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Influence of the Damping in the Sandwich Bar on the Dynamic Stability

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Abstract

In the paper influence of the damping in the sandwich bar on its dynamic stability is studied using analytical methods. This paper presents an analysis of dynamic buckling of a sandwich bar compressed by a periodically variable force. In order to determine the dynamic stability of the bar equations of its transverse vibration were formulated. From the equations of motion, differential equations interrelating of the dynamic deflection with space and time were derived. The partial differential equations were solved using the method of separation of variables (Fourier's method). Then the ordinary differential equation describing the bar vibrations was solved. An analysis of the solution became the basis for determining the regions of sandwich bar motion instability. Finally, the value of the compressive force at which dynamic stability occurs have been calculated.

Keywords: sandwich bars, stability.

1. Introduction

Sandwich constructions are characterized by light weight and high strength. Such features are highly valuable in aviation, building engineering and automotive applications. The primary aim of using sandwich constructions is to obtain properly strong and rigid structures with vibration damping capacity and good insulating properties. Figure 1 shows a scheme of a sandwich construction which is composed of two thin faces and relatively thick core [4, 5, 6, 7, 8]. The core, made of plastic and metal sheet or foil, transfers transverse forces and maintains a constant distance between the faces. Sandwich constructions are classified into bars, plates and beams. A major problem in the design of sandwich constructions is the assessment of their stability under axial loads causing their buckling or folding. The existing methods of calculating such structures are limited to the assessment of their stability under loads constant in time [7, 8]. There are no studies dealing with the analysis of parametric vibration and dynamic stability.

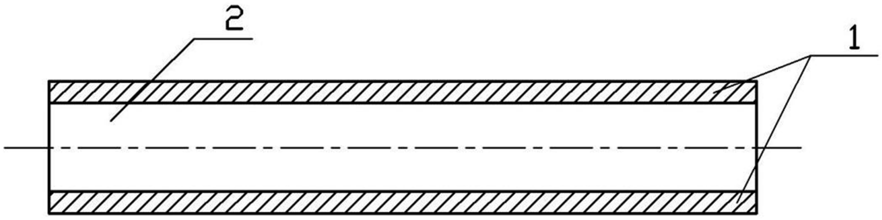

Figure 1. Scheme of sandwich construction: 1 – faces, 2 – core

This paper presents a dynamic analysis of a sandwich bar compressed by a periodically variable force, assuming that the core is linearly viscoelastic. Differential equations describing the dynamic flexural buckling of bars are derived and regions of instability are identified. The dynamic analysis of sandwich constructions is of great importance for vehicles and aeroplanes, since most of the loads which occur in them have the form of time-dependent forces.

2. Equation of sandwich bar motion

The basis for describing the dynamic buckling of a sandwich bar is the differential equation of sandwich beam centre line.

The equation can be written as

$$E_t I \frac{\partial^4 y}{\partial x^4} = q - k \frac{E_t I}{S} \cdot \frac{\partial^2 q}{\partial x^2} \tag{1}$$

where:

- E_t – Young's modulus of the plate,
- I – moment of inertia of the plates,
- y – deflection of the bar,
- q – load intensity,
- k – a coefficient representing the influence of the transverse force on the deflection of the bar,
- S – transverse rigidity of the bar,
- x – coordinate signifying position of the cross section of the sandwich bar.

The cross section of the sandwich bar is shown in Fig. 2.

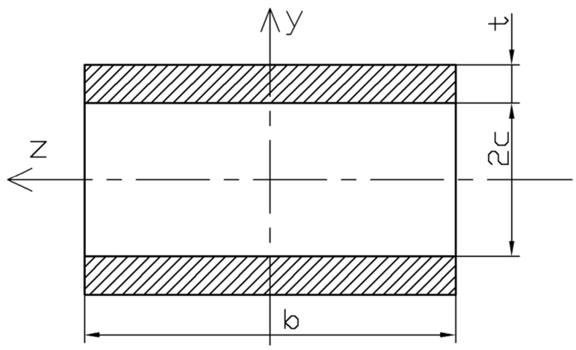

Figure 2. Cross section of sandwich bar

In sandwich constructions a coefficient $k = 1$.

$$S = 2bcG_c \tag{2}$$

where:

b, c – dimensions of the core (Fig. 2),

G_c – modulus of the rigidity of the core material.

Load intensity q can be written in the form:

$$q = q_1 + q_2 + q_3 \tag{3}$$

$$q_1 = -F\frac{\partial^2 y}{\partial x^2},\ q_2 = -\mu\frac{\partial^2 y}{\partial t^2},\ q_3 = -\eta_r\frac{\partial y}{\partial t} \tag{4}$$

where:
F – compressive force
μ – unit mass of the sandwich bar
η_r – damping coefficient of the core material.

Force F can be expressed as follows:

$$F = F_1 + F_2 \cdot \cos pt \tag{5}$$

where:
F_1 – constant component of the compressive force,
F_2 – amplitude of the variable component of the compressive force,
p – frequency of the variable component F_2,
t – time.

After substituting equations (3) into differential equation (1) the following differential equation is obtained:

$$E_t I\left(1-\frac{F}{S}\right)\frac{\partial^4 y}{\partial x^4} + F\frac{\partial^2 q}{\partial x^2} - \frac{E_t I}{S}\mu\frac{\partial^4 y}{\partial x^2 \partial t^2} + \mu\frac{\partial^2 y}{\partial t^2} + \eta_r\frac{\partial y}{\partial t} - \frac{E_t I}{S}\eta_r\frac{\partial^3 y}{\partial x^2 \partial t} = 0 \tag{6}$$

The above equation is a fourth-order homogenous equation with time-dependent coefficients. It was solved by the method of separation of variables (Fourier's method). The solution can be presented in the form of an infinite series:

$$y = \sum_{n=1}^{\infty} X_n(x) T_n(t) \tag{7}$$

where:
$X_n(x)$ – eigenfunctions,
$T_n(t)$ – functions dependent on the time t.

The eigenfunctions $X_n(x)$, satisfying the boundary conditions at the supports of the bar at its ends, have the following form:

$$X_n(x) = A_n \sin\frac{\pi n x}{l} \tag{8}$$

where:
l – length of the bar.

Having substituted equations (7) and (8) into equation (6), one gets the following ordinary differential equation describing functions $T_n(t)$.

$$\ddot{T}_n + 2h\dot{T}_n + {\omega_{0n}}^2(1 - 2\psi_n \cos pt)T_n = 0 \tag{9}$$

where:

$$2h = \frac{\eta_r}{\mu},\ 2\psi_n = \frac{F_2\left(\frac{\pi n}{l}\right)^2}{\mu\omega_{0n}{}^2} \tag{10}$$

The square of frequency ω_{0n} can be expressed as follows:

$$\omega_{0n}{}^2 = \omega_0^2 - \frac{F_1\left(\frac{\pi n}{l}\right)^2}{\mu} \tag{11}$$

where:

ω_0 – the natural frequency of vibration of the bar when $F_1 = 0$, $F_2 = 0$, $\eta_r = 0$.

The square of frequency ω_0 can be expressed as follows:

$$\omega_0{}^2 = \frac{E_t I\left(\frac{\pi n}{l}\right)^4}{\mu\left[1+\frac{E_t I}{S}\left(\frac{\pi n}{l}\right)^2\right]} \tag{12}$$

Differential equation (9) is Hill's equation in the form [1, 2, 3, 4, 5, 6]:

$$\ddot{T}_n + 2h\dot{T}_n + \Omega_n{}^2[1 - f(t)]T_n = 0 \tag{13}$$

where h and Ω_n are coefficients.

By solving of equation (13) the boundary lines of the first region of instability has been obtained (Fig. 3).

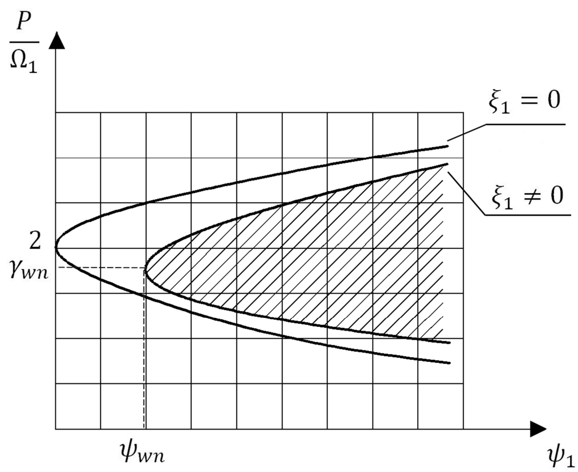

Figure 3. First region of instability (ξ_1=0 – without damping, ξ_1≠0 – with damping)

Hence the "wedge" of the first region instability has the coordinates:

$$\psi_{w1} = 2\sqrt{\xi_1 - 2\xi_1{}^2},\ \gamma_{w1} = 2\sqrt{1 - 3\xi_1} \tag{14}$$

where:

$$\xi_1 = \left(\frac{h}{\Omega_1}\right)^2 \tag{15}$$

From formula (14) the boundary value of coefficient ψ_{w1} at which parametric resonance occurs has been obtained. If $\psi_1 < \psi_{w1}$, no parametric resonance arises. It follows from the above that there exists compressive force F_1 and F_2 at which the bar does not lose stability. Then the component F_2 satisfies a condition:

$$F_2 < \frac{2\sqrt{\xi_1 - 2\xi_1{}^2} \cdot \mu \omega_{0n}{}^2}{\left(\frac{\pi n}{l}\right)^2} \tag{16}$$

3. Example of calculations

Calculations of the sandwich bar has been performed for the following data assumed:

$b = 25$ mm, $c = 7{,}5$ mm, $t = 0{,}5$mm, $l = 50$ cm, $E_t = 7 \cdot 10^4$ MPa, $G_c = 70$ MPa
$\mu = 7 \cdot 10^{-2}$ kg $\cdot$ m^{-1},

$\xi_1 = 0{,}01$:

If $F_1 = 800$ N, then $F_2 < 78{,}87$ N
If $F_1 = 1000$ N, then $F_2 < 39{,}6$ N
If $F_1 = 1100$ N, then $F_2 < 20{,}6$ N.

$\xi_1 = 0{,}1$:
If $F_1 = 800$ N, then $F_2 < 225{,}34$ N
If $F_1 = 1000$ N, then $F_2 < 113{,}14$ N
If $F_1 = 1100$ N, then $F_2 < 58{,}86$ N.

4. Conclusions

Stability of sandwich bar depends on damping in the core. Damping reduces the areas of instability sandwich bar compressed by periodically variable force. Exist values of damping in which motion of sandwich bar is stability. It follows from the above that there exist compressive force components F_1 and F_2 at which the sandwich bar does not lose stability.

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