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Mathematical modeling of mudflow dynamics

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Abstract: Despite the on-going efforts of scientists, there are still few scientifically justified mathematical models that give a practical prediction of the origin, dynamics and destructive force of mudflow. Many problems related to the study of mudflows, and especially their dynamics, are not extensively studied due to the complexity of the process. The contributions of Gagoshidze (1949; 1957; 1962; 1970), Natishvili et al. (1976; 1963; 1969), Tevzadze (1971), Beruchash-vili et al. (1958; 1969; 1979), Muzaev, Sozanow (1996), Gavardashvili (1986), Fleshman (1978), Vinogradov (1976) towards the study of the hydrology of mudflows deserves atten-tion. In the scientific works of Voinich-Sianozhensky et al. (1984; 1977) and Obgadze (2016; 2019), many different mathematical models have been developed that accurately reflect the dynamics of a mudflow caused by a breaking wave. It should also be noted that many inter-esting imitation models have been developed by the team of Mikhailov and Chernomorets (1984). In mountainous districts, the first hit of a mudflow is taken on by lattice-type struc-tures offered by Kherkheulidze (1984a; 1984b) that release the flow from fractions of large stones and floating trees. After passing through the lattice-type structures, the mudflow is re-leased from large fractions and turns into a water-mud flow. In order to simulate this flow, a mathematical model based on the baro-viscous fluid model offered by Geniev-Gogoladze (1987; 1985) has been developed, where the averaging formula of Voynich-Sianozhencki is used for the particle density, and for the concentration of the solid phase, the diffusion equa-tion is added to the system dynamics equations. In the given article, for the constructed math-ematical model, the exact solution of the one- dimensional flow in the mudflow channel is considered. The problem of stratification of the fluid density under equilibrium conditions is discussed. In the riverbed of the Kurmukhi River, for two-dimensional currents, the problem of flow around the bridge pier with an elliptical cross-section is considered. The Rvachev-Obgadze variation method (1982; 1989a; 1989b) is used to solve the streamlined problem.

Keywords: water-mud flow, The Rvachev-Obgadze variation method, Geniev-Gogoladze's baro-viscous fluid

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Introduction

Mudflow, along with other devastating natural disasters in a mountainous region, such as floods, rockfalls, landslides, avalanches and earthquakes, pose a great danger to people and engineering structures. Stony-muddy, water-stony and water-muddy mudflows in practice are formed suddenly and represent a flow moving in the bed of a mountainous region, consisting of stones torn off the slopes and a water, rock-crushed mass. Such flows, in a short time, destroy bridges, roads, hydroelectric power plants and cover arable lands, canals and settlements with silt.

Despite the on-going efforts of scientists, there are still few scientifically based mathematical models that can provide a practical prediction of the origin, dynamics and destructive loads of mudflows. Many problems related to mudflows, especially their dynamics, are not extensively studied due to the complexity of the process.

1. Conceptual model of the formation of a mudflow

Formation of a mudflow is associated with the loss of stability on the slopes of mountain rivers, which leads to their collapse, landslides and blocking of the riverbed. The accumulated water breaks through the natural dam and, together with the crushed-stone mass, flows at high velocity along the riverbed. This type of flow is a mudflow, which can cause great damage due to the high kinetic energy. Therefore, where the riverbed narrows, lattice-type reinforced concrete structures are built (I. Kherkheulidze), capturing large fractions of the mudflow (large stones, bushes, trees, etc.), which reduces the kinetic energy of the flow, however these structures are partially destroyed during the passage of a mudflow, e.g. Figure 1 – Valley of the Duruji River in the Kvareli region.

Fig. 1. I. Kherkheulidze's lattice-type mudflow-catching structure (*own photo*)

Thereafter, the mudflow becomes a stream of fine gravel and mud, which can wash away the river bed, pollute fields with sediment and destroy engineering structures. In order to prevent damage, the downstream banks of the riverbed are reinforced with grid-type structures, dams, etc. (Fig. 2).

Fig. 2. Coast protection grid and dam (*own photo*)

The mechanism for generating a mudflow can be represented by the diagram presented in Figure 3:

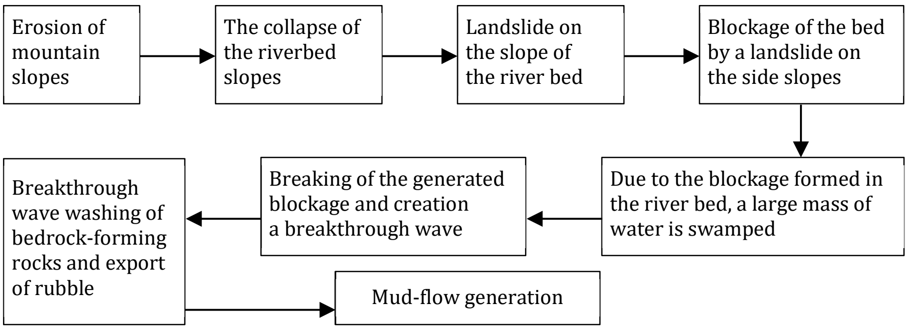

Fig. 3. Scheme of mudflow occurrence (*own study*)

Despite the above protective measures, mudflows create a great danger for head engineering structures. In order to assess the expected damage and optimally design a structure in a mountainous area, it is important to estimate the expected structural loads with practical precision.

According to the abovementioned, it is necessary to develop an adequate mathematical model of a mudflow.

2. Mathematical model of a mudflow

A mudflow is a stream consisting of gravel and water-mud components. As a result of interaction with engineered structures, high pressure is created in the mudflow. Under high-pressure conditions, the rheology of the fluid is well described by the Geniev-Gogoladze mathematical model of the baro-viscous fluid (Geniev, 1985; Geniev & Gogoladze, 1987).

$$\sigma_{ij} = -p\delta_{ij} + (\mu_0 + \lambda p)(v_{i,j} + v_{j,i}). \tag{1}$$

It is obvious that the rheological equation of Geniev-Gogoladze is a generalization of the Newtonian fluid model.

If $\lambda = 0$, then the rheological equation of a baro-viscous fluid coincides with the rheological equation of a non-compressed Newtonian fluid:

$$\sigma_{ij} = -p\delta_{ij} + \mu_0(v_{i,j} + v_{j,i}). \tag{2}$$

The rheological equation of a baro-viscous fluid was firstly developed by Professor G. Geniev, and a detailed study of this model was carried out by his graduate student R. Gogoladze. He showed that this model works well under high-pressure conditions.

A mudflow is a mixture of water and suspended crushed stones. We represent the mudflow in the form of a stream containing, on the one hand, the non-compressed Geniev-Gogoladze fluid, which makes it possible to take into account the properties of the high-pressure fluid that has arisen when flowing around engineered structures, and on the other hand, to describe the dynamic properties of the water flow and take into account the interaction of these two components using the diffusion equation written for the suspended component.

When using the Geniev-Gogoladze baro-viscous fluid model, it should be taken into account that in the case of a mudflow, the baro-viscosity coefficient λ depends on the s concentration of the mud component, i.e. $\lambda = f(s)$; This means that if we use the Taylor equation, we can express the baro-viscosity coefficient using the quality series of the mud component

$$\lambda = f(s) = f(0) + \frac{f'(0)}{1!}s + 0(s^2) \tag{3}$$

If the concentration of the mud component is equal to zero, then we have a water model $\lambda = 0$, which means that $f(0) = 0$ and $\frac{f'(0)}{1!} = \lambda_0$ and $f(s) \cong \lambda_0 s$.

Selecting the appropriate scale ($\lambda_0 = 1$), we can get that $\lambda = s$. Then the modified rheological equation of Geniev-Gogoladze will look like:

$$\sigma_{ij} = -p\delta_{ij} + (\mu_0 + sp)(v_{i,j} + v_{j,i}). \tag{4}$$

Each small volume V of the mudflow consists of the water density ρ_w and the mud density ρ_s, i.e. the mass m of this volume is the sum of the masses of water $\rho_w V(1-s)$ and mud solution $\rho s V s$, placed in the corresponding volume: $\rho V = \rho_w V(1-s) + \rho_s V s$. This means that the density of the mudflow is expressed by the corresponding densities of water and mud particles as follows: $\rho = s\rho_s + + (1-s)\rho_w$.

Therefore, the dynamics equations $\sigma_{ji,j} + \rho b_i = \rho \dot{v}_i$ will look like:

$$-p_{,j}\delta_{ij} + (s_{,j}p + p_{,j}s)(v_{i,j} + v_{j,i}) + (\mu_0 + sp)(v_{i,jj} + v_{j,ij}) + \rho b_i = \rho \dot{v}_i. \quad (5)$$

Since the density of the mixture is $\rho = s\rho_s + (1-s)\rho_w = s(\rho_s - \rho_w) + \rho_w$, we write the continuity equation $\frac{\partial \rho}{\partial t} + v_k \frac{\partial \rho}{\partial x_k} + \rho v_{i,i} = 0$ in the following way:

$$\frac{\partial s}{\partial t} + v_k \frac{\partial s}{\partial x_k} + \frac{s\rho_s + (1-s)\rho_w}{\rho_s - \rho_w} v_{i,i} = 0. \quad (6)$$

The system of differential equations is closed using the diffusion equation:

$$\frac{\partial s}{\partial t} + v_k \frac{\partial s}{\partial x_k} = D s_{i,i}. \quad (7)$$

where D is the coefficient of mechanical diffusion.

Thus, the mathematical model of the mudflow dynamics (5), (6), (7) has been built, consisting of five equations and containing five unknowns: v_1; v_2; v_3; p; s.

We must then add the homogeneous Dirichlet conditions to this system of equations:

$$v_{i_{|\partial\Omega}} = 0; \quad (8)$$

Initial conditions:

$$v_{i_{|t=0}} = v_i(x_1, x_2, x_3); \quad s_{|t} = 0 = s(x_1, x_2, x_3); \quad (9)$$

and based on the content of the problem, the kinematic conditions, e.g. the pressure on the free surface is equal to the atmospheric pressure:

$$p_{|\partial\Omega_0} = p_{atm}; \quad (10)$$

3. The problem of stratification of particle density of solid mud fraction

Based on the mudflow model, we consider the equilibrium equations of the mixture in order to determine the law of distribution of particles of the mud solution fraction by depth (stratification). In a state of equilibrium, the velocity of motion is equal to zero. Therefore, from equations (5)-(7) we obtain the system:

$$-p_{,i} + \rho b_i = 0; \quad (11)$$

$$\frac{\partial s}{\partial t} = 0; \tag{12}$$

$$\frac{\partial^2 s}{\partial x_3^2} = 0. \tag{13}$$

In order to calculate the concentration distribution, it is necessary to solve the Dirichlet problem for the one-dimensional case, where the boundary condition will be the condition of sediment deposition at the bottom of the water-mud mixture near the dam and the absence of solid particles on the open surface, i.e.:

$$\begin{cases} s|_{x_3=h} = 1 \\ s|_{x_3=0} = 0 \end{cases} \tag{14}$$

Therefore, the depth distribution of the solid fraction will be:

$$s = \frac{1}{h} x_3. \tag{15}$$

Then we get that the pressure distribution in a stationary flow of the mixture, which provides the following weakly quadratic law:

$$p = p_{atm.} + \rho\, gx + \frac{x_3^2}{2h}(\rho - \rho)g. \tag{16}$$

Based on the equation (16), the pressure at the bottom of the stationary water--mud mixture will be maximum and its value will be:

$$p = p_{atm.} + \rho_w gh + \frac{h}{2}(\rho_s - \rho_w)g. \tag{17}$$

Thus, we have obtained the equations of the mudflow equilibrium, which make it possible to find: the distribution of the concentrations of solid components and pressures over depth.

If the liquid is homogeneous, i.e. $\rho_s = \rho_w$, then equation (17) turns into the classical hydrostatic law:

$$p = p_{atm.} + \rho_w gh. \tag{18}$$

4. Stationary one-dimensional dynamics of a mudflow in an inclined channel

Let us consider the motion of a steady one-dimensional mudflow on an inclined plane. During the movement along an inclined plane, the mudflow is accelerated until the slope of the channel is equal to 10-15°. During the descent from the mountain, the slope of the river bed gradually decreases, and when the slope is equal to

5-7°, the flow velocity goes into a constant mode; The flow in this section of the mudflow conduit is one-dimensional: $v = v(y)$; $u = u(y)$; $v = w = 0$. Therefore, if we turn to the traditional variables x; y; z and accordingly to the corresponding designations, where $x_1 = x$; $x_2 = y$, then the corresponding system of equations for the governing parameters will look like:

$$0 = \frac{p}{\rho} L_{11}(u, v, w, s) + \frac{s}{\rho} L_{12}(u, v, w, p) + \frac{\mu_0 + sp}{\rho} \Delta u + g\ sin\psi; \tag{19}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} - g \cos \psi; \tag{20}$$

$$0 = D \frac{\partial^2 s}{\partial y^2}; \tag{21}$$

where:

$$\rho = s(\rho_v - \rho_w) + \rho_w; \tag{22}$$

$$L_{11}(u, s) = \frac{\partial u}{\partial y}\frac{\partial s}{\partial y};$$

$$L_{12}(u, p) = \frac{\partial p}{\partial y}\frac{\partial u}{\partial y};$$

Accordingly,

$$s = 1 - \frac{1}{h} y. \tag{23}$$

Taking into account the equation (23), we obtain that the density of the mixture is calculated by the following equation:

$$\rho = \left(1 - \frac{1}{h} y\right)(\rho_s - \rho_w) + \rho_w. \tag{24}$$

Integration of the system of equations gives the following pressure distribution:

$$p = -\left[\left(y - \frac{1}{2h} y^2\right)(\rho_s - \rho_w) + \rho_w y\right] gcos\psi + p_{atm} + \frac{1}{2} h(\rho_s + \rho_w) gcos\psi. \tag{25}$$

For the distribution of the velocity field, we have the following equation:

$$\frac{d^2 u}{dy^2} \cdot \frac{\mu_0 + sp}{\rho} + \frac{du}{dy}\left[\frac{p}{\rho}\frac{ds}{dy} + \frac{s}{\rho}\frac{dp}{dy}\right] + g\ sin\psi = 0; \tag{26}$$

the numerical solution of which is shown in Figure 4.

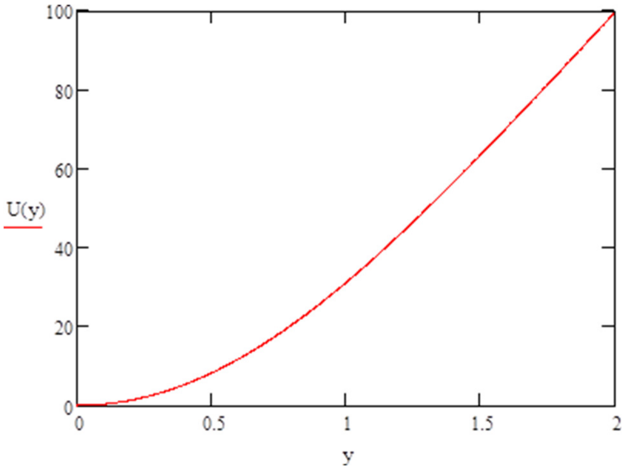

Fig. 4. Variation of the mudflow's velocity according to the depth (*own study*)

Thus, we have obtained that in the case of a one-dimensional stationary flow of a water-mud liquid along an inclined plane, the concentration of the coarse fraction changes linearly, the pressure value, taking into account the depths, increases in a square, while the velocity field varies from the surface according to a quasi-square law.

5. Water-mud flow around the bridge pier of an elliptical cross-section on the Kurmukhi River (Saingilo)

To determine the parameters describing the physical properties of the mudflow, we will use the compositional model in the case of two-dimensional stationary flows, i.e. the following system of differential equations:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+G\cdot p\cdot L_{11}+s\cdot G\cdot L_{12}+ \\ +\left(\frac{1}{Re}+s\cdot G\cdot p\right)\cdot\left(\Delta u+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}\right)+\frac{1}{Fr^2}\cdot sin\psi=0; \tag{27}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+G\cdot p\cdot L_{21}+s\cdot G\cdot L_{22}+ \\ +\left(\frac{1}{Re}+s\cdot G\cdot p\right)\cdot\left(\Delta v+\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}\right)+\frac{1}{Fr^2}\cdot cos\psi=0; \tag{28}$$

$$u\frac{\partial s}{\partial x}+v\frac{\partial s}{\partial y}+\left(s+\frac{\rho_w}{\rho_s-\rho_w}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0; \tag{29}$$

$$u\frac{\partial s}{\partial x}+v\frac{\partial s}{\partial y}=\frac{1}{Pe}\cdot\Delta s. \tag{30}$$

Where we can turn towards the dimensional values using the following equations:

$$x' = l_0 x;\ y' = l_0 y;\ p = \rho v^2 p_0';\ u' = v_0 u;\ v' = v_0 v;\ b_i' = g b_i; \qquad (31)$$

It is obvious, that $\rho(s) = s + \frac{\rho_w}{\rho_v - \rho_w}$; $2l_0$ is the average width of the composite fluid channel; the dimensionless numbers of: $G = \frac{\lambda v_0}{l_0}$ Geniev-Gogoladze, $Re = \frac{\mu_0}{v_0 l_0 \rho}$ Reynolds, $Fr = \frac{v_0}{\sqrt{g l_0}}$ Froude, $Pe = \frac{v_0 l_0}{D}$ Péclet, and dimensionless multiples look like:

$$L_{11}(u, v, w, s) = 2\frac{\partial u}{\partial x}\frac{\partial s}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial s}{\partial y} + \frac{\partial v}{\partial x}\frac{\partial s}{\partial y};$$

$$L_{12}(u, v, w, p) = 2\frac{\partial p}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial p}{\partial y}\frac{\partial u}{\partial y} + \frac{\partial p}{\partial y}\frac{\partial v}{\partial x};$$

$$L_{21}(u, v, w, s) = \frac{\partial v}{\partial x}\frac{\partial s}{\partial x} + 2\frac{\partial v}{\partial y}\frac{\partial s}{\partial y} + \frac{\partial u}{\partial y}\frac{\partial s}{\partial x};$$

$$L_{22}(u, v, w, p) = \frac{\partial p}{\partial x}\frac{\partial v}{\partial x} + 2\frac{\partial p}{\partial y}\frac{\partial v}{\partial y} + \frac{\partial p}{\partial x}\frac{\partial u}{\partial y}.$$

Let us consider the problem of flow around an elliptical section of a bridge pier using the two- dimensional option. When the mudflow has already entered a relatively flat landscape and it becomes important to determine the nature of the flow around the bridge pier and the quality of filling the river bed with solid sediments, since, according to the exact solutions, we already know the depth distribution of the concentration and pressure of the solid fraction, and we have already calculated the depth distribution of the velocity field.

Let us build the diagram of the problem, describing the flow around the bridge pier of an elliptical cross-section (Fig. 5). The lengths of the semi-axes of the ellipse:

a := 2 b := 0.2

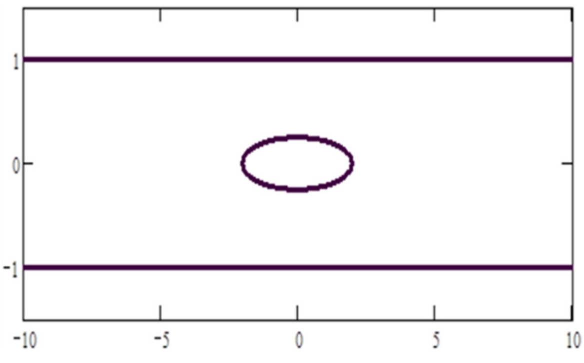

Fig. 5. Scheme of flow around a bridge pier with the shape of an elliptical cross-section (*own study*)

