

ISSUES OF DESCRIPTIVE GEOMETRY COMPRISED IN THE WORKS „GEOMETRIA WYKREŚLNA Z ZASTOSOWANIEM DO PERSPEKTYWY, CIENIÓW, KAMIENIARSTWA, CIESIOŁKI I INNYCH KONSTRUKCYY, WYPRACOWANA DLA UŻYTKU SZKOŁY WOYSKOWEY APPLIKACYJNEJ”¹ AND „ZASTOSOWANIA GEOMETRYI WYKREŚLNEJ”² BY FRANCISZEK SAPALSKI

Andrzej KOCH¹, Tomasz WIEJA²

^{1/} Faculty of Applied Mathematics AGH University of Science and Technology
30-059 Kraków, al.Mickiewicza 30, Poland
¹ mail: twieja@poczta.fm
² email: makoch28@interia.pl

Abstract. In the article the theory and practice of Descriptive Geometry applications comprised in the first textbook on Descriptive Geometry printed on the Polish land in 1822 in Warsaw (volume I) and in Cracow (volume II, journal I) have been presented. The substantial contents of the presented textbook reflects well on the world high – level of the at that time didactics of Descriptive Geometry in Poland. The assessment and essential analysis contain references to of the time methodology of teaching Descriptive Geometry and to the form of recording of three – dimensional construction as a plane drawing.

Keywords: Franciszek Sapalski, Descriptive Geometry, applications, methodology, journal

1 The life and works of Franciszek Sapalski

Franciszek Sapalski was born on April 1, 1791 in Warsaw. He graduated from the famous Krzemieniec High School. In 1809 he began to work in the Ministry of Public Income and Treasury. He possessed outstanding mathematical skills which he decided to use during his military service. In 1810 he joined the ranks of the armed forces of the Warsaw Grand Duchy as an artillery sergeant. After graduation from a military school of artillery and military engineers he was promptly promoted to the rank of second lieutenant and next to lieutenant. During the time of his military service he begins to work on writing Descriptive Geometry textbook and attends, as a private person, lectures by Jan Joachim Liwet – first professor of the subject at application school of the Warsaw Grand Duchy. In 1812 he submits a concept behind the first, written in the Polish language, textbook on Descriptive Geometry, for assessment of the Society of Friends of Sciences in Warsaw. Unfortunately, the work was not completed because in 1812 he took part as an adjutant major in the Napoleonic Campaign. In 1813 he left the army because of bad health and devoted himself to scientific career. Then he goes to Paris attending public lectures on: mathematics given by Lacroix, optics by Biot, physics by Gay – Lussac, chemistry by Thenard, astronomy by Delambre and Argo and mechanics by Poisson. Having obtained support from the Russian ambassador count Pozzo di Bordo he starts studying Descriptive Geometry at the technical university (l’Ecole

¹ Descriptive Geometry with Application to Perspective, Shadows, Masonry, Carpentry and Other Constructions, Worked for the Use of Military Application School

² Applications of Descriptive Geometry

Politechnique) under Hachett. In 1816 he was appointed deputy professor of Descriptive Geometry and Mechanics at the Jagellonian University (UJ). In the same year he becomes a member of Cracow Scientific Society. On November, 1817 he prepares a scientific treatise on "Stereotomia or Descriptive Geometry Theory". As a result of the treatise F. Sapalski was appointed regular professor of Descriptive Geometry and Mechanics at the Jagellonian University. Till 1833 he gives lectures in Polish and then goes into retirement. At that time he holds numerous scientific and civic offices. In 1820 he is elected a representative of UJ to the Assembly of National Representation and in 1823 takes up the post of the Dean of the Faculty of Mathematics at UJ. In 1824 he becomes a corresponding member of Warsaw Scientific Society and a year later is appointed Senator at the Ruling Senate of Free City of Cracow. In 1822 he publishes the first volume of his work "Descriptive Geometry". For this work he is rewarded with a diamond ring by Tsar Alexander I. He dies on April 2, 1838 in Cracow.

2 Theoretical Geometric Problems comprised in the 1st volume of the work "Geometria Wykreślna"

The textbook worked out by F. Sapalski consists of descriptive and graphical (lithographs) parts. The subject of geometry is divided into eight chapters:

First part – "*O położeniu punktu, linii prostej, płaszczyzny i figur z nich złożonych*" (On location of point, straight line, plane and figures composed of them).

Second part – "*Zagadnienia dotyczące się punktu, prostej i płaszczyzny*" (Problems concerning point, straight line and plane).

Third part – "*O liniach krzywych, ich stycznych i normalnych*" (On curved lines, tangent and normal to them)

Fourth part – "*Opisanie powierzchni krzywych, ich wykreślenia na płaszczyznach rzutów i niektóre zagadnienia dotyczące się powierzchni*" (Description of curved surfaces, their drawings on projection planes and some problems concerning surfaces).

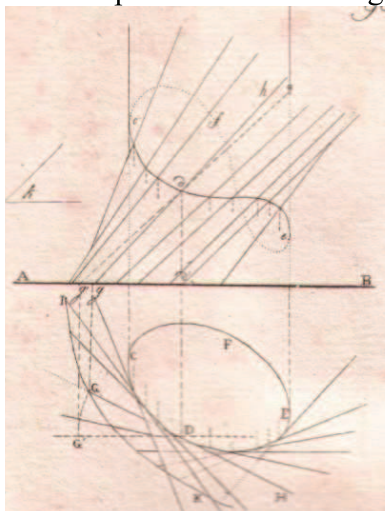


Figure 1: Construction of a twist surface with one given directrix. Generating line is tangent to cylindrical surface belonging to the given directrix and inclined at given angle to a plane perpendicular to the generators

Fifth part – "*O płaszczyznach stycznych i liniach normalnych do powierzchni krzywych*" (On planes tangent and lines normal to curved surfaces).

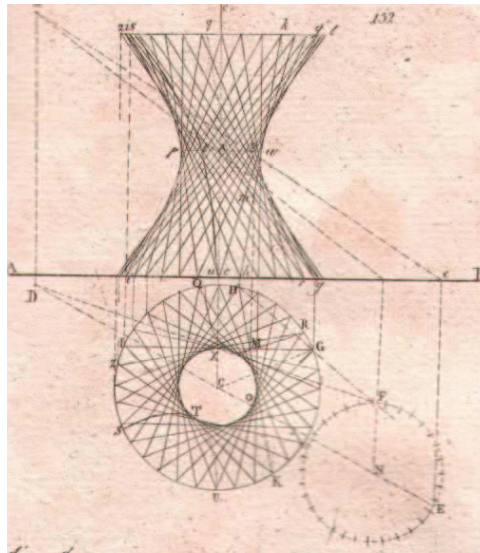


Figure 2: Construction of a regular star inscribed into a circle belonging to given point and tangent to given surface – hyperboloid of two sheets

Sixth part – „*O przecięciach się powierzchni i liniach stycznych tych przecięć*” (On intersections of surfaces and lines tangent to these intersections).

Seventh part – „*O rozwinięciu powierