Scientific Journals
of the Maritime University of Szczecin

Zeszyty Naukowe
Akademii Morskiej w Szczecinie

2016, 45 (117), 114–119
ISSN 1733-8670 (Printed)
ISSN 2392-0378 (Online)
DOI: 10.17402/094

Received: 31.08.2015
Accepted: 15.12.2015
Published: 25.03.2016

Application of multistate systems safety modelling in maritime transport

Krzysztof Kołowrocki, **Joanna Soszyńska-Budny**

Gdynia Maritime University
81–87 Morska St., 81-225 Gdynia, Poland
e-mails: k.kolowrocki@wn.am.gdynia.pl, joannas@am.gdynia.pl, corresponding author

Key words: safety, multistate system, ageing, operation process, maritime transport, risk

Abstract

Basic concepts of the safety analysis of ageing multistate systems are introduced. The system components and the system multistate safety functions are defined. The mean values and variances of the multistate systems' lifespans in the safety state subsets and the mean values of their lifespans in the particular safety states are defined. The multistate system risk function and the moment of exceeding the critical safety state are introduced. A series safety structure and a parallel-series safety structure of the multistate systems with ageing components are defined and their safety functions are determined. The multistate system safety models are applied to the prediction of safety characteristics of a maritime ferry.

Introduction

Taking into account the importance of the safety and operating process effectiveness of real technical systems it seems reasonable to expand the two-state approach in system safety analysis to a multi-state approach (Amari & Misra, 1977; Xue 1985; Kołowrocki, 2004; Kołowrocki & Soszyńska-Budny, 2011.) The assumption that the systems are composed of multi-state components with safety states degrading in time (Kołowrocki, 2004; Kołowrocki, Soszyńska-Budny, 2010; 2011) gives the possibility for more precise analysis of their safety and operational processes' effectiveness. This assumption allows us to identify a system safety critical state which to exceed is either dangerous for the environment or does not assure the necessary level of operation process effectiveness. Then, an important system safety characteristic is the time to the moment of exceeding the system safety critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state safety function that is a basic characteristics of the multi-state system. The safety models of the typical multistate system structures considered here can be applied in the safety analysis of real complex technical systems. They may be successfully applied, for instance, to safety analysis, identification, prediction and optimization of maritime transportation systems.

Multistate approach to safety analysis

In the multistate safety analysis, to define a system composed of n, $n \in N$ ageing components we assume that:

- E_i, $i = 1,2,\ldots,n$, are components of a system;
- all components and a system under consideration have the set of safety states $\{0,1,\ldots,z\}$, $z \geq 1$;
- the safety states are ordered, the state 0 is the worst and the state z is the best;
- the component and the system safety states degrade with time t;
- $T_i(u)$, $i = 1,2,\ldots,n$, $n \in N$ are independent random variables representing the lifetimes of components E_i in the safety state subset $\{u,u+1,\ldots,z\}$ while they were in the safety state z at the moment $t = 0$;

- $T(u)$ is a random variable representing the lifetime of a system in the safety state subset $\{u,u+1,\ldots,z\}$, while it was in the safety state z at the moment $t = 0$;
- $s_i(t)$ is a component E_i safety state at the moment t, $t \in \langle 0,\infty)$, given that it was in the safety state z at the moment $t = 0$;
- $s(t)$ is the system safety state at the moment t, $t \in \langle 0,\infty)$, given that it was in the safety state z at the moment $t = 0$.

The above assumptions mean that the safety states of the ageing system and components may be changed in time only from better to worse.

Definition 1. A vector:

$$S_i(t,\cdot) = [S_i(t,0), S_i(t,1),\ldots, S_i(t,z)]$$
$$\text{for } t \in \langle 0,\infty),\ i = 1,2,\ldots,n \qquad (1)$$

where:

$$S_i(t,u) = P(s_i(t) \geq u \mid s_i(0) = z) = P(T_i(u) > t \qquad (2)$$

for $t \in \langle 0,\infty)$, $u = 0,1,\ldots,z$, is the probability that the component E_i is in the safety state subset $\{u,u+1,\ldots,z\}$ at the moment t, $t \in \langle 0,\infty)$, while it was in the safety state z at the moment $t = 0$, is called the multistate safety function of a component E_i.

Definition 2. A vector:

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,0),\boldsymbol{S}(t,1),\ldots,\boldsymbol{S}(t,z)],\ t \in \langle 0,\infty) \qquad (3)$$

where:

$$\boldsymbol{S}(t,u) = P(s(t) \geq u \mid s(0) = z) = P(T(u) > t) \qquad (4)$$

for $t \in \langle 0,\infty)$, $u = 0,1,\ldots,z$, is the probability that the system is in the safety state subset $\{u,u+1,\ldots,z\}$ at the moment t, $t \in \langle 0,\infty)$, while it was in the safety state z at the moment $t = 0$, is called the multi-state safety function of a system.

The safety functions $S_i(t,u)$ and $\boldsymbol{S}(t,u)$, $t \in \langle 0,\infty)$, $u = 0,1,\ldots,z$, defined by (2) and (4) are called the coordinates of the components and the system multistate safety functions $S_i(t,\cdot)$ and $\boldsymbol{S}(t,\cdot)$ given by respectively (1) and (3). It is clear that from Definition 1 and Definition 2, for $u = 0$, we have $S_i(t,0) = 1$ and $\boldsymbol{S}(t,0) = 1$.

Definition 3. A probability:

$$\boldsymbol{r}(t) = P(s(t) < r \mid s(0) = z) = P(T(r) \leq t)$$
$$t \in \langle 0,\infty) \qquad (5)$$

that the system is in the subset of safety states worse than the critical safety state r, $r \in \{1,\ldots,z\}$ while it was in the safety state z at the moment $t = 0$ is called a risk function of the multi-state system (Kołowrocki & Soszyńska-Budny, 2011).

Under this definition, from (4), we have:

$$\boldsymbol{r}(t) = 1 - P(s(t) \geq r \mid s(0) = z) = 1 - \boldsymbol{S}(t,r)$$
$$t \in \langle 0,\infty) \qquad (6)$$

and if τ is the moment when the system risk exceeds a permitted level δ, then $\tau = \boldsymbol{r}^{-1}(\delta)$ where $\boldsymbol{r}^{-1}(t)$ is the inverse function of the system risk function $\boldsymbol{r}(t)$.

Safety of series and parallel-series systems

Now, after introducing the notion of the multistate safety analysis, we may define basic multi-state safety structures.

Definition 4. A multistate system is called series if its lifetime $T(u)$ in the safety state subset $\{u,u+1,\ldots,z\}$ is given by:

$$T(u) = \min_{1\leq i\leq k}\{\max_{1\leq j\leq l_i}\{T_{ij}(u)\}\},\ u = 1,2,\ldots,z.$$

The number n is called the system structure shape parameter.

The above definition means that a multi-state series system is in the safety state subset $\{u,u+1,\ldots,z\}$ if and only if all its n components are in this subset of safety states. This definition is very close to that of a two-state series system considered in a classical reliability analysis that is not failed if all its components are not failed. This fact justifies the safety structure scheme for a multistate series system presented in Figure. 1.

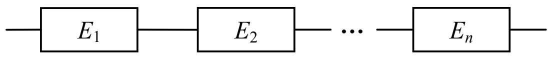

Figure 1. The scheme of a series system safety structure

It is easy to work out that the safety function of the multi-state series system is given by the vector (Kołowrocki & Soszyńska-Budny, 2011):

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1),\ldots, \boldsymbol{S}(t,z)] \qquad (7)$$

with the coordinates:

$$\boldsymbol{S}(t,u) = \prod_{i=1}^{n} S_i(t,u),\ t \in \langle 0,\infty), u = 1,2,\ldots z \qquad (8)$$

Hence, if the system components have exponential safety functions, i.e.:

$$S_i(t,\cdot) = [1,S_i(t,1),\ldots,S_i(t,z)]$$
$$t \in \langle 0,\infty),\ i = 1,2,\ldots,n \qquad (9)$$

where:

$$S_i(t,u) = \exp[-\lambda_i(u)t],\ t \in \langle 0,\infty),$$
$$u = 1,2,\ldots,z,\ i = 1,2,\ldots,n \qquad (10)$$

the formula (14) takes the following form:

$$\boldsymbol{S}(t,u) = \prod_{i=1}^{n} \exp[-\lambda_i(u)t$$
$$t \in \langle 0,\infty),\ u = 1,2,\ldots,z \qquad (11)$$