Since a viscous liquid adheres to the boundary walls; accordingly, we have the following homogeneous boundary adhesion conditions:

$$u_{|\,\partial G} = v_{|\,\partial G} = 0, \tag{32}$$

where

$$\partial G = \left\{\left((x;y)|(1-y^2=0) \wedge \left(\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\right)\right)\right\}. \tag{33}$$

Let us create an RO function that matches these boundary conditions. In this case, we have two support sets:

$$\Omega_1 = \{((x;y)|(1-y^2=0))\};$$

$$\Omega_2 = \left\{\left((x;y)|\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\right)\right)\right\}.$$

Using these sets, we write the boundary multiplication equation in the Boolean Algebra of Sets:

$$\Omega = \Omega_1 \cup \Omega_2.$$

We obtain the corresponding predicate equation by a natural homomorphism, i.e., we will have the following type of predicate equation:

$$p = x_1 \vee x_2,$$

where:

$$x_1 = 1 - y^2;$$

$$x_2 = \frac{x^2}{a^2}+\frac{y^2}{b^2}-1.$$

Then, based on the Obgadze homomorphism, we will have the following *RO* function:

$$RO = (x_1 \cdot x_2)$$

i.e.

$$RO = (1-y^2)\cdot\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right). \tag{34}$$

Obviously, in the area of our problem $RO \geq 0$, which, according to Rector's theorem means, that $\boldsymbol{\varphi_{ij}}\}_{i,j=1}^{\infty}$ **is a Schauder basis, then** $\{\boldsymbol{RO\cdot\varphi_{ij}}\}_{i,j=1}^{\infty}$ **system is also a Schauder basis**.

Therefore, we can look for the components of the velocity of solving our problem as follows:

$$\begin{cases} u(x;y;a) = RO \cdot \sum_{i=0}^{n} \sum_{j=0}^{n} \alpha_{ij}\, \varphi_{ij} \\ v(x;y;\beta) = RO \cdot \sum_{i=0}^{n} \sum_{j=0}^{n} \beta_{ij}\, \varphi_{ij} \end{cases} \tag{35}$$

Taking into account that the concentration of the solid component is equal to unity at the boundary and in a case of stationary flow varies in proportion towards the distance from the boundary, we can formulate the corresponding boundary conditions:

$$s(x;y)|_{\partial G} = 1. \tag{36}$$

Then the concentration distribution of the solid component should be searched by the following way:

$$s(x;y;\gamma) = 1 + RO \cdot \sum_{i=0}^{n} \sum_{j=0}^{n} \gamma_{ij}\, \varphi_{ij}\,; \tag{37}$$

and, accordingly, we seek the distribution of the pressure field as follows:

$$p(x;y;\xi) = \sum_{i=0}^{n} \sum_{j=0}^{n} \xi_{ij}\, \varphi_{ij}. \tag{38}$$

In order to solve the problem, we need: for the system of equations, to compose the error function after inserting into it the defining parameters (36), (37), (38).

Let us select the following polynomial system of basic functions:

$$\varphi_{ij}(x,y) = x^i y^j. \tag{39}$$

After the derivation of the distribution functions of the determining parameters, we can calculate the corresponding error distribution function for each equation of the built system of differential equations:

$$R1(x,y,\alpha,\beta,\gamma,\xi);\; R2(x,y,\alpha,\beta,\gamma,\xi);\; R3(x,y,\alpha,\beta,\gamma);\; R4(x,y,\alpha,\beta,\gamma).$$

Thus, we get a four-dimensional error vector:

$$R(R1;R2;R3;R4). \tag{40}$$

Our goal is to minimize the norm of this vector in the Ω area of the flow, i.e. we are looking for a generalized solution to the problem from the following condition:

$$I(\alpha,\beta,\gamma,\xi) = \iint_{\Omega} \sqrt{(R1)^2 + (R2)^2 + (R3)^2 + (R4)^2}\, dxdy \to min. \tag{41}$$

The calculation results reveal that the following principle is applied: **when the flow goes around the bridge pier, the accumulation of the soil component of the mudflow occurs in its boundary layer in such a way that in combination with the pier's shape, it forms a shape with minimum resistance for a given two-component fluid.**

Calculations show that this principle is also applicable to structures of a different, more complex geometric configuration.

