Parametric identification of dynamic imprecise models based on relational fuzzy cognitive maps

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In the paper certain approach to the selection of identification parameters of relational fuzzy cognitive map's dynamic model, based on appropriate computational algorithms of identification, is described. On the selected simulation results there is presented the comparison of the efficiency of different methods of parameters selection (adaptation) in fuzzy relations depending on the adaptation assumed goal.

1. Introduction

The identification problem the is an important element of synthesis and analysis of models of dynamic systems [1]. At solving it important role plays: selecting the model's type, the information on its structure and topology and completeness of the information on its characteristics (obtained e. g. on the way of measurement) and other essential parameters.

In the paper the problem of identification of fuzzy relational cognitive maps dynamic models [2, 4, 5, 6] is stated. Specificity of such constructions lies in a fact that the fuzzyfication subjects not only values of concepts but also relations between them. In such a structure, regardless of parametric identification (power coefficients and dispersion coefficients in relations), also functional identification – connected with possible (not only linear) relational connections between concepts [2, 6] is needed.

Certain approach to the solving of such type identification problems is described. In this paper, due to complexity of the problem of comprehensive adaptation of fuzzy relations, parametrical identification is focused.

2. Formulating the problem

Let fuzzy relational cognitive map type (1) is given:

$$\langle \mathbf{X}, \mathbf{R} \rangle \tag{1}$$

where: $\mathbf{X} = [X_1, ..., X_n]^T$ – fuzzy values of concepts; $\mathbf{R} = \{R_{i,j}\}$ – fuzzy relations matrix.

At it a fuzziness of Gauss type with member function (2)-(3) was selected.

$$\mu_{X_i}(x) = e^{-\left(\frac{x-\overline{X}_i}{\sigma_i}\right)^2} \quad (2)$$

$$\mu_{R_{i,j}}(x_1, x_2) = e^{-\left(\frac{x_2-r_{i,j}(x_1)}{\sigma_{i,j}}\right)^2} \quad (3)$$

where: μ_{X_i} – member function of the i-th concept's fuzzy value; $\mu_{R_{i,j}}$ – member function of fuzzy relation between concepts i and j; $\overline{X}_i$ – center of the member function of the i-th concept's fuzzy value; σ_i – dispersion (fuzziness) coefficient of the i-th concept; x_1, x_2 – axes of a fuzzy relation's universum; $\sigma_{i,j}$ – dispersion coefficient of the fuzzy relation between concepts i and j; $r_{i,j}$ – power coefficient of the fuzzy relation between concepts i and j ($r_{i,j}(1)$ corresponds with power of the relation in equivalent crisp model)

There can be defined following vectors of unknown (exact to experts' knowledge) parameters:

$$\mathbf{Q} = [Q_1, Q_2]^T \quad (4)$$

where: $Q_1 = [\overline{X}_1, ..., \overline{X}_n, \overline{\sigma}_1, ..., \overline{\sigma}_n]^T$; $Q_2 = [\{r_{i,j}\}, \{\sigma_{i,j}\}]^T$; $i, j = 1, ..., n$.

Dynamic dependencies between concepts for X_i (i = 1, ..., n) are described by following fuzzy relational equations [2]:

$$X_i(t+1) = X_i(t) \oplus \bigoplus_{\substack{j=1 \\ j\neq i}}^{n} [X_j(t) \ominus X_j(t-1)] \circ R_{j,i} \quad (5)$$

where: i - number of considered concept (i = 1, ..., n), t - discrete time (t = 0, 1, ..., T), $\oplus$ - fuzzy addition operation, $\ominus$ - fuzzy subtraction operation, $R_{j,i}$–individual fuzzy relation between fuzzy concepts numbered j and i, –maxmin fuzzy composition operation.

The task of the identification of parameters vector **Q** can be divided into few problems presented in Fig. 1.