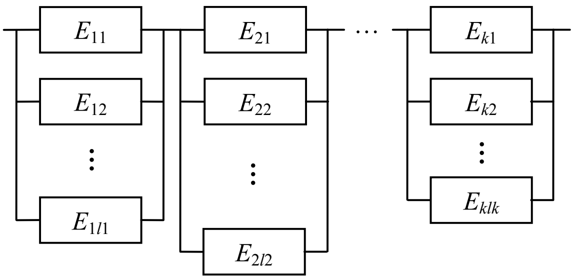

Figure 2. The scheme of a parallel-series system

Definition 5. A multistate system is called parallel-series if its lifetime $T(u)$ in the safety state subset $\{u,u+1,\ldots,z\}$ is given by

$$T(u) = \min_{1\le i\le k}\{\max_{1\le j\le l_i}\{T_{ij}(u)\}\},\ \ u = 1,2,\ldots,z.$$

The above definition means that the multistate parallel-series system is composed of k multistate parallel subsystems and is in the safety state subset $\{u,u+1,\ldots,z\}$ if and only if all its k parallel subsystems are in this safety state subset. In this definition l_i, $i = 1,2,...,k,$ denote the numbers of components in the parallel subsystems. The numbers k and l_1, l_2,..., l_k are called the system structure shape parameters. The scheme of a multistate parallel-series system is given in Figure 2.

The safety function of the multi-state parallel-series system is given by the vector (Kołowrocki & Soszyńska-Budny, 2011):

$$\boldsymbol{S}_{k;l_1,l_2,\ldots,l_k}(t,\cdot) = [1, \boldsymbol{S}_{k;l_1,l_2,\ldots,l_k}(t,1),\ldots,\boldsymbol{S}_{k;l_1,l_2,\ldots,l_k}(t,z)] \tag{12}$$

with the coordinates

$$\boldsymbol{S}_{k;l_1,l_2,\ldots,l_k}(t,u) = \prod_{i=1}^{k}\left[1-\prod_{j=1}^{l_i}[1-S_{ij}(t,u)]\right]$$
$$t \in \langle 0,\infty),\ \ u = 1,2,\ldots,z \tag{13}$$

where k is the number of its parallel subsystems linked in series and l_i, $i = 1,2,...,k$, are the numbers of components in the parallel subsystems.

Hence, if the system components have exponential safety functions, i.e.:

$$S_{ij}(t,\cdot) = [1,S_{ij}(t,1),\ldots,S_{ij}(t,z)],\ t \in \langle 0,\infty)$$
$$i = 1,2,\ldots,k,\ j = 1,2,\ldots,l_i \tag{14}$$

where

$$S_i(t,u) = \exp[-\lambda_{ij}(u)t],\ \ t \in \langle 0,\infty)$$
$$u = 1,2,\ldots,z,\ \ i = 1,2,\ldots,k,\ \ j = 1,2,\ldots,l_i \tag{15}$$

the formula (13) takes the following form:

$$\boldsymbol{S}_{k;l_1,l_2,\ldots,l_k}(t,u) = \prod_{i=1}^{k}\left[1-\prod_{j=1}^{l_i}[1-\exp[-\lambda_{ij}(u)t]]\right]$$
$$t \in \langle 0,\infty),\ \ u = 1,2,\ldots,z \tag{16}$$

Safety of maritime ferry technical system

Ferry technical system description

The considered maritime ferry is a passenger Ro-Ro ship operating in the Baltic Sea between Gdynia and Karlskrona ports on a regular everyday line. We assume that the ferry is composed of a number of main subsystems having an essential influence on its safety. These subsystems are illustrated in Figure 3.

On the scheme of the ferry presented in Figure 3, the following subsystems are identified:

S_1 – a navigational subsystem;
S_2 – a propulsion and controlling subsystem;
S_3 – a loading and unloading subsystem;
S_4 – a stability control subsystem;
S_5 – an anchoring and mooring subsystem;
S_6 – a protection and rescue subsystem;
S_7 – a social subsystem.

In the safety analysis of the ferry, we omit the protection and rescue subsystem S_6 and the social subsystem S_7 and consider only the strictly technical subsystems S_1, S_2, S_3, S_4 and S_5, herein after called the ferry technical system.