Since we already know the law of the concentration of a large fraction of soil in a one-dimensional mudflow and the distribution of pressures (according to the corresponding exact solutions), as well as the distribution of velocity over depth, we can easily determine the image of a three-dimensional diagram of the stationary flow of the pier and the corresponding diagram of loads on the bridge pier (Figs. 6-8).

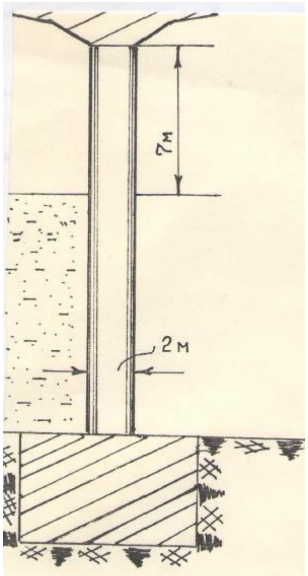

Fig. 6. Scheme of a bridge pier on the Kurmukhi River (Sayngilo) (*own study*)

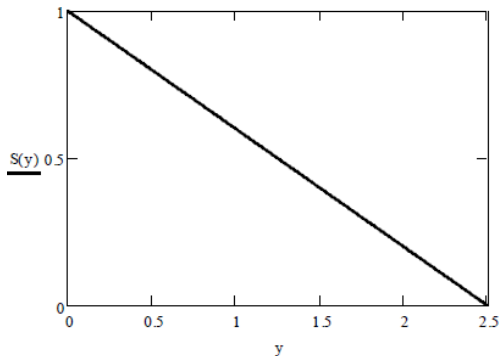

Fig. 7. Distribution of the concentration of the solid fraction in the flow along the depth (*own study*)

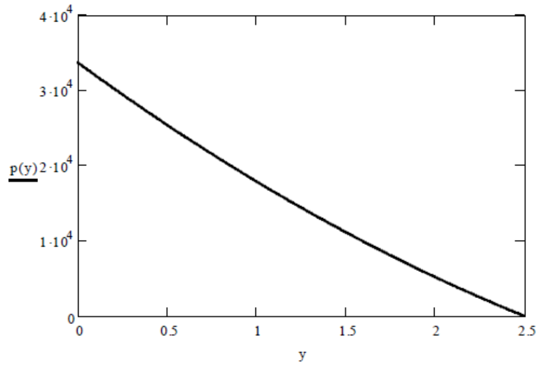

Fig. 8. Distribution of heads in the active layer of mudflow (before approaching the bridge pier) along the depth (*own study*)

In order to determine the load force acting on the bridge pier, let us use the following equation:

$$F = \oiint_{\partial G} \sigma_{ij} n_j df, \tag{42}$$

where integration takes place along the flooded surface f of the bridge pier, n_j is the external normal of a cylindrical pier with an elliptical cross-section, and σ_{ij} – averaged stress tensor component matrix of mudflow, i.e.:

$$\sigma_{ij} = -p\delta_{ij} + (\mu_0 + s\lambda p)(v_{i,j} + v_{j,i}). \tag{43}$$

Due to the symmetry of the problem, the set moment of forces acting on the pier is too small to be taken into account.

It is obvious, that if we substitute (43) into equation (42), then we get:

$$F = -\oiint_{\partial G} p n_i df + \oiint_{\partial G} (\mu_0 + s\lambda p)(v_{i,j} + v_{j,i}) n_j df \tag{44}$$

If we turn over to dimensionless values in equation (104), then we get:

$$\begin{aligned} F &= \rho_{\text{საშ.}} \cdot v_0^2 \cdot S_0 \cdot \oiint_{\partial G} p n_i df \\ &+ \oiint_{\partial G} \left(\frac{\mu_0 \cdot v_0}{l_0} \cdot S_0 + S\lambda \rho_{\text{საშ}} \cdot v_0^2 \cdot \frac{v_0}{l_0} S_0 p \right) (v_{i,j} + v_{j,i}) n_j df = \\ &= C_p \cdot \frac{\rho_{\text{საშ.}} \cdot v_0^2}{2} \cdot S_0 \cdot C_\tau \cdot \frac{\rho_{\text{საშ.}} \cdot v_0^2}{2} \cdot S_0; \end{aligned} \tag{45}$$

where S_0 is the mid-section area of a streamlined structure; $\rho_{aver.}$ – average density of mudflow, v_0 – average flow velocity in the immediate vicinity of the construction;

$C_p = 2 \cdot \oiint_{\partial G} p n_i df$ – coefficient of static load; and $C_\tau = 2 \cdot \oiint_{\partial G} \left(\frac{1}{Re} S_0 + s \cdot G \cdot \cdot S_0 p\right) (v_{i,j} + v_{j,i}) n_j df$ – coefficient of dynamic load.

Thus, the value of the total force (in Newton) acting on engineered structures along the perimeter is the sum of static and dynamic effects and is calculated using the following equation:

$$F = C_p \cdot \frac{\rho_{ცპ} \cdot v_0^2}{2} \cdot S_0 + C_\tau \cdot \frac{\rho_{ცპ} \cdot v_0^2}{2} \cdot S_0. \tag{46}$$

In this equation, C_p and C_τ, respectively, are dimensionless coefficients of static and dynamic actions, which depend on the geometric configuration of the flow around structure relatively to the flow velocity.

The load force acting on the pier depends on the concentration of the solid mud solution fraction s in the flow, which is in contact with the streamlined structure; on the Geniev-Gogoladze and Reynolds numbers *Re* (Figs. 9 and 10).

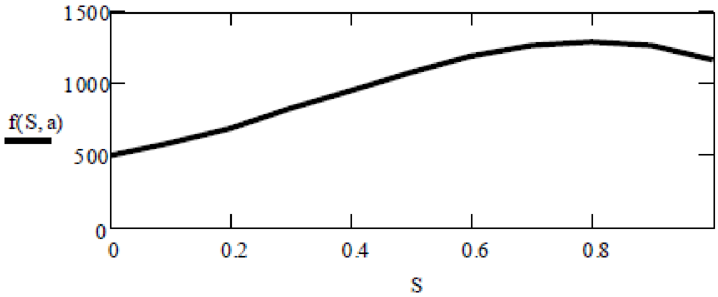

Fig. 9. Dependence of the total load on the average concentration of the mud part (*own study*)

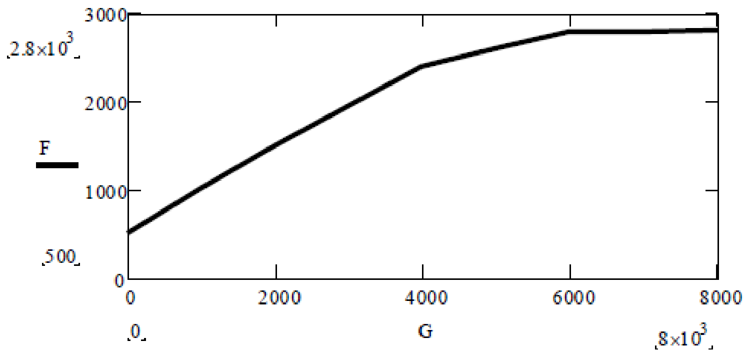

Fig. 10. Dependence of the full load on the Geniev-Gogoladze's number (*own study*)

Thus, the algorithm developed, which is based on the Rvachov-Obgadze variation method, allows the calculation of the expected loads on engineered structures caused by mudflows, using the mathematical model of a mudflow built in the article and carrying out the appropriate numerical implementation and simulation of the process depending on various environmental conditions.

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