Alexandr BARABANOV[*], Vadym KRAMAR[*], Antoni ŚWIĆ[**],
Victor TARANENKO[**]

DESIGN OF EQUIVALENT ONE-TIME MODEL OF MULTIVARIABLE MULTI-STAGE CONTROL SYSTEM

Abstract

If a control system contains a few digital chains of treatment of information with different periods of quantum, its research is strongly complicated. In the case of rational commensurable periods of quantum transformation of a multirate system it is possible to design an equivalent one-time system with enhanceable dimension. The general going is offered to the design of an equivalent one-time system, allowing to transform the vectorial-matrix model of the initial multirate system into a vectorial-matrix one-time model of the system, characteristic for vectorial-matrix models of multidimensional continuous systems. Due to that, it becomes possible, in principle, to transfer of methods of analysis of continuous multidimensional systems onto the class of digital-analogue multirate systems.

1. INTRODUCTION

The usage of microprocessors or IBM in the measuring and processing channels along with continuously working arrangement is typical for modern automatic control systems. As a rule, similar control systems carry out measurements and processing of some signals sharing

---

[*]   prof. Alexandr Barabanov, Sevastopol National Technical University, Universyteckaya Str.
   33, 99-053, Sevastopol, Ukraine

[*]   doc. Vadym Kramar, Sevastopol National Technical University, Universyteckaya Str. 33,
   99-053, Sevastopol, Ukraine

[**] prof. Antoni Świć, Lublin University of Technology, Institute of Technological Systems
   of Information, Nadbystrzycka Str. 36, 20-618, Lublin, Poland

[**] prof. Victor Taranenko, Lublin University of Technology, Institute of Technological
   Systems of Information, Nadbystrzycka Str. 36, 20-618, Lublin, Poland

time. Mathematical models of such systems are represented in a multivariate multistep uninterruptedly discrete automatic control systems. Investigations of similar systems lead to the need of their model development in a complex range, with transfer functions as main modelling elements. The problem of linear digital-analogue (uninterruptedly discrete) multistep systems contains two essential complexity aspects within the system multiextent variety of quantum circuits; second aspect characterizes the problem as unsolvable. In the case of commensurable periods (their multiplicity to some "efficient" period), the second aspect becomes in general equal to the first, significantly strengthening it by great multiplicity of numbers of "efficient" period quantum periods, due to which the system can be converted into a monostep multivariate impulse system of extensive dimension. Also, to obtain CAY models in the form of transfer functions, methods using signal graph have found a wide application. Signal graph provides evident representation of system variables and their interaction. It is well known that to determine transfer functions of linear permanent systems the Manson equation can be applied. In [1], [2] approaches of Manson equation application in one-time uninterruptedly discrete systems are observed. If there are some various times, the Coffi and Williams matrix approach should be applied. This article gives a description of the matrix approach to the design of an equivalent model of multivariate multirate uninterruptedly discrete system and also to the creation of a signal graph one-time uninterruptedly discrete system for which the Manson equation can be applied.

2. MATHEMATICAL MODULE OF INVESTIGATED SYSTEM

Consider a multivariate linear system with digital and analogue control circuits adjusted to certain transfer function. Let $u(s)$ be an object governing vector, and $y(s)$ - object output vector. Dimensions of the vectors are m and p, respectively. Consider $x_1(s), x_2(s), \ldots, x_r(s)$ to be variables, quantified in $T_1, T_2, \ldots, T_r$ times (periods) accordingly (among which there can be equal ones) and consider

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_r \end{bmatrix}$$

to be the appropriate vector.

Equation of uninterrupted object and system analogue circuits from the object to the key quantification can be identified as follows:

$$y(s) = W_0(s)u(s),$$
$$x(s) = E(s)y(s) + B(s)u(s), \tag{1}$$

where $W_0(s), E(s), B(s)$ are transfer function matrixes of the correspondent dimensions.

Thus

$$x(s) = U(s)u(s), \tag{2}$$

2

where: $U(s) = E(s)W_0(s) + B(s)$.

Suppose the quantum periods $T_1, T_2, \ldots, T_r$ equal some rate, i.e. are shown in the form

$$T_1 = n_1 T, \, T_2 = n_2 T, \, \ldots, T_r = n_r T, \tag{3}$$

where: $n_1, n_2, \ldots, n_r$ are identity numbers.

Consider

$$x_{1T_1}^*(s), \, x_{2T_2}^*(s), \ldots, x_{rT_r}^*(s)$$

to be Laplace discrete transformation quantified in periods $T_1, T_2, \ldots, T_r$ with variables $x_1(t), \, x_2(t), \ldots, x_r(t)$ respectively.

Each quantified signal $x_i(k_i T_i)$, $k_i = 0, 1, \ldots$, $i \in \overline{1, r}$ is converted by a certain digital circuit and summarized with two similar signals, resulting in control effect formation. Besides digital circuits, in the control effect formation analogue circuits can also be applied (from object outputs). Taking that into account, we can identify the equation for k-th component of control vector as follows:

$$u_k(s) = -\sum_{i=1}^{r} d_{ki}(s) x_{iT_i}^*(s) - \sum_{i}^{p} f_{ki}(s) y_i(s) + u_{\kappa 3}(s),$$

where: $d_{ki}(s)$, $f_{ki}(s)$ are transfer functions of parallel digital and analogue circuits, and $u_{\kappa 3}(s)$ is effect setting formed by digital circuit. In most cases of summarizing and digital-analogue signal conversion we can state that

$$d_{ki}(s) = a(s) W_{kiT_i}^*(s), \tag{4}$$

$$u_{\kappa 3}(s) = a(s) u_{\kappa 3 T}^*(s), \tag{5}$$

where: $a(s) = \dfrac{1 - e^{-Ts}}{s}$, and $W_{kiT_i}^*(s)$, $u_{\kappa 3}^*(s)$ are periodic functions (with periods $2\pi j / T_i$ and $2\pi j / T$, respectively) characterizing digital and digital analogue conversion.

Let us take into consideration matrix $W_*(s)$ with elements $W_{kiT_i}^*(s)$, and also vectors $x_*(s)$, $v^*(s)$ with elements $x_{iT_i}^*(s)$ and $u_{\kappa 3 T}^*(s)$. Symbol * beneath will be used hereafter to define periodicity properties of matrix $W_*(s)$ and vector $x_*(s)$ at particular periods. At the same time, all these elements in equation (3) meet equation $x(s + 2\pi j / T) = x(s)$, and for a given matrix $W_*(s)$ and vector $x_*(s)$ the following equations are possible

$(cb_*)_T^* = c_T^* b_*$ (or $(b_* c)_T^* b_* c_T^*$).

Considering (4), (5), and writing equations for control effects in vector-matrix form

$$u(s) = -a(s)W_*(s)x_*(s) - F(s)y(s) + a(s)v^*(s),$$

where: $F(s)$ - matrix $m \times p$ with elements f_{ki}, is obtained due to equation (1) and (2)

$$u(s) = -G(s)W_*(s)x_*(s) + G(s)v^*(s), \tag{6}$$

$$x(s) = -C(s)W_*(s)x_*(s) + C(s)v^*(s), \tag{7}$$

$$y(s) = -L(s)W_*(s)x_*(s) + L(s)v^*(s), \tag{8}$$

where:

$$G(s) = a(s)(I + F(s)W_0(s))^{-1},$$

$$C(s) = U(s)G(s),$$

$$L(s) = W_0(s)G(s).$$

3. EQUAL MODEL OF THE SYSTEM

The multirate system model can be converted into a one-time system model with quantum period T which is the biggest common devisor of quantum time $T_1, T_2, \ldots T_r$. Let us rely on such equations [3]

$$y_{nT}^*(s) = \frac{1}{n}\sum_{k=1}^{n} y_T^*\left(s + \frac{2 \cdot j}{nT}(k-1)\right), \qquad (xy_{nT}^*)_T^* = x_T^* y_{nT}^*.$$

Further on, symbol T will be avoided in defining a certain impulse conversion. Considering this equation (7) we see that

$$x^*(s) = -C^*(s)W_*(s)x_*(s) + C^*(s)v^*(s).$$

Changing s into $s + \frac{2\pi j}{NT}(v-1)$, where N, v - some natural numbers, we obtain

$$x^{*v}(s) = -C^{*v}(s)W_*^v(s)x_*^v(s) + C^{*v}(s)v^{*v}(s), \tag{9}$$

where for each function $\varphi(s)$ we apply

$$\varphi^v(s) = \varphi\left(s + \frac{2\pi j}{NT}(v-1)\right).$$

Suppose that in equation (9) $v = 1, \ldots, N$, thus we obtain the equation

$$\hat{x}^*(s) = -\hat{C}^*(s)\hat{W}_*(s)\hat{x}_*(s) + \hat{C}^*(s)\hat{v}^*(s), \tag{10}$$

4

where:

$$\hat{x}(s) = (x^1{}'(s), x^2{}'(s), ..., x^N{}'(s))' ,$$ (11)

$$\hat{x}_*(s) = (x_*{}^1{}'(s), x_*{}^2{}'(s), ..., x_*{}^N{}'(s))' ,$$ (12)

$$\hat{v}(s) = (v^1{}'(s), v^2{}'(s), ..., v^N{}'(s))' ,$$ (13)

$$\hat{C}(s) = diag(C^1(s), C^2(s), ..., C^N(s)),$$ (14)

$$\hat{W}(s) = diag(W^1(s), W^2(s), ..., W^N(s)) .$$ (15)

Consider vector components $x^v_*(s)$

$$x^{*v}_{iT_i}(s) = \frac{1}{n_i} \sum_{k=1}^{n_i} x_i^*(s + \frac{2\pi j}{NT}(v-1) + \frac{2\pi j}{n_i T}(k-1)) .$$ (16)

Let N be the least common multiple $n_1, n_2, ..., n_r$, such as

$$N = v_1 n_1, \ N = v_2 n_2, \ ... \ , N = v_r n_r,$$

where $v_1, v_2, ... , v_r$ - natural numbers. Thus, equation (16) can be defined as

$$x^{*v}_{iT_i}(s) = \frac{1}{n_i} \sum_{k=1}^{n_i} x_i^*(s + \frac{2\pi j}{NT}[v + v_i(k-1) - 1]) .$$ (17)