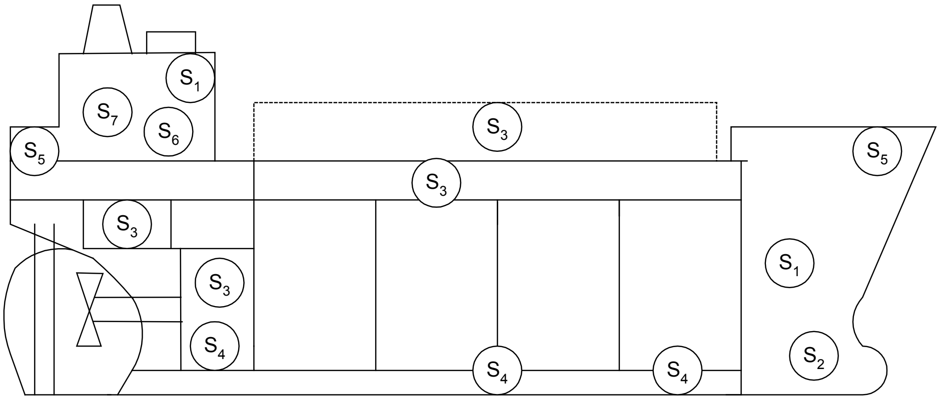

Figure 3. Subsystems having an essential influence on the ferry safety

The navigational subsystem S_1 is composed of one general component $E_{11}^{(1)}$, which is equipped with GPS, AIS, speed log, gyrocompass, magnetic compass, echo sounding system, paper and electronic charts, radar, ARPA, communication system and other subsystems.

The propulsion and controlling subsystem S_2 is composed of:

- subsystem S_{21} which consist of 4 main engines $E_{11}^{(2)}$, $E_{12}^{(2)}$, $E_{13}^{(2)}$, $E_{14}^{(2)}$;
- subsystem S_{22} which consist of 3 thrusters $E_{21}^{(2)}$, $E_{22}^{(2)}$, $E_{31}^{(2)}$;
- subsystem S_{23} which consist of twin pitch propellers $E_{41}^{(2)}$, $E_{51}^{(2)}$;
- subsystem S_{24} which consist of twin directional rudders $E_{61}^{(2)}$, $E_{71}^{(2)}$.

The loading and unloading subsystem S3 is composed of:

- subsystem S_{31} which consist of 2 remote upper trailer decks to main deck $E_{11}^{(3)}$, $E_{21}^{(3)}$;
- subsystem S_{32} which consist of 1 remote forward car deck to main deck $E_{31}^{(3)}$;
- subsystem S_{33} which consist of passenger gangway to Gdynia Terminal $E_{41}^{(3)}$;
- subsystem S_{34} which consist of passenger gangway to Karlskrona Terminal $E_{51}^{(3)}$.

The stability control subsystem S_4 is composed of:

- subsystem S_{41} which consist of an anti-heeling system $E_{11}^{(4)}$, which is used in port during loading operations;
- subsystem S_{42} which consist of an anti-heeling system $E_{21}^{(4)}$, which is used at sea to stabilizing ships rolling.

The anchoring and mooring subsystem S_5 is composed of:

- subsystem S_{51} which consist of aft mooring winches $E_{11}^{(5)}$;
- subsystem S_{52} which consist of forward mooring and anchor winches $E_{21}^{(5)}$;
- subsystem S_{53} which consist of forward mooring winches $E_{31}^{(5)}$.

The subsystems S_1, S_2, S_3, S_4, S_5, described above form a general series safety structure of the ferry technical system presented in Figure 4.

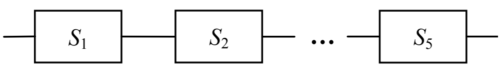

Figure 4. The general scheme of the ferry technical system safety structure

Ferry technical system safety

After discussion with experts, taking into account the safety of the operation of the ferry, we determine the following five safety states (z = 4) of the ferry technical system and its components:

- safety state 4 – the ferry operation is fully safe;
- safety state 3 – the ferry operation is less safe and more dangerous because of the possibility of environmental pollution;
- safety state 2 – the ferry operation is less safe and more dangerous because of the possibility of environmental pollution and small accidents;
- safety state 1 – the ferry operation is much less safe and much more dangerous because of the possibility of serious environmental pollution and extensive accidents;
- safety state 0 – the ferry technical system is destroyed.

