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NONLINEAR EFFECTS COMPUTED IN GAUSSIAN BEAMS PROPAGATING IN LIQUIDS

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The existing theory of focused Gaussian nonlinear beams is based on the parabolic approximation which is not valid near to the sound source, and is restricted to weakly concave transducers. The authors have solved the exact problem of nonlinear propagation of focused Gaussian beams in lossy media without the above limitations. For this purpose a numerical model of the first author was used.

INTRODUCTION

In many ultrasonic applications piezoelectric concave transducers are used at high pressures causing nonlinear effects in a liquid medium or in human tissue where the water content is equal to about 70%. The Gaussian ultrasonic field has attracted wide attention due to its interesting radiation characteristics and simple mathematical description. Gaussian beams radiated by weakly focused piezoelectric transducers were applied for example in determining the temperature elevation caused at various insonation times during medical ultrasonographic inspection of patients (Filipczyński et al. 1993). Gaussian beams have also been found in ultrasonic microscopy (Germain and Cheeke, 1988).

1. THEORETICAL DESCRIPTION OF GAUSSIAN FOCUSED BEAMS

The theory and experimental confirmation of properties of weakly focused ultrasonic Gaussian beams was first given by Filipczyński and Etienne (1973). However it was restricted to linear theory only. Du and Breazeale (1987) using the parabolic approximation and a perturbation approach, were able to calculate the second harmonic in a weakly focused nonlinear Gaussian beam. Recently Du and Wang (1995) also using the parabolic approximation showed that in a weakly focused Gaussian field all harmonics maintain a Gaussian profile in the axial direction at any distance.

This general statement seems to be limited due to the parabolic approximation used which does not allow determination of the field near to the sound source.

The purpose of the present paper is to explore whether the parabolic approximation applied by the authors mentioned above introduces any modifications when compared with the exact solution.

2. NUMERCAL METHOD

In view of the limitations of existing nonlinear propagation theory the authors applied a numerical method for the exact determination of the harmonic generation at various pressure levels in Gaussian beams. Therefore a numerical model of the first author (Wojcik 1998) was used, which has been verified experimentally (Filipczyński et al. 1999), based on the nonlinear propagation procedure of Christopher and Parker (1991) and justified mathematically by Wójcik (2000).

For numerical determination a plane boundary pulse was transformed by a concave parabolic lens to obtain a focused beam. The details of this procedure were published in our previous papers (Filipczyński et al. 1999).

Ultrasonic pulses with the carrier frequency of 3 MHz composed of 4 cycles were used in the numerical calculations. The radii of the radiating circular transducers were equal to 1.5 cm and to 0.75 cm while the geometrical focus was at 10 cm and the physical one was 6 cm. Two values of the initial pressure amplitudes at the centre of the transducer were chosen to be p_0 = 1.4 MPa and 0.22 MPa. The numerical calculations were performed for ultrasonic beams in blood and in water.

It should be noted that in our work the knowledge of small signal attenuation coefficients is sufficient. In the applied numerical code the characteristic function $G_a(x)$ of the nonlinear increase of the absorption is used. So the obtained numerical results are valid for various pressures. This function of the nonlinear increase of the absorption was derived by Wójcik (1998) and verified experimentally (Filipczyński et al. 1999).

To calculate the exact pressure distribution in the beam it is necessary to know the boundary conditions on the transducer surface (for $z = 0$). Various pressure distributions at the transducer surface were assumed (uniform, approximated polynomial and Gaussian). However, in this presentation only the Gaussian distribution will be considered.

3. COMPUTED GAUSSIAN BEAMS

The results of computations may be presented in the grey-scale (in z, r coordinates), as the amplitude distributions in linear and logarithmic scales (along z coordinate) and finally as the amplitude distributions in linear scale (along the r coordinate) for z from 0 to 180 mm from the transducer.

We have found numerically the ultrasonic field generated by the Gaussian velocity on the transducer surface in a lossy nonlinear fluid medium (blood) without limitations of weakly focus, performing calculations up to the third harmonic. The particle velocity on the transducer surface was assumed to be of the form

$$\exp\left(-r^2 / R_0^2\right)$$

where r denote the radial coordinate and R_0 the radius for which the velocity decreased to 1/e = 0.37 of its maximum value. We assumed such a Gaussian distribution, for which $R_0 = 0.5\,a$, and two values of the radius a of the circular transducer equal to 15 mm and 7.5 mm.

From the many computed cases we can observe the influence of the initial pressure obtained for $z = r = 0$ and also compare the results from the standpoint of the attenuation of the medium.

The pressure distributions as a function of axial distance z presented in Fig. 1 demonstrate the existence of minima which are the evidence of nongaussian beam

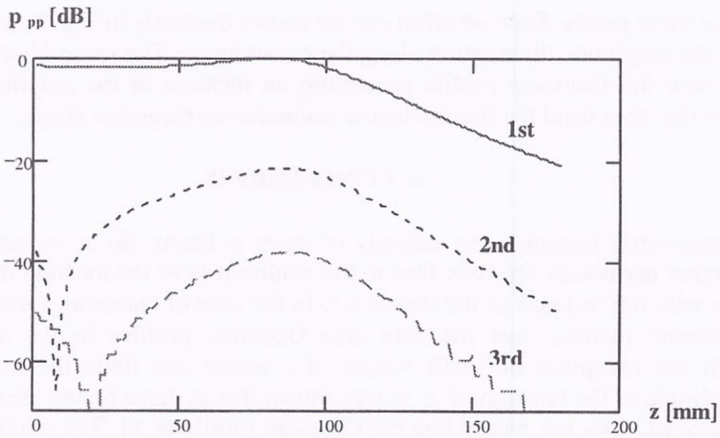

Fig. 1 Pressure distributions computed along the beam axis *z* for the 1-st (solid line), 2-nd (dashed line) and 3-rd (dotted line) harmonics presented in logarithmic scale.

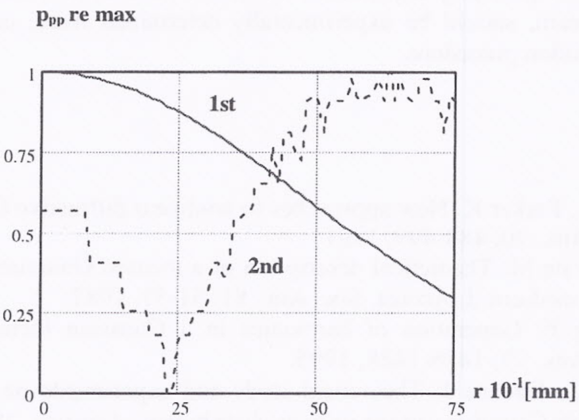

Fig. 2 Shapes of the pressure amplitudes for the 1-st (solid line) and 2-nd (dashed line) harmonics computed at the distance of $z = 9$ mm in relation of maximum values.

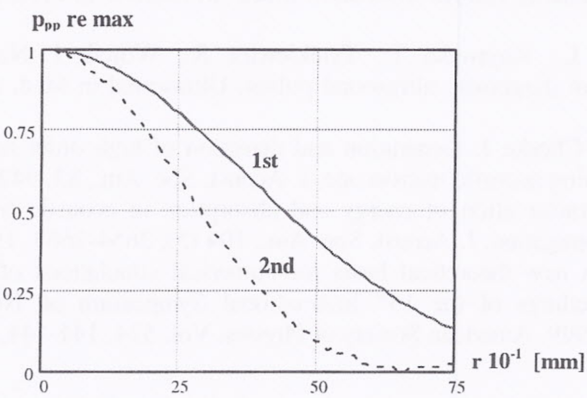

Fig. 3 Shapes of the pressure amplitudes for the 1-st (solid line) and 2-nd (dashed line) harmonics computed at the distance of $z = 30$ mm in relation of maximum values.

distributions at these points. Such an effect can be shown distinctly in Fig. 2 when computing for $z = 9$ mm the amplitude distribution along the r coordinate. The second harmonic deviates significantly from the Gaussian profile presenting an increase of the amplitude with the r coordinate. On the other hand the first harmonic maintains its Gaussian shape.

4. CONCLUSIONS

The piezoelectric transducer in majority of cases is linear. So at its surface (z=0) the second and higher harmonics are zero. Due to the nonlinearity of the medium their amplitudes are increasing with the increase of the distance z. In the case of Gaussian focused waves they show a monotonic increase and maintain also Gaussian profiles in the axial direction. However with the exception of small values of z where one finds the minimum of the harmonic amplitude as the function of z. It was shown that at those points the beam profile is completely changed, does not resembling the Gaussian profile at all. The minima are situated at different small values of z depending on the order of harmonic and on the value of initial pressure. This problem may be important for measurement procedures in the vicinity of the transducer surface. The boundary conditions, which are critical for the description of the whole propagating beam, should be experimentally determined there, and then taken into account in the computation procedure.

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