In Fig. 1 the identification goals are also schematically presented. In approaches from points a) and c) the identification (adaptation) goal is obtaining such parameters of a cognitive map, so as selected (the i-th) concept's value, in a specific moment of discrete time, obtains a level, which fits into a canal with given width, where Fig. 1a) concerns defuzzyfied value and Fig. 1c) – fuzzy value. An approach from Fig. 1b) presents the situation where as exact mapping of the i-th concept's value as possible is significant.

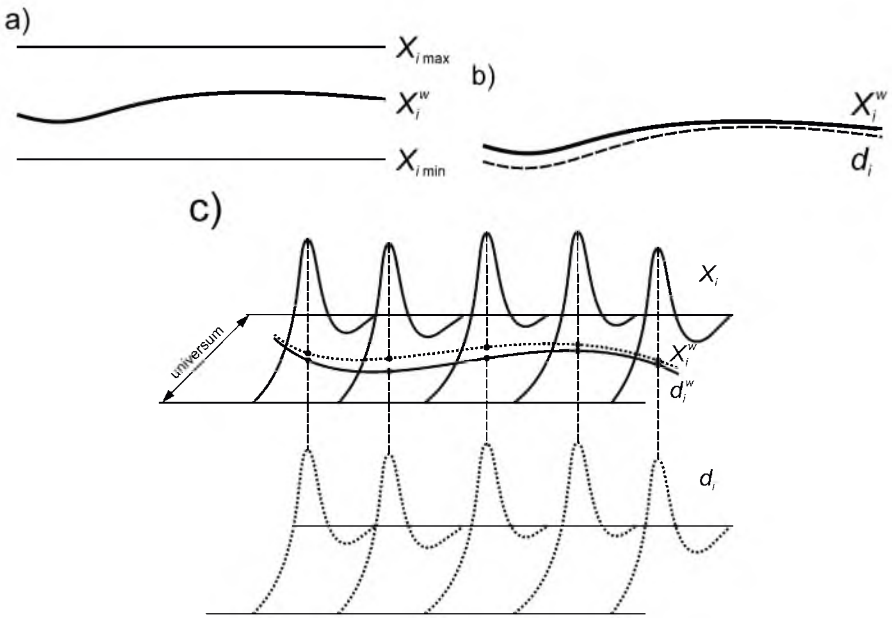


Fig. 1. Methods of preparation of the identification criterion for defuzzified (*a)*, *b)*) and fuzzy (*c)*) values of concepts based on expert knowledge, $X_{i\,\max}$, $X_{i\,\min}$ – assumed (on the base of expert knowledge) limits of the i-th concept's value, d_i – assumed (crisp) course of the i-th concept's value, X_i^w – defuzzyfied value of the i-th concept

Approaches presented in Fig. 1 can be taken as follow:

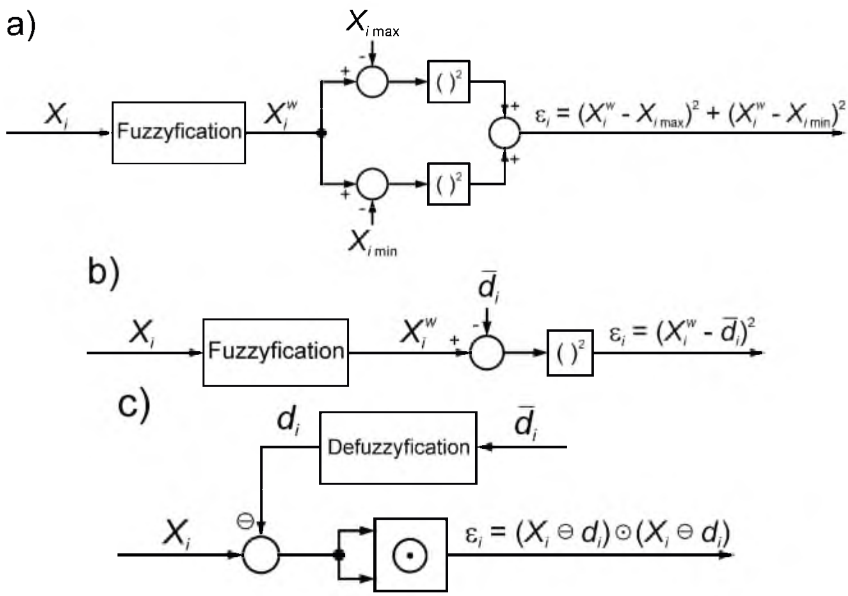


Fig. 2. Diagrams of the identification criterion from Fig. 1 in crisp (*a)*, *b)*) and fuzzy (*c)*) form