Moreover, according to expert opinions, we assume that the only possible transitions between the components' safety states are from better to worse and we assume that the system and its components' critical safety state is $r = 2$.

From the above, the subsystems S_υ, $\upsilon = 1,2,3,4,5$, are composed of five-state, i.e. $z = 4$, components $E_{ij}^{(\upsilon)}$, $\upsilon = 1,2,3,4,5$, having the safety functions:

$$S_{ij}^{(\upsilon)}(t,\cdot) = [1, S_{ij}^{(\upsilon)}(t,1), S_{ij}^{(\upsilon)}(t,2), S_{ij}^{(\upsilon)}(t,3), S_{ij}^{(\upsilon)}(t,4)]$$

with the coordinates that by the assumption are exponential of the forms:

$$S_{ij}^{(\upsilon)}(t,1) = \exp[-\lambda_{ij}^{(\upsilon)}(1)t],\ S_{ij}^{(\upsilon)}(t,2) = \exp[-\lambda_{ij}^{(\upsilon)}(2)t]$$
$$S_{ij}^{(\upsilon)}(t,3) = \exp[-\lambda_{ij}^{(\upsilon)}(3)t],\ S_{ij}^{(\upsilon)}(t,4) = \exp[-\lambda_{ij}^{(\upsilon)}(4)t]$$

The subsystem S_1 consists of one component $E_{ij}^{(1)}$, $i = 1$, $j = 1$, i.e. we may consider it either as a series system composed of $n = 1$ components or for instance as a parallel-series system with parameters $k = 1$, $l_1 = 1$ with the exponential safety functions on the basis of data coming from experts and given below.

The coordinates of the subsystem S_1 component five-state safety function are:

$$S_{11}^{(1)}(t,1) = \exp[-0.033t],\ \ S_{11}^{(1)}(t,2) = \exp[-0.04t]$$
$$S_{11}^{(1)}(t,3) = \exp[-0.045t],\ \ S_{11}^{(1)}(t,4) = \exp[-0.05t]$$

Thus, the subsystem S_1 safety function is identical to the safety function of its component, i.e.:

$$\boldsymbol{S}^{(1)}(t,\cdot) = [1, \boldsymbol{S}^{(1)}(t,1), \boldsymbol{S}^{(1)}(t,2), \boldsymbol{S}^{(1)}(t,3), \boldsymbol{S}^{(1)}(t,4)]$$
$$t \in \langle 0,\infty) \qquad (17)$$

where, according to the formulae (18)–(19), we have:

$$\boldsymbol{S}^{(1)}(t,u) = \boldsymbol{S}_{1;1}(t,u) = \prod_{i=1}^{1}\left[1 - \prod_{j=1}^{1}\left[1 - S_{ij}^{(1)}(t,u)\right]\right] =$$
$$= S_{11}^{(1)}(t,u),\quad t \in \langle 0,\infty),\quad u = 1,2,3,4 \qquad (18)$$

and particularly:

$$\boldsymbol{S}^{(1)}(t,1) = \boldsymbol{S}_{1;1}(t,1) = \exp[-0.033t] \quad (19)$$

$$\boldsymbol{S}^{(1)}(t,2) = \boldsymbol{S}_{1;1}(t,2) = \exp[-0.04t] \quad (20)$$

$$\boldsymbol{S}^{(1)}(t,3) = \boldsymbol{S}_{1;1}(t,3) = \exp[-0.045t] \quad (21)$$

$$\boldsymbol{S}^{(1)}(t,4) = \boldsymbol{S}_{1;1}(t,4) = \exp[-0.05t] \quad (22)$$