Let us define that for each $l > N$ we will have, considering $l = fN + v,$

$$x_i^*(s + \frac{2\pi j}{NT} l) = x_i^*(s + \frac{2\pi j}{NT} v)$$ (18)

according to periodic functions $x_i^*(s)$ with s in times $2\pi j / T$. Due to equation (18), the right side of equation (17) contains only $x(s)$, meaning that there is a linear vector conversion $\hat{x}^*(s)$ into $\hat{x}_*(s)$, $\hat{x}_*(s)$, i.e. we obtain

$$\hat{x}_*(s) = \prod \hat{x}^*(s) ,$$ (19)

where $\Pi - rN \ x \ rN$ number matrix. Substituting equation (19) into equation (10), we define

$$\hat{x}^*(s) = (I_{rN} + \hat{C}^*(s)\hat{W}_*(s)\prod)^{-1} \hat{C}^*(s)\hat{v}^*(s) .$$ (20)

Defining the equation thus

$$\hat{y}(s) = -\hat{L}(s)\hat{W}_*(s)\hat{x}_*(s) + \hat{L}(s)\hat{v}^*(s)$$

where:

$$\hat{y}(s) = (y^1{}'(s), y^2{}'(s), ..., y^N{}'(s))', \quad \hat{L}(s) = diag(L^1(s), L^2(s), ..., L^N(s)),$$ (21)

5

and using equation (19), (20), we obtain

$$\hat{y}(s) = \hat{L}(s)\{-\hat{W}_*(s)\prod(I_{rN} + \hat{C}^*(s)\hat{W}_*(s)\prod)^{-1}\hat{C}^*(s) + I_{mN}\}\hat{v}^*(s) \ . \tag{22}$$

By applying identity

$$I - A(I+BA)^{-1}B = (I+AB)^{-1} \tag{23}$$

we define also

$$\hat{y}(s) = \hat{L}(s)(I_{mN} + \hat{W}_*(s)\prod\hat{C}^*(s))^{-1}\hat{v}^*(s)$$

and

$$\hat{y}^*(s) = \hat{L}^*(s)(I_{mN} + \hat{W}_*(s)\prod\hat{C}^*(s))^{-1}\hat{v}^*(s) . \tag{24}$$

Let us define matrix Π as block one in the form of $\Pi = \{\Pi_{\upsilon p}\}$, $\upsilon \in 1,..., N$, where $\Pi_{\upsilon l} - r$ x r - matrix and, giving the definition

$$W^* = \hat{W}_* \prod \hat{C}^* \tag{25}$$

define the elements of block representation of this matrix

$$W^*(s) = \{W^*_{\upsilon p}(s)\}, \upsilon, l \in 1,...,N \ , \tag{26}$$

$$W^*_{\upsilon l}(s) = W^{\upsilon}_*(s)\prod\nolimits_{\upsilon l} C^{*l}(s), \upsilon, l \in 1,...,N \ . \tag{27}$$

In this representation

$$W^{\upsilon}_*(s) = W_*(s + \frac{2\pi j}{NT}(\upsilon - 1)), \upsilon \in 1,...,N,$$

$$C^{*l}(s) = C^*(s + (l-1)\frac{2\pi j}{NT}), l \in 1,...,N \ .$$

Block elements $\Pi_{\upsilon p}$, $\upsilon, l \in 1,...,N$ have the form of

$$\prod\nolimits_{\upsilon l} = diag(\pi^1_{\upsilon l}, \pi^2_{\upsilon l},...,\pi^r_{\upsilon l}) . \tag{28}$$

Carrying out simple actions in such a representation, from equation (27) we obtain for the matrix elements $W^*_{\upsilon l}$

$$W^{*\sigma\mu}_{\upsilon l}(s) = \sum_{i=1}^{r} \pi^i_{\upsilon l} W_{*\sigma i}(s + (\upsilon-1)\frac{2\pi j}{NT})C^*_{i\mu}(s + (l-1)\frac{2\pi j}{NT})$$

The created matrixes (26) of open equivalent one-time system allow to find out outputs of the closed initial system, quantified in points kT, $k=0,1,...$ defining for this purpose the matrix

$$H^*(s) = (I_{mN} + W^*(s))^{-1}$$

as block type

$$H^* = \{H^*_{\upsilon p}\}, \ \upsilon, l \in 1, ... , N$$

6

From representation (24) we find out

$$y^*(s) = \sum_{l=1}^{N} L^*(s) H_{1l}^*(s) u^{*l}(s) , \qquad (29)$$

where: $y^*(s) = y^{*1}(s), L^*(s) = L^{*1}(s)$.

4. STRUCTURAL INVARIANT OF QUANTUM CIRCUITS

In the common case of multivariate multistep control systems description, a complex chain of mutual quantum process effect in close system digital circuits with different quantum steps takes place. Typical description feature of this chain is matrix Π, i.e. transformation (19).

The dimension and complexity of this matrix is conditioned by mutual number characteristics of digital circuits on numbers $n_1,\ldots,n_r$ determining mutual relation between quantum steps and N - the least common multiple of given numbers. Non-zero elements of this matrix determine only numbers $1/n_i, i = 1,\ldots,r$, making sense for relative quantum density (relations of «rear» number counting out to «frequent» numbers). As in system structure particularly digital circuits and quantum keys arrangement does not adversely affect matrix Π, so it is a quantum circuit structure invariant. To complete the formation of the equivalent one-step system model, let us now point out the way of Π matrix element calculation. Let us find them in the form of matrix (18). We examine the equation

$$\sum_{l=1}^{N} \pi_{vp}^{i} x_i^{*l} = \frac{1}{n_i} \sum_{k=1}^{n_i} x_i^{*v+(k-1)v_i} , \qquad (30)$$

which takes place according to (19) and (17) with indication

$$x_i^{*l}(s) = x_i^*(s+(l-1)\frac{2\pi j}{NT}), \ i = 1,\ldots,r ,$$

which was mentioned above. Note that, considering equation (30), periodicity ratio should be concerned

$$x_i^{*N+p} = x_i^{*p}, \ l = 1,2,\ldots$$

as the abovementioned function property $x_i^*(s), i = 1,\ldots,r$.

For this purpose let us consider the right side of equation (30) and numbers multitude

$$p(k) = v + (k-1)v_i, \ k = 1,\ldots,n_i \qquad (31)$$

with fixed index value v, i. It is evident that numbers of this multitude pose property

$$p(k'') > p(k'), \ k'' > k' .$$

Let p_{max} be the greatest from numbers $p(k,v,i) = v + (k-1)v_i$ with free index $k \in \overline{1,n_i}$; $v \in \overline{1,N}$; $i \in \overline{1,r}$.

$$p_{max} = \underset{i \in \overline{1,r}}{max}(N + (n_i - 1)v_i) = N + N - \overline{v} = N + \overline{\gamma},$$

where: $\overline{v}$ - the least number v_i, $i \in \overline{1,r}$. According to the ratio $N = n_i v_i$, if even one of all numbers n_i, $i \in \overline{1,r}$ is less than N (even one of all numbers n_i, $i \in \overline{1,r}$ is bigger than 1), it means $\overline{\gamma} < N$. Owing to this, all numbers from (31), bigger than N, may be $N + \gamma$, $0 \le \gamma \le N - 1$. Now at one's own choosing k'', k', k so that $k'' > k' > k$ we obtain

$$p(k) = v + (k - 1)v_i = \gamma \le N,$$

$$p(k') = v + (k' - 1)v_i = N + \gamma',$$

$$p(k'') = v + (k'' - 1)v_i = N + \gamma''.$$

Subtracting we find out

$$\gamma'' - \gamma' = (k'' - k')v_i, \ \ \gamma - \gamma' = (k - k'')v_i + N = (n_i + k - k'')v_i.$$

Thus

$$\gamma' < \gamma'' < \gamma.$$

On account of equation (30), the following rule can be derived to calculate the elements of the quantum density π_{vl}^{i}, $l = 1,\ldots,N$ matrix Π by fixed indexes v, i

$$\pi_{vl}^{i} = \begin{cases} 1/n_i, \ l = l(k) = p(k), \ p(k) \le N \\ 1/n_i, \ l = l(k) = p(k) - N, \ p(k) > N \\ 0 \quad \ l \ne l(k), \end{cases}$$

where: value $p(k)$, $k = 1,\ldots,n_i$ is calculated in (31).

Thus there is an algorithm for which we can create a Table

Table 1. Table of algorithm

1	1	2		*		*		N
$\delta(l)$			...0	1	0...0	1	0...	

8

The upper row of the table is for identity numbers from 1 to N, some of them are marked with a star. The lower one is for values of $\delta(l)$, equal to 1 below marked numbers and 0 on the contrary. The rule of marking the upper row is the following. Values are calculated as $p(k) = v + (k - 1)v_i$, $k = 1,\ldots,n_i$. If $p(k) \le N$, we mark the number $l_* = p(k)$, if $p(k) = N + \gamma(k)$, then number $l_* = \gamma_k$. In this way the lower row determines the value of $\delta(l)$, $l = 1,\ldots,N$, and $\pi_{vl}^i = \dfrac{1}{n_i}\delta(l)$, $l = 1,\ldots,N$.

Thus, in this way we define the whole matrix row Π, i.e. the diagonal elements with fixed indexes v, i of all big matrixes, equal to either $1/n_i$ or 0. Suppose $v = 1$, then using algorithm r times in values of $i = 1,\ldots,r$ we find the first row of Π matrix

$$\Pi_{11},\ \Pi_{12},\ \ldots,\ \Pi_{1N-1},\ \Pi_{1N}.$$

This row, by means of series circle permutation, determines the whole matrix Π. The lower row looks as follows

$$\Pi_{1N},\ \Pi_{11},\ \ldots,\ \Pi_{1N-2},\ \Pi_{1N-1}$$

etc. E.g., by $N = 3$ the whole matrix Π is given by

$$\Pi = \left\|\begin{array}{c:c:c} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ \hdashline \Pi_{13} & \Pi_{11} & \Pi_{12} \\ \hdashline \Pi_{12} & \Pi_{13} & \Pi_{11} \end{array}\right\|.$$

This conclusion is easy to come to if we compare π_{vl}^i and π_{v+1l}^i, corresponding to $k \in \overline{1, n_i}$. According to equation (31), $p(k, v+1) = p(k, v) + 1$. Thus, if $l(k) < N$, then $l'(k) = l(k) + 1$, and if $l(k) = N$, then $l'(k) = 1$. For block matrixes which are defined by $k \in \overline{1, n_i}$, such number will be represented as $\Pi_{v+1l'} = \Pi_{vl+1}$, $l < N$; $\Pi_{v+1l'} = \Pi_{v1}$, $l = N$, which is the rule definition.

It is also essential to emphasize the opportunity to form prescribed matrix blocks Π. Let us consider block Π_{vl}. Diagonal elements π_{vl}^i, $i = 1,\ldots,r$ should be defined with values v, l.

Let us set $i \in \overline{1,\ldots,r}$, then:

1. If there is $k \in \overline{1, n_i}$, satisfying any equation

$$v + (k-1)v_i = l, \quad v + (k-1)v_i - N = l, \tag{32}$$

then

$\pi_{vl}^i = 1/n_i$ and

2. If there is no $k \in \overline{1, n_i}$, then

$$\pi_{vl}^i = 0.$$

Example. Suppose $r = 2,\ n_1 = 1,\ n_2 = 3$.

Thus, $N = 3,\ \nu_1 = 3,\ \nu_2 = 1$. For block Π_{11} there is an equation

$$i = 1 \quad 1 + (k-1)3 = 1,\ \ 1 + (k-1)3 - 3 = 1,\ k \in \overline{1,3};$$
$$i = 2 \quad 1 + (k-1)1 = 1,\ \ 1 + (k-1)1 - 3 = 1,\ k \in \overline{1,3}.$$

That is why $\pi_{11}^1 = 1,\ \pi_{11}^2 = 1/3$.

For block Π_{12}:

$$i = 1 \quad 1 + (k-1)3 = 2,\ \ 1 + (k-1)3 - 3 = 2,\ k \in \overline{1,3};$$
$$i = 2 \quad 1 + (k-1)1 = 2,\ \ 1 + (k-1)1 - 3 = 2,\ k \in \overline{1,3}.$$

Thus, $\pi_{12}^1 = 0,\ \pi_{12}^2 = 1/3$.

Finally, for block Π_{13}:

$$i = 1 \quad 1 + (k-1)3 = 3,\ \ 1 + (k-1)3 - 3 = 3,\ k \in \overline{1,3};$$
$$i = 2 \quad 1 + (k-1)1 = 3,\ \ 1 + (k-1)1 - 3 = 3,\ k \in \overline{1,3}.$$

Thus, $\pi_{13}^1 = 0,\ \pi_{12}^2 = 1/3$.

The whole matrix is given by

$$\Pi = \left\|\begin{array}{cc:cc:cc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 1/3 & 0 & 1/3 \\ \hdashline 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 1/3 & 0 & 1/3 \\ \hdashline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 1/3 & 0 & 1/3 \end{array}\right\|.$$

Differences in quantum density are seen in matrix Π in the form of inhomogeneous filling of its blocks. When quantum density is equal in all the circuits, i.e. $n_1 = n_2 = \cdots = n_r = n,\ N = n$ and $\nu_1 = \nu_2 = \cdots = \nu_r = 1$, then matrix Π is fully filled and all block elements $\Pi_{\nu l},\ \nu, l = 1, \ldots, n$ are equal to

$$\Pi_{\nu l} = \frac{1}{n} I_r,$$

where: I_r - - single matrix $r \times r$.

It is clear that the first equation (32), by $\nu = 1,\ \nu_i = 1$ by any $l \in \overline{1,n}$, has solution $k = l$, which means that $\pi_{\nu l}^i = 1/n,\ i \in \overline{1,r}$.

5. FORMATION OF EQUIVALENT MODELS OF MULTIRATE SYSTEMS IN THE FORM OF SIGNAL GRAPH

To obtain a CAY model in the form of transfer functions, methods using signal graph have found a wide application. Signal graph provides obvious introduction of system variables and their interaction. It is well known that to define transfer functions of linear uninterrupted systems the Manson formula can be used. In [1], [2] approaches using Manson formula in one time uninterrupted discrete systems are considered. In this article the formation of signal graph multirate uninterrupted discrete system for which Manson formula is applicable is observed.

While constructing signal graph of multirate uninterrupted discrete system, let us consider the symbol system of Sodper and Becky [2]. White knot in the graph is used to define uninterrupted variable system. Black knot is used to define discrete variable and quantum operations, variable significance represented by any black knot being discrete form of sum transformation of all variables, being in the knot, according to a certain rate. Because in uninterrupted discrete, as a rule, it is impossible to outline inlet variable in presentation for, thus it is worthwhile to coordinate inlet influence. It is carried out by introduction of branches with transfer function equal to inlet variables so that entire transfer function belonging to the single output, transfer function and output become equal.

Linear uninterrupted discrete system with some quantifiers with discrete time $T_1, \ldots, T_N$ can be described by the system of linear algebraic equations in certain fields

$$A(s)x(s) = \sum_{i=1}^{N} B_i x^{T_i}(s) + R , \qquad (33)$$

where: $x(s) - n$ is vector of variable systems represented according to Laplace;

$A(s) - n \times n$ uninterrupted transfer function matrix; $x^{T_i}(s)$ - discrete transformation in times T_i, $i = 1, \ldots, N$ vector $x(s)$; $R - n$ – standardized input vector which, according to system linearity, can be considered as vector $R' = [1\ 0\ 0\ \ldots\ 0]'$ where ' – here and further means transportation; $B_i - n \times n$ – matrixes characterizing quantifier presence with various discrete times in system, they consist of 0 and 1 and represent quantification.

To simplify (33) it is worthwhile to write first of all equations for uninterrupted, and then for quantum variables by turns for each discrete time $T_i, i = 1, \ldots, N$ [1].

$A(s)$ is of block type

$$A(s) = \begin{bmatrix} W(s) & \Phi_1(s) & \cdots & \Phi_N(s) \\ 0 & I_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I_N \end{bmatrix}, \qquad (34)$$

where: N – quantity of different measure point; $W(s) - l \times l$ – transfer function matrix; l – uninterrupted variables number; $\Phi_i(s) - l \times m_i$ – transfer function matrix; m_i – quantifier number with discrete time $T_i, i = 1, \ldots, N$; I_i – identity matrixes $m_i \times m_i$.

Let us define

$$y_i^{T_i}(s) = B_i x^{T_i}(s), \ i = 1,\ldots,N. \tag{35}$$

Vector $y_i^{T_i}(s)$ consists of the m_i vector component x^{T_i}, complying with quantifier outputs with discrete time T_i.

From (33), taking into account (35) (considering the presence of $A^{-1}(s)$), we obtain

$$x(s) = A^{-1}(s)\sum_{j=1}^{N} y_j^{T_j}(s) + A^{-1}(s)R. \tag{36}$$

Elements a_{ij}^{-1} of matrix A^{-1} are transfer functions from j knot to i knot of initial system with all open quantifiers. Actually, considering quantifiers and vectors $y_i^{T_i}(s)$ to be independent input effects which can be defined as zero, from (4) we obtain

$$x_i(s) = \sum_{j=1}^{N} a_{ij}^{-1} r_j \ , \tag{37}$$

where: r_j – vector elements of normalized input R.

On the basis of (36), uninterrupted discrete system 'complex graph' can be created, which gives us system transfer functions by applying the Manson formula. A complex graph is uninterrupted discrete system initial graph and discrete graph combination, made up $y_i^{T_i}(s)$.

In (36), variables $y_i^{T_i}(s)$ are input signals of keys which we will define as input variables. Having formed N discrete graph for $y_i^{T_i}(s)$, we may delete quantifiers from the initial graph and connect input knots of keys with equivalent knots of discrete graphs by means of branches with identity intensification coefficients. In this way, we obtain a complex system graph to which the Manson formula is applicable.

Consider uninterrupted discrete system with N quantizers, so that discreteness es are as follows

$$\frac{T_i}{T_{i+1}} = \frac{b_i}{q_i} \qquad i = 1,\ldots,N-1. \tag{38}$$

The system of linear algebraic equations with the time rate for this system is shown in (36).

Multiplying (36) to the correspondent B_i, we obtain a ratio system

$$y_i(s) = B_i A^{-1}(s)\sum_{j=1}^{N} y_j^{T_j} + B_i A^{-1}(s)R \quad i = 1,\ldots,N \ . \tag{39}$$

Let T – discrete time – be equal to the least common multiple time of all N times T_i. We can carry out discrete conversion of vector $x(s)$ to least common multiple – time T

$$x^T(s) = (A^{-1}(s)\sum_{j=1}^{N} y_j^{T_j}(s))^T + (A^{-1}(s)R)^T .$$

(40)

Assuming the meaning of $\dfrac{T}{T_i} = n_i$, and using discrete transformer property

$$[g(s)z^T(s)]^{nT} = \sum_{i=o}^{n-1} (g(s)e^{-iTs})^{nT} (z(s)e^{iTs})^{nT} ,$$

we can write

$$x^T(s) = \sum_{i_1=0}^{n_1-1} (A^{-1}(s)e^{-i_1 T_1 s})^T (y_1(s)e^{i_1 T_1 s})^T + \ldots +$$

$$+ \sum_{i_N=0}^{n_N-1} (A^{-1}(s)e^{-i_N T_N s})^T (y_N(s)e^{i_N T_N s})^T + .$$

(41)

$$+ (A^{-1}(s)R)^T$$

Let us identify

$$Y_{1(i_1)} = y_1(s)e^{i_1 T_1 s} \ldots Y_{N(i_N)} = y_N(s)e^{i_N T_N s}; \; i_j = 0,\ldots,(n_j - 1); \quad j = 1,\ldots,N.$$

We obtain equation of vectors $Y_{1(i)}^T,\ldots,Y_{N(i)}^T$, then multiply in (39) each equation to

$e^{i_j T_i s}$ $(i_j = 0,\ldots,(n_j - 1), \quad j = 1,\ldots,N)$.

$$\begin{cases} Y_{1(i_1)} = (B_1 A^{-1}(s)e^{i_1 T_1 s}\sum_{j=1}^{N} y_j^{T_j}(s)) + (B_1 A^{-1}(s)Re^{i_1 T_1 s}) & i_1 = 0,\ldots,(n_1 - 1) \\ \qquad\qquad\qquad\qquad\vdots \\ Y_{N(i_N)} = (B_N A^{-1}(s)e^{i_N T_N s}\sum_{j=1}^{N} y_j^{T_j}(s)) + (B_N A^{-1}(s)Re^{i_N T_N s}) & i_N = 0,\ldots,(n_N - 1) \end{cases}$$

42)