Irrespective of chosen method, identification of a cognitive map's parameters amounts to the such a selection of them, which allows to minimize given criterion. This criterion depends on the identification goals and on used approach. At this, it can be assumed, that its base will be the value of only one – selected – concept, or aggregated values of all concepts. Criteria from Fig. 2, in reference to all concepts (for selected moment t), can be written in the form:

– crisp:

$$J_1^w = \frac{1}{2}\sum_{i=1}^{n}\left[\left(X_i^w - X_{i\max}\right)^2 + \left(X_i^w - X_{i\min}\right)^2\right] \Rightarrow \min_{Q} \tag{6}$$

$$J_2^w = \frac{1}{2}\sum_{i=1}^{n}\left(X_i^w - \bar{d}_i\right)^2 \Rightarrow \min_{Q} \tag{7}$$

– fuzzy:

$$J^R = \bigoplus_{i=1}^{n}[(X_i \ominus d_i) \odot (X_i \ominus d_i)] \Rightarrow \min_{Q} \tag{8}$$

<u>Remark:</u>

If criteria (6)-(8) refers to average distance between given intervals and defuzzyfied values of the model's concepts, then these criteria should be completed with additional summing with respect to consecutive steps of discrete time t.

Criteria (6)-(8) allows, theoretically, applying any optimization algorithms [3, 7] (with restrictions or without restrictions dependently on changes of values of vector **Q**). Typical gradient, no-gradient, genetic and other methods can be rated among such algorithms. They can be also completed with additional teaching procedures (similar to neural networks – e.g. by Hebb method) [4].

3. Selected algorithms of identification of model (5)

To optimize criterion (6), (7) or (8) one can use (as mentioned in chapter II) different optimizing algorithms, which may be presented in following general form:

$$\underset{\sim}{Q}^{new} = \underset{\sim}{Q}^{old} \oplus \eta \odot \Delta J \tag{9}$$

where: $\underset{\sim}{Q}^{old}$ – previous approximation; $\underset{\sim}{Q}^{new}$ – next approximation of parameters vector $\underset{\sim}{Q}$;

$$\Delta J = \begin{cases} \Delta J_1^w \\ \Delta J_2^w \\ \Delta J^R \end{cases} \tag{10}$$

– different type of the criterion (e.g. gradient or random value) „increase" estimation; $\oplus$ – addition operation (fuzzy – for fuzzy optimization or normal – for crisp optimization); $\odot$ – multiplication operation (fuzzy – for fuzzy optimization or normal – for crisp optimization); $\eta > 0$ – algorithm's step coefficient.

Computer realization of random version of algorithm (9)-(10) for criterion (7) showed certain efficiency of the identification process. Selected results of the research are presented in the next chapter.

4. Selected results of simulation research

The research subject was exemplary fuzzy relational cognitive map composed of 5 concepts, general diagram of which is shown in Fig. 3.

The simulation purpose was checking the work of the algorithm of identification of two parameters of fuzzy relations: relation power ($r_{i,j}$) and fuzziness coefficients ($\sigma_{i,j}$). The identification process goal was to select parameters of fuzzy relations, so as concepts in analyzed model of cognitive map could achieve values (after defuzzyfication) corresponded with reference values. Analyzed map has closed structure, so time courses of concepts' values can oscillate and achieve relative stabilization after certain number of discrete time steps.