The subsystem S_2 is a five-state parallel-series system composed of components $E_{ij}^{(2)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, $k = 7$, $l_1 = 4$, $l_2 = 2$, $l_3 = 1$, $l_4 = 1$, $l_5 = 1$, $l_6 = 1$, $l_7 = 1$, with the exponential safety functions identified on the basis of data coming from experts given below. The coordinates of the subsystem S_2 components' five-state safety functions are:

$$S_{1j}^{(2)}(t,1) = \exp[-0.033t],\ S_{1j}^{(2)}(t,2) = \exp[-0.04t]$$
$$S_{1j}^{(2)}(t,3) = \exp[-0.05t],\ S_{1j}^{(2)}(t,4) = \exp[-0.055t]$$
$$j = 1,2,3,4,$$

$$S_{2j}^{(2)}(t,1) = \exp[-0.066t],\ S_{2j}^{(2)}(t,2) = \exp[-0.07t]$$
$$S_{2j}^{(2)}(t,3) = \exp[-0.075t],\ S_{2j}^{(2)}(t,4) = \exp[-0.08t]$$
$$j = 1,2,$$

$$S_{31}^{(2)}(t,1) = \exp[-0.066t],\ S_{31}^{(2)}(t,2) = \exp[-0.07t]$$
$$S_{31}^{(2)}(t,3) = \exp[-0.075t],\ S_{31}^{(2)}(t,4) = \exp[-0.08t]$$

$$S_{i1}^{(2)}(t,1) = \exp[-0.033t],\ S_{i1}^{(2)}(t,2) = \exp[-0.04t]$$
$$S_{i1}^{(2)}(t,3) = \exp[-0.045t],\ S_{i1}^{(2)}(t,4) = \exp[-0.05t]$$
$$i = 4,5,6,7.$$

Hence, according to the formulae (18)–(19), the subsystem S_2 safety function is given by:

$$\boldsymbol{S}^{(2)}(t,\cdot) = [1, \boldsymbol{S}^{(2)}(t,1), \boldsymbol{S}^{(2)}(t,2), \boldsymbol{S}^{(2)}(t,3), \boldsymbol{S}^{(2)}(t,4)]$$
$$t \in \langle 0,\infty) \quad (23)$$

where

$$\boldsymbol{S}^{(2)}(t,u) = \boldsymbol{S}_{7;4,2,1,1,1,1,1}(t,u) = \prod_{i=1}^{7}\left[1 - \prod_{j=1}^{l_i}\left[1 - S_{ij}^{(2)}(t,u)\right]\right]$$
$$t \in \langle 0,\infty),\ u = 1,2,3,4 \quad (24)$$

and particularly (25)–(28).

Proceeding in an analogous way for the remaining subsystems S_3, S_4 and S_5, we find their safety functions (Kołowrocki & Soszyńska-Budny, 2011).

Next considering that the ferry technical system is a five-state series system, after applying (7)–(8), its safety function is given by:

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)],\ t \geq 0 \quad (29)$$

whereby (19)–(22), (25)–(28) and results given in (Kołowrocki, Soszyńska-Budny, 2011) we have:

$$\boldsymbol{S}(t,u) = \boldsymbol{S}_5(t,u) =$$
$$= \boldsymbol{S}^{(1)}(t,u)\, \boldsymbol{S}^{(2)}(t,u)\, \boldsymbol{S}^{(3)}(t,u)\, \boldsymbol{S}^{(4)}(t,u)\, \boldsymbol{S}^{(5)}(t,u)$$
$$\text{for } u = 1,2,3,4$$

and particularly (30)–(33).

The safety function of the ferry five-state technical system is presented in Figure 5.

As the critical safety state is $r = 2$, then the system risk function, according to (6), is given by:

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,2) = 1 - [12\exp[-0.815t] +$$
$$+8 \exp[-0.925t] + 6 \exp[-0.895t] +$$
$$-16\exp[-0.855t] - 6\exp[-0.885t] - 3\exp[-0.965t]]$$
$$\text{for } t \geq 0 \quad (34)$$

Hence, the moment when the system risk function exceeds a permitted level, for instance $\delta = 0.05$, is: $\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.077$.