Let us carry out discrete transformation (42) to the least common multiple – time T

$$\begin{cases} Y^T_{1(i_1)} = \sum_{k_1=0}^{n_1-1} (\overline{W}_{1(i_1)} e^{-k_1 T_1 s})^T Y^T_{1(k_1)} + \sum_{k_2=0}^{n_2-1} (\overline{W}_{1(i_1)} e^{-k_2 T_2 s})^T Y^T_{2(k_2)} + \ldots + \\ \qquad\qquad + \sum_{k_N=0}^{n_N-1} (\overline{W}_{1(i_1)} e^{-k_N T_N s})^T Y^T_{N(k_N)} + \Phi^T_{(1)} \quad i_1 = 0,\ldots,(n_1-1) \\ \qquad\qquad\qquad\qquad\qquad\qquad \vdots \\ Y^T_{N(i_N)} = \sum_{k_1=0}^{n_1-1} (\overline{W}_{N(i_N)} e^{-k_1 T_1 s})^T Y^T_{1(k_1)} + \sum_{k_2=0}^{n_2-1} (\overline{W}_{N(i_N)} e^{-k_2 T_2 s})^T Y^T_{2(k_2)} + \ldots + \\ \qquad\qquad + \sum_{k_N=0}^{n_N-1} (\overline{W}_{N(i_N)} e^{-k_N T_N s})^T Y^T_{N(k_N)} + \Phi^T_{(N)} \quad i_N = 0,\ldots,(n_N-1) \end{cases} \tag{43}$$

where:

$$\begin{aligned} \overline{W}_{j(i_j)} &= B_j A^{-1}(s) e^{i_j T_j s} \quad j = 1,\ldots,N, \\ \Phi_j &= B_j A^{-1}(s) R e^{i_j T_j s} \quad j = 1,\ldots,N. \end{aligned} \tag{44}$$

Calculating together (43) and (41) and introducing Y_i we may obtain input-output relation for all variable systems.

This system contains

$$n + \sum_{j=1}^{N} m_j n_j$$

equation, where: n – vector dimension $x(s)$; m_j – number of keys with times T_j; n_j – number equal to the relation of the least common multiple time to time T_j.

Ratio (41), after introducing Y_i, can be accepted as discrete system description, inputs of which along with outputs of initial uninterruptedly discrete system are variables $Y^T_{j(i_j)}$ $j = 1,\ldots,N;$ $i_j = 0,\ldots,(n_j - 1)$. As their representation is

$$y^{T_j}_j = \sum_{i_j=0}^{n_j-1} e^{-i_j T_j s} [y_j e^{i_j T_j s}]^T = \sum_{i_j=0}^{n_j-1} e^{-i_j T_j s} Y^T_{j(i_j)} . \tag{45}$$

it can be considered to be a result of input signals parallelization of keys with time T_j in the initial system on n_j branches, each of which has transfer coefficient

$$e^{-i_j T_j s} \quad i_j = 0,\ldots,(n_j - 1).$$

Then, the initial system in (9) can be specified by the discrete graph in which keys are open and their outputs are input signals into the system along with initial input signals. At the same time, each of these new inputs from quantifiers T_j ($j=1,\ldots,N$) can be represented as a set

14

of inputs $Y^T_{j(i_j)}$ $j = 1,...,N;$ $i_j = 0,...,(n_j - 1)$ and, by connecting certain knots by identity connections, we can formulate the input-output ratio according to the final discrete graph.

Discrete graphs for $Y^T_{j(i_j)}$ are built on the basis of (43). There should be

$$k = \sum_{j=1}^{N} n_j$$

discrete graphs.

The construction of algorithm for uninterruptedly discrete system signal graph on the basis of equations (41) – (43) will be as follows:

1. On the basis of structure scheme an uninterruptedly discrete system initial graph is formed. All quantifiers in it, T_j $j = 1,...,N$, are considered to be regulated by diminution open, at the same time output signals of keys are considered to be input ones into the system where $\sum_{j=1}^{N} m_j$ is the quantity of black knots and m_j – key number with time T_j.

2. Form n_j $j = 1,...,N$ of the discrete graph, corresponding to time T_j on the following procedures:

a1) $j = 1;$

a) in the initial graph we compose only knots corresponding to quantifiers input signals with time T_j and connected with them input knots (initial and from quantifiers). As a result we obtain the intermediate graph;

a2) $i_j = 0;$

b) in the intermediate graph we replace all transfer functions of links W with $We^{i_j T_j s}$;

c) we make input signals parallel from keys T_k, $k = 1,...,N$, $k \neq j$ to n_k branches, substituting   each   transfer   function   of   the   branch W for $We^{-l_k T_k s}$ $l_k = 0,...,(n_k - 1)$, $k = 1,...N$, $k \neq j$.

d) we substitute knots for black, and transfer functions for their discrete transformation in time T. We enter input knots in the Table through $Y^T_{k(i_k)}$, then state conformity between knots x^T and $Y^T_{k(i_k)}$ by means of identity connections, where it is necessary. We obtain a discrete graph corresponding to time T_j.

e) points b) ÷ d) are now executed n_j times, supposing $i_j = i_j + 1$; this results in discrete graph n_j, corresponding to time T_j, $i_j = 0,...,(n_j - 1);$

f) j – we increase by one and repeat points a) ÷ e) for discrete graph construction for all N quantifiers, i.e. $j = 1,...,N$.

3. Now we turn back to the initial graph and make all input signals parallel from keys $y_j^{T_j}$, $j = 1,...,N$, into n_j corresponding branches $j = 1,...,N$.