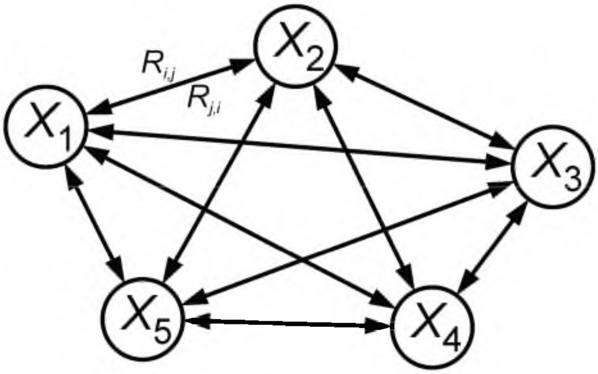


Fig. 3. Exemplary cognitive map being the research object. $X_1 - X_5$ – values of concepts; $R_{i,j}$, $R_{j,i}$ – general denotations of fuzzy relations between concepts; $i,j = 1, ..., 5$.

4.1. Reference courses

Reference values were obtained on the simulation way, by building crisp cognitive map with the structure from Fig. 3 and freely set values of relation powers. Values of these powers are put into Table 1.

Table 1. Values of relation powers in cognitive map crisp model, which was a source of reference values of concepts

r	X_1	X_2	X_3	X_4	X_5
X_1	0	0,5	0,4	0,2	0,4
X_2	0,3	0	0,3	0	0
X_3	0	0	0	0,4	0,4
X_4	0	-0,6	-0,4	0	0
X_5	-0,4	-0,4	0	-0,3	0

Crisp model with relations from Tab. 1 was examined for initial values of concepts from Table 2.

Table 2. Initial values (crisp) of concepts in analyzed model

Concept	X_1	X_2	X_3	X_4	X_5
Value	0,6	0,3	0	0	0,3

Time courses of concept values in crisp reference model (reference courses) are shown in Fig. 4.

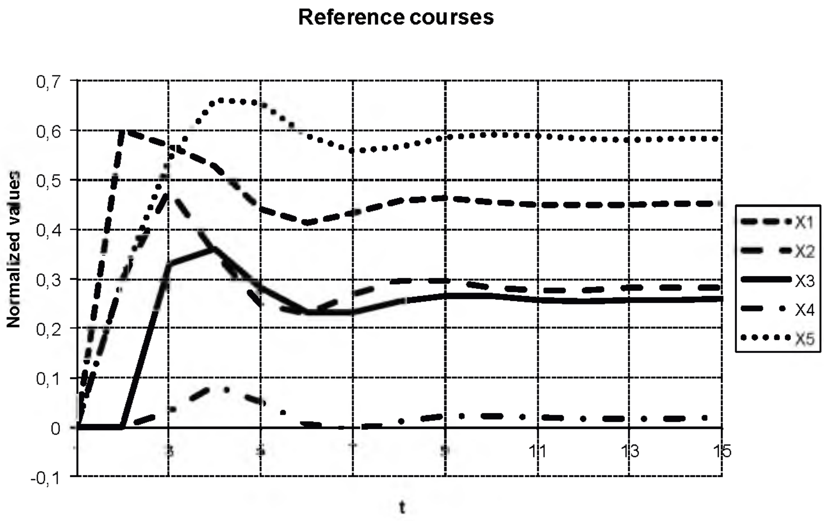


Fig. 4. Reference courses of the concepts' values

4.2. Selection of the identification method

For the identification purposes criterion (7) was chosen. It should be remembered that equation (7) in principle presents only certain idea of the criterion. Detailed solutions can be different, depending on current needs and possibilities. From this reason three tests were conducted for following approaches to the criterion problem:

1. The value in selected – the 15-th step of discrete time was used as a reference value. In this approach it's assumed that value of each course in selected point of discrete time has fundamental meaning. Additionally, instead total criterion for all concepts, partial criteria for each concept were used, what resulted that the algorithm's action lad in the achieving local minima for consecutive concepts and the identification precision was increased by multiple repeating the adaptation cycle.
2. Full time course of given concept's value was a reference course and coefficients y_i, consistent with equation (11), were consecutively minimized.