The graph of the risk function $\boldsymbol{r}(t)$ of the ferry technical system is presented in Figure 6.

$$\boldsymbol{S}^{(2)}(t,1) = \boldsymbol{S}_{7;4,2,1,1,1,1,1}(t,1) =$$
$$= [6[\exp[-0.033t]]^2\, [1 - \exp[-0.033t]]^2 + 4[\exp[-0.033t]]^3\, [1 - \exp[-0.033t]] + [\exp[-0.033t]]^4]\cdot$$
$$\cdot\, [1 - [1 - \exp[-0.033t]]^2]\exp[-0.066t]\, \exp[-0.033t]\, \exp[-0.033t]\, \exp[-0.033t]\, \exp[-0.033t] =$$
$$= 12\exp[-0.33t] + 8\exp[-0.429t] - 16\exp[-0.363t] - 3\exp[-0.462t] \quad (25)$$

$$\boldsymbol{S}^{(2)}(t,2) = \boldsymbol{S}_{7;4,2,1,1,1,1,1}(t,2) =$$
$$= [6[\exp[-0.04t]]^2\, [1 - \exp[-0.04t]]^2 + 4[\exp[-0.04t]]^3\, [1 - \exp[-0.04t]] + [\exp[-0.04t]]^4]\cdot$$
$$\cdot[1 - [1 - \exp[-0.07t]]^2]\exp[-0.07t]\, \exp[-0.04t]\, \exp[-0.04t]\, \exp[-0.04t]\, \exp[-0.04t] =$$
$$= 12\exp[-0.38t] + 8\exp[-0.49t] + 6\exp[-0.46t] -16\exp[-0.42t]-6\exp[-0.45t] - 3\exp[-0.53t] \quad (26)$$

$$\boldsymbol{S}^{(2)}(t,3) = \boldsymbol{S}_{7;4,2,1,1,1,1,1}(t,3) =$$
$$= [6[\exp[-0.05t]]^2\, [1 - \exp[-0.05t]]^2 + 4[\exp[-0.05t]]^3\, [1 - \exp[-0.05t]] + [\exp[-0.05t]]^4]\cdot$$
$$\cdot[1 - [1 - \exp[-0.075t]]^2]\exp[-0.075t]\, \exp[-0.045t]\, \exp[-0.045t]\, \exp[-0.045t]\, \exp[-0.045t] =$$
$$= 12\exp[-0.43t] + 8\exp[-0.555t] + 6\exp[-0.53t] -16\exp[-0.48t]-6\exp[-0.505t] - 3\exp[-0.605t] \quad (27)$$

$$\boldsymbol{S}^{(2)}(t,4) = \boldsymbol{S}_{7;4,2,1,1,1,1,1}(t,4) =$$
$$= [6[\exp[-0.055t]]^2\, [1 - \exp[-0.055t]]^2 + 4[\exp[-0.055t]]^3\, [1 - \exp[-0.055t]] + [\exp[-0.055t]]^4]\cdot$$
$$\cdot[1 - [1 - \exp[-0.08t]]^2]\exp[-0.08t]\, \exp[-0.05t]\, \exp[-0.05t]\, \exp[-0.05t]\, \exp[-0.05t] =$$
$$= 12\exp[-0.47t] + 8\exp[-0.605t] + 6\exp[-0.58t] -16\exp[-0.525t]-6\exp[-0.55t] - 3\exp[-0.66t] \quad (28)$$

$$\boldsymbol{S}(t,1) = \exp[-0.033t]\ [12\exp[-0.33t] + 8\exp[-0.429t] - 16\exp[-0.363t] - 3\exp[-0.462t]]\ \exp[-0.139t] \cdot \cdot \exp[-0.083t]\ \exp[-0.099t] = 12\exp[-0.684t] + 8\exp[-0.783t] - 16\exp[-0.717t] - 3\exp[-0.816t] \quad (30)$$

$$\boldsymbol{S}(t,2) = \exp[-0.040t]\ [12\exp[-0.38t] + 8\exp[-0.49t] - 6\exp[-0.46t] - 16\exp[-0.42t] - 6\exp[-0.45t] + -3\exp[-0.53t]]\ \exp[-0.175t]\ \exp[-0.100t]\ \exp[-0.12t] = 12\exp[-0.815t] + 8\exp[-0.925t] + 6\exp[-0.895t] + -16\exp[-0.855t] - 6\exp[-0.885t] - 3\exp[-0.965t] \quad (31)$$

$$\boldsymbol{S}(t,3) = \exp[-0.045t]\ [12\exp[-0.43t] + 8\exp[-0.555t] + 6\exp[-0.53t] - 16\exp[-0.48t] - 6\exp[-0.505t] + -3\exp[-0.605t]]\ \exp[-0.200t]\ \exp[-0.110t]\ \exp[-0.145t] = 12\exp[-0.930t] + 8\exp[-1.055t] + +6\exp[-1.030t] - 16\exp[-0.980t] - 6\exp[-1.005t] - 3\exp[-1.105t] \quad (32)$$