We transfer each function from n_j branches W and substitute for $We^{-k_j T_j s}$ $k_j = 0,...,(n_j - 1)$, $j = 1,...N$, respectively. We now enter the input knots in the Table through variables $Y_{j(i_j)}^T$, $i_j = 0,...,(n_j - 1)$.

We connect to the marked inputs all of discrete graphs, got in item 2, by single connections. We set, where necessary, other single connections, proper identical knots. We get the component graph.

4. We substitute all the knots with black, and transfer functions on their discrete transformations on time the least common multiple – on time of T. We get the final discrete graph of the system.

5. By applying the Mason rule we determined the necessity of input-output correlation for the system variables of x^T for the obtained graph of the system.

Now we can formulate the algorithm of receipt of signal graph of multivariate of the continuously-discrete system for the case of multiple times.

1. On the basis of the flow diagram of the system, an initial graph containing white and black knots is formed. Black knots have indexes of Ti, $I = 1,..,N$, proper to the value of time of discreteness. Times are considered well-organized on a decrease. In the initial graph keys are considered broken; here the output signals of keys are considered entrance knots in the system and black knots correspond to them.

2. On the initial graph of the system, through the application of the Mason algorithm, the discrete graph of 1st level is formed in conformance with the following:

All the knots, being by an entrance for the keys with the smallest time of *TN,* get out in the initial graph. They are considered output knots. All the entrance knots, related to the indicated output, get out then. Entrance knots can be the entrance signals of the initial system, and also outputs of keys with large times. In the Mason algorithm, connections between these knots are determined and the intermediate count of 1st level is formed.

Further, all white knots of the intermediate graph are replaced with black knots, with the index of variables of *TN*, which corresponds to discrete transformation of variables in time of TN, and the transmission functions of connections are replaced by their discrete transformations in time of *TN*. Black knots that are proper discrete variables for large times remain unchanged. The discrete count of the system of 1st level is formed in the same manner. We set, where necessary, single connections for proper identical knots.

3. The component graph of 1st level is formed. It turns out to be a combination of the initial graph of the system and discrete graph of 1st level. Thus the proper black knots of the graph, being weakened signals of keys Ti, are united by single connections, $i = 1,..,N$. A component graph of 1st level is the basis for the construction of discrete graph of 2nd level.

4. Algorithms for the discrete graph of 2nd level, et cetera till *N*-th number of levels 2, are formed as per item a, with the change that in place of keys with time of TN, keys are utilized accordingly with time of *TN-1* et cetera, till time of T_1. As a result of the expounded procedure, *N* number of discrete counts of the system will be formed.

5. The final count of the system is further formed, which turns out to be a combination of the initial and N of discrete counts of the system.

By single connections, the accordance of knots of discrete counts is set with the entrance knots of the initial count.

16

6. In the algorithm of Mason, the input-output correlations are determined for the system variables of multirate continuously-discrete system with multiple times.

The offered method provides a formalised procedure of construction of input-output correlations of multirate continuously-discrete systems.

6. ABOUT ANALYSIS OF MULTIRATE SYSTEMS PROCESSES

Observed multirate systems applications in the form of equivalent one-time models give common reasons for process analysis in given one-time systems. While observing resultant correlation, it is not difficult to see that in all cases impulse images of equivalent models outputs have the form of rational functions of given variables. Turning to Z–images we will have an equation defining the output images as of z. They correspond to:

$$Y(kT) = \frac{1}{2\pi j} \oint f(z) z^{k-1} dz \ .$$
(46)

The closed contour of integration in equation (46) covers all poles $f(z)$. Thus, the problem of process analysis in given multirate systems becomes reduced to the problem of rational functions of pole distribution analysis $f(z)$ concerning a single circle.

References

[1] SEDLAR M. BEKEY G. *Signal Flow Graphs of Sampled–Data Systems*: A New formulation//IEEE Trans. on Automatic Control, Vol. AC-12,2,April 1967, pp.154-161.

[2] KUO B. *Teorija i proektirovanie cifrovykh sistem upravlenija* (in Russian).- M.:Mashinostroenie, 1986.

[3] COFFEY T.C., WILLIAMS I.J. *Stability Analysis of Multiloop, Multirate, Sampled Systems* // AIAA Journal, Vol.-4, December 1968, pp.129-144.

[4] DZHURI E.H. *Impulsnye sistemy avtomaticheskogo regulirovanija*. (in Russian)- M.:Fizmatgiz,1963.

[5] GARTMAHER F.R. *Teoria matrix* (in Russian).- M.:Nauka, 1977. 576 s.

[6] ALBERTOS P., *Block Multirate Input–Output Model for Sampled–Data Control Systems*/IEEE Trans. On Autom. Control, vol.35, No.9, pp.1085-1088, 1990.

[7] ARAKI M., YAMAMOTO K., *Multivariable multirate sampled-data systems: state-space description, transfer characteristics, and Nyquist criterion*// IEEE Trans. on Control, vol.31, pp.145-154, 1986.

[8] DEZOER CH. VIDJASAGAR M. *Sistemy s obratnojj svjazju: vkhod-vykhodnye sootnoshenija*. (in russian)- M.:Nauka,1983. 380 s.

[9] MACIEJOWSKI, J.M. *Multivariable Feedback design*// Addison-Wesley 1989.

[10] SKOGESTAD S., POSTLETHWAITE I. *Multivariable Feedback Control. Analysis and Design*// Wiley, 1996.