$$y_i = \sqrt{\frac{\sum_{t=1}^{T}\left(X_i^w(t) - \bar{d}_i(t)\right)^2}{T}} \tag{11}$$

where: t – discrete time step; T – discrete time being the test subject; $X_i^w(t)$ – defuzzyfied value of the i-th concept in the t-th step of discrete time; $\bar{d}_i(t)$ – value of reference course of the i-th concept in the t-th step of discrete time.

3. The value in selected – the 15-th step of discrete time was used as a reference value, but the criterion was directly connected with equation (7).

Tests, carried out for each of above mentioned approaches, showed, that, for the result accuracy, chosen method of the final result achieving has significant meaning. In principle two approaches can be applied. In the first one the series of the adaption cycles is made, checking, after finishing each of them, concept values only in one, final, point of discrete time (moment T). Such a method is faster but it doesn't give possibility to achieve good mapping of full time course. The second, more complex, approach is based on achieving local optima for consecutive steps of discrete time, at this in each next step if this time the adaptation starting point is the optimum achieved for the previous step. So, it's a certain type of recursion, which is generally presented in equation (12).

$$\mathbf{Q}(t) = \begin{cases} \mathbf{Q}_{init} & \text{dla} \quad t = 1 \\ f\left(\mathbf{Q}(t-1)\right) & \text{dla} \quad t \in [2, T] \end{cases} \tag{12}$$

where: f – function of the criterion optimization consistent with (9); T – final step of discrete time.

The tests proved that approach according to (12), despite it's much more time-consuming, allows achieving much better mapping of full time courses of individual concepts with keeping expected values in target point.

4.3. Parameters adaptation algorithm

During the adaptation of fuzzy relations' parameters it is difficult to find analytical method of determining optimal partial changes. It results from the nature of applied fuzzy numbers. It should be remembered, that functional changes influence on universum and deform it. On the other hand, for the correctness of maxmin operations on fuzzy numbers and relations, keeping the universum invariability has crucial meaning. The method to reconcile these two contradictions can be making frequent defuzzyfication and repeated fuzzyfication of concepts' values on the correct universum, but such an approach could false the results. For this reason, on current stage of the research, for fuzzy relations' parameters adaptation purposes, method of „trial and error" was chosen. The method lies in making, according certain algorithm, little changes of powers as well as fuzziness coefficients of relations with simultaneous observing results of these changes. For positive results, the changes are fixed, for negative – cancelled. The process is conducted till it achieves planned identification (limiting criterion value) accuracy or till it achieves the state, when consecutive cycles don't bring any changes. The algorithm has built-in additional mechanism of ending based on executing the set, limiting for given stage, number of the adaptation cycles.

4.4. Simulation results

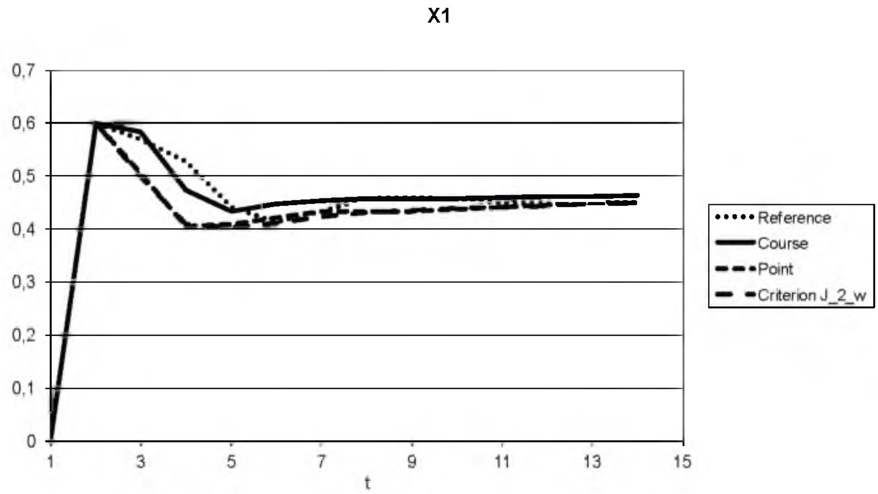
The process of fuzzy relation's parameters identification has supervised teaching nature, based on a comparison with reference values. Initial conditions (initial values of identified parameters) can be, in principle, assumed in any way, but here there can be distinguished two main methods depending on knowledge level in modeled system. If this level is relatively high, one can use expert knowledge and initially determine approximate relation parameters. The adaptation process goal can be then boil down to „specify" individual values. Such a process would be less time- and labor-consuming. In situation of little or none knowledge of the modeled system's structure, assuming zero initial values of the relations' powers is safest because finally they can achieve positive and negative levels as well. As for fuzziness coefficients, assuming too low as well as too high values would be equally adverse. Series of tests results, that their initial values can be assumed between 0.3 and 0.6. The method of fuzzyfication of concepts is a separate problem. Here crucial role plays expert knowledge.

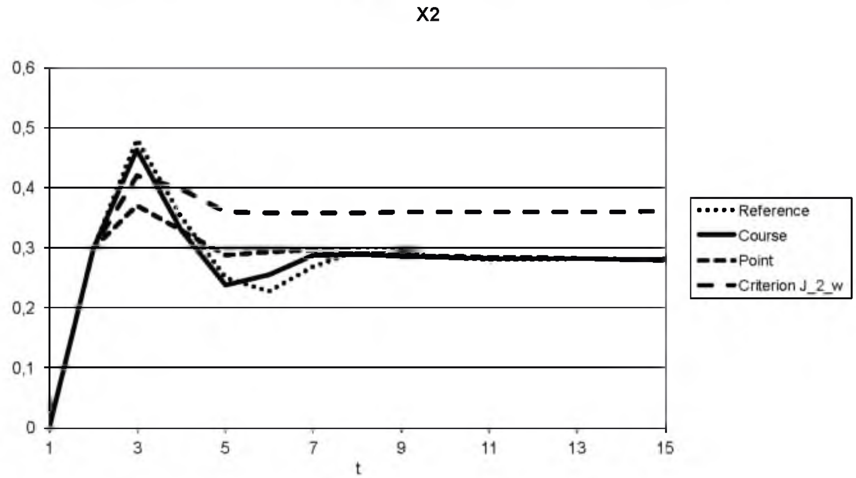
For the analyzed model needs it was assumed that concepts as well as relations are fuzzyfied on the base of Gauss type functions, where for concepts the collective coefficient $\sigma = 0.6$ was assumed, while for relations identical initial values of this coefficient equal to 0.4 were taken. For the symmetry keeping, the universum was expanded to the range [-2, 2]. On this universum 17 sampling points, being concurrently centers of linguistic values' membership functions, were evenly

located. There were also assumed zero initial values of powers for all relations. Three tests were executed and their results were marked:
– "point" – for method 1 from point 4.2,
– "course" – for method 2 from point 4.2,
– "criterion J_2_w" – for method 3 from point 4.2.

In each test different final values of parameters in fuzzy relations were obtained, what means, that similar courses of defuzzyfied values of concepts can be resulted from models that are differed each other.

The simulation was executed using computer application which was created especially to this purpose. The results are presented in Fig. 5.