$$\boldsymbol{S}(t,4) = \exp[-0.05t]\ [12\exp[-0.47t] + 8\exp[-0.605t] + 6\exp[-0.58t] - 16\exp[-0.525t] - 6\exp[-0.55t] + -3\exp[-0.66t]]\ \exp[-0.230t]\ \exp[-0.120t]\ \exp[-0.165t] = 12\exp[-1.035t] + 8\exp[-1.170t] + +6\exp[-1.145t] - 16\exp[-1.090t] - 6\exp[-1.115t] - 3\exp[-1.225t] \quad (33)$$

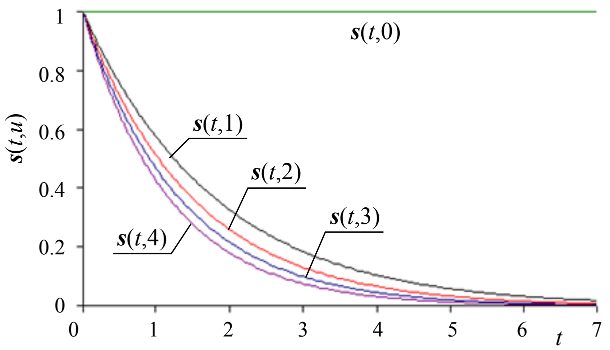

Figure 5. The graph of the ferry technical system safety function $s(t,\cdot)$ coordinates

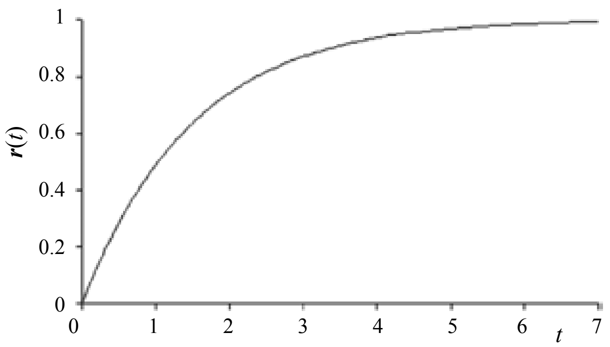

Figure 6. The graph of the risk function $r(t)$ of the ferry technical system

Conclusions

The proposed model for safety evaluation and prediction of the typical multistate system structures considered here are applied to a safety analysis of a maritime ferry technical system operating in the Baltic Sea. The safety function, the risk function and other safety characteristics of the system considered are found. The system safety structures are fixed generally without a high degree of accuracy in details concerning the subsystems' structures because of their complexity and, concerning the components' safety characteristics, because of the lack of statistical data necessary for their estimation. However, the results presented in this paper suggest that it seems reasonable to continue the investigations focusing on the methods of safety analysis for other more complex multi-state systems.

Acknowledgements

The paper presents the results developed in the scope of the EU-CIRCLE project titled "A pan – European framework for strengthening Critical Infrastructure resilience to climate change" that has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No. 653824.

http://www.eu-circle.eu/

References

1. Amari, S.V. & Misra R.B. (1977) Dynamic reliability analysis of coherent multistate systems. *IEEE Transactions on Reliability* 46. pp. 460–461.
2. Kołowrocki, K. & Soszyńska-Budny, J. (2010) Safety and risk evaluation of a ferry technical system. *Journal of Polish Safety and Reliability Association*, *Summer Safety & Reliability Seminars* 4 (1). pp. 159–172.
3. Kołowrocki, K. & Soszyńska-Budny, J. (2011) *Reliability and Safety of Complex Technical Systems and Processes: Modeling – Identification – Prediction – Optimization*. Springer.
4. Kołowrocki, K. (2004) *Reliability of Large Systems*. Elsevier.
5. Kołowrocki, K., Soszyńska-Budny, J., Jurdziński, M. & Dziula, P. (2007) On multi-state safety analysis in shipping. *International Journal of Reliability, Quality and Safety Engineering. Special Issue: System Reliability and Safety* 14 (6). pp. 547–567.
6. Xue, J. (1985) On multi-state system analysis. *IEEE Transactions on Reliability* 34, 329–337.