tag{108}$$

$i = 1,2,...,\nu'$, $j = 1,2,...,w.$

In the case when the processess $Z'(t)$ and $C(t)$ are independent the expession (6.108) for the transient probabilities can be expressed in the following way

$$p'q_{ij}(t) = P(Z'C(t) = z'c_{ij}) = P(Z'(t) = z'_i \cap C(t) = c_j)$$
$$= P(Z'(t) = z'_i) \cdot P(C(t) = c_j) = p'_i(t) \cdot q_j(t),$$
$$t \in <0,+\infty), \ i = 1,2,...,\nu', \quad j = 1,2,...,w, \tag{109}$$

where

$$p'_i(t) = P(Z'(t) = z_i), \ t \in <0,+\infty), \tag{110}$$

are the transient probabilities of the operation process $Z'(t)$ defined in Chapter 3 and

$$q_j(t) = P(C(t) = c_j), \ t \in <0,+\infty), \tag{111}$$

$j = 1,2,...,w,$

are the transient probabilities of the climate-weather change process $C(t)$ defined in Chapter 4.

The limit values of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the operation states $z'c_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, can be found from [Kołowrocki, Soszyńska-Budny, 2011]

$$p'q_{ij} = \lim_{t\to\infty} \frac{\pi_{ij} M'N_{ij}}{\sum_{i=1}^{\nu'} \sum_{j=1}^{w} \pi_{ij} M'N_{ij}}, \ i = 1,2,...,\nu', \tag{112}$$

$j = 1,2,...,w,$

where $M'N_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, are given by (6.107), while the steady probabilities π_{ij}, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, of the vector $[\pi_{ij}]_{1x\nu'w}$ satisfy the system of equations

$$\begin{cases} [\pi_{ij}][p'q_{ij\,kl}] = [\pi_{ij}] \\ \sum_{i==1}^{\nu'} \sum_{j=1}^{w} \pi_{ij} = 1, \end{cases} \tag{113}$$

where $p'q_{ij\,kl}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, $k = 1,2,...,\nu'$, $l = 1,2,...,w$, are given by (6.53).

In the case of a periodic system operation process, the limit transient probabilities $p'q_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, at the operation states given by (6.112), are the long term proportions of the critical infrastructure operation process $Z'C_{ij}(t)$ sojourn times at the particular operation states $z'c_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w.$

Other interesting characteristics of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ possible to obtain are its total sojourn times $\hat{\theta}'\hat{C}_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, at the particular operation states $z'c_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, during the fixed system opetation time. It is well known [Kołowrocki, Soszyńska-Budny, 2011] that the system operation process total sojourn times $\hat{\theta}'\hat{C}_{ij}$, at the particular operation states $z'c_{ij}$, for sufficiently large operation time θ, have approximately normal distributions with the expected value given by

$$\hat{M}'\hat{N}_{ij} = E[\hat{\theta}'\hat{C}_{ij}] = p'q_{ij}\theta, \tag{114}$$

$i = 1,2,...,\nu'$, $j = 1,2,...,w,$

where $p'q_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, are given by (6.112).

7.2. Critical infrastructure operation process related to operating environment threats and extreme weather hazard – dependent critical infrastructure operation process and climate-weather change process

The mean values of the conditional sojourn times $\theta'C_{ij\,kl}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, $k = 1,2,...,\nu'$, $l = 1,2,...,w$, at the operation state $z'c_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,\backslash w$, when the next operation state is $z'c_{kl}$, $k = 1,2,...,\nu'$, $l = 1,2,...,w$, are defined by [Kołowrocki, Soszyńska-Budny, 2011]

$$M'N_{ij\,kl} = E[\theta' C_{ij\,kl}] = \int_0^\infty t dH' C_{ij\,kl}(t) dt \quad (115)$$

$$= \int_0^\infty t h' c_{ij\,kl}(t) dt,$$

$i = 1,2,\ldots,\nu'$, $j = 1,2,\ldots,w$, $k = 1,2,\ldots,\nu'$, $l = 1,2,\ldots,w.$

Since from the formula for total probability, it follows that the unconditional distribution functions of the conditional sojourn times $\theta' C_{ij}$, of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the operation states state $z'c_{ij}$, $i = 1,2,\ldots,\nu'$, $j = 1,2,\ldots,w$, are given by

$$H'C_{ij}(t) = \sum_{k=1}^{\nu'}\sum_{l=1}^{w} p'_{ij\,kl} H'C_{ij\,kl}(t), \quad (116)$$

$t \in <0,+\infty)$, $i = 1,2,\ldots,\nu'$, $j = 1,2,\ldots,w$,

Hence, the mean values $E[\theta' C_{ij}]$ of the unconditional distribution functions of the conditional sojourn times $\theta' C_{ij}$, of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the operation states $z'c_{ij}$, $i = 1,2,\ldots,\nu'$, $j = 1,2,\ldots,w$, are given by

$$M'N_{ij} = E[\theta' C_{ij}] = \sum_{k=1}^{\nu'}\sum_{l=1}^{w} p'_{ij\,kl} M'N_{ij\,kl}, \quad (117)$$

$i = 1,2,\ldots,\nu'$, $j = 1,2,\ldots,w$,

where $M'N_{ij\,kl}$ are defined by the formula (6.115).

The transient probabilities of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the operation states $z'c_{ij}$, $i = 1,2,\ldots,\nu'$, $j = 1,2,\ldots,w$, can be defined by

$$p'q_{ij}(t) = P(Z'C(t) = z'c_{ij}),\ t \in <0,+\infty), \quad (118)$$

$i = 1,2,\ldots,\nu'$, $j = 1,2,\ldots,w.$

In the case when the processess *Z'*(*t*) and *C*(*t*) are dependent the transient probabilities can be expresed either by

$$p'q_{ij}(t) = P(Z'C(t) = z'c_{ij}) = P(Z'(t) = z'_i \cap C(t) = c_j)$$

$$= P(Z'(t) = z'_i) \cdot P(C(t) = c_j \mid Z'(t) = z'_i)$$

$$= p'_i(t) \cdot q_{j/i}(t),\ t \in <0,+\infty), \quad (119)$$

$i = 1,2,\ldots,\nu'$, $j = 1,2,\ldots,w$,

where

$$p'_i(t) = P(Z'(t) = z'_i),\ t \in <0,+\infty),$$

(120)

$i = 1,2,\ldots,\nu'$,

are transient probabilities of the operation process *Z'*(*t*) defined in Chapter 3 and

$$q'_{j/i}(t) = P(C(t) = c_j \mid Z'(t) = z'_i),$$

(121)

$t \in <0,+\infty)$, $i = 1,2,\ldots,\nu'$, $j = 1,2,\ldots,w$,

are conditional transient probabilities of the climate-weather change process *C*(*t*) defined in Chapter 4 in case they are not conditional
or by

$$p'q_{ij}(t) = P(Z'C(t) = z'c_{ij}) = P(Z'(t) = z'_i \cap C(t) = c_j)$$

$$= P(C(t) = C_j) \cdot P(Z'(t) = z'_i \mid C(t) = c_j)$$

$$= q_j(t) \cdot p'_{i/j}(t),\ t \in <0,+\infty), \quad (122)$$

$i = 1,2,\ldots,\nu'$, $j = 1,2,\ldots,w$,

where

$$q_j(t) = P(C(t) = c_j),\ t \in <0,+\infty), \quad (123)$$

$j = 1,2,\ldots,w$,

are transient probabilities of the climate-weather change process *C*(*t*) defined in Chapter 4 and

$$p'_{i/j}(t) = P(Z'(t) = z'_i \mid C(t) = c_j),$$

(124)

$t \in <0,+\infty)$, $i = 1,2,\ldots,\nu'$, $j = 1,2,\ldots,w$,

are conditional transient probabilities of the operation process *Z'*(*t*) defined in Chapter 3 in case they are not conditional.

The limit values of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the operation states $z'c_{ij}$, $i = 1,2,\ldots,\nu'$, $j = 1,2,\ldots,w$, can be found from [Kołowrocki, Soszyńska-Budny, 2011]

$$p'q_{ij} = \lim_{t\to\infty} \frac{\pi_{ij} M'N_{ij}}{\sum_{i=1}^{\nu'}\sum_{j=1}^{w} \pi_{ij} M'N_{ij}}, \quad (125)$$

$i = 1,2,...,\nu', \quad j = 1,2,...,w,$

where $M'N_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, are given by (16), while the steady probabilities π_{ij}, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, of the vector $[\pi_{ij}]_{1x\nu' w}$ satisfy the system of equations

$$\begin{cases} [\pi_{ij}][p'q_{ij\,kl}] = [\pi_{ij}] \\ \sum_{i-=1}^{\nu'} \sum_{j=1}^{w} \pi_{ij} = 1. \end{cases} \tag{126}$$

In the case of a periodic system operation process, the limit transient probabilities $p'q_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, at the operation states given by (7.24), are the long term proportions of the critical infrastructure operation process $Z'C_{ij}(t)$ sojourn times at the particular operation states $z'c_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$.
Other interesting characteristics of the critical infrastructure operation process related to operating environment threats and extreme weather hazards possible to obtain are its total sojourn times $\hat{\theta}'\hat{C}_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, at the particular operation states $z'c_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, during the fixed system opetation time. It is well known [Kołowrocki, Soszyńska-Budny, 2011] that the system operation process total sojourn times $\hat{\theta}'\hat{C}_{ij}$, at the particular operation states $z'c_{ij}$, for sufficiently large operation time θ, have approximately normal distributions with the expected value given by

$$\hat{M}'\hat{N}_{ij} = E[\hat{\theta}'\hat{C}_{ij}] = p'q_{ij}\theta,$$
(127)
$i = 1,2,...,\nu', \quad j = 1,2,...,w,$

where $p'q_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, are given by (125).

8. Conclusions

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