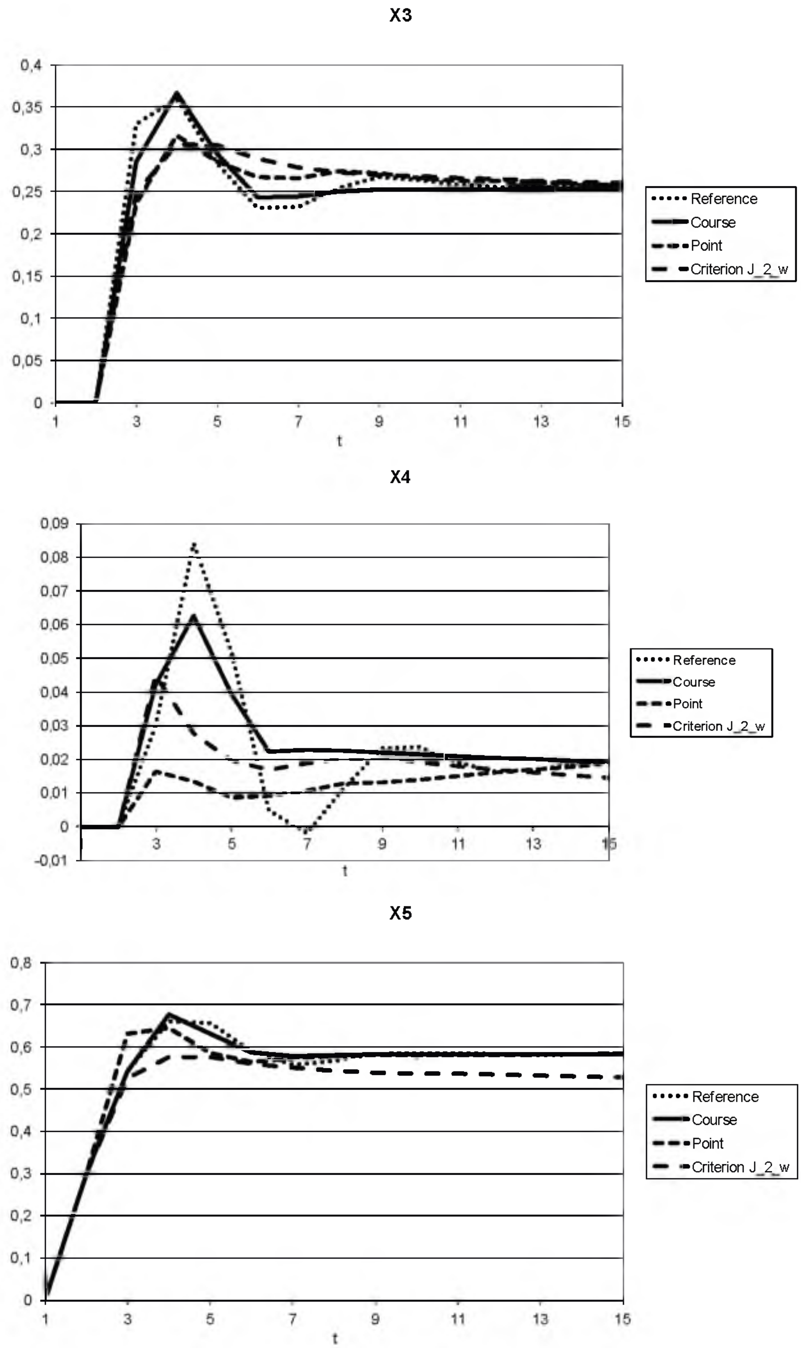


Fig. 5. Comparison of results of fuzzy relational cognitive map's parameters identification, made by three different methods

5. Conclusions

The paper is devoted to the advance of certain approach to solving the problem of parametrical and functional identification of dynamic models of fuzzy relational cognitive maps. The identification problem in forms: crisp and fuzzy was presented. Identification criteria were also described. General algorithm of parametrical identification (9)-(10) was adduced. On the selected example there was conducted analysis of the identification algorithm based on one of the proposed criterions for three different approaches. From carried out comparison (Fig. 5) consequents that the method of comparison individual outputs courses with reference courses gives best results. It delivers expected values of concepts in given step of discrete time as well as whole course shape closest to the reference course. It means that this method can be applied not only to the inquiring current state of modeled systems but it also can find application for the simulation of complex objects work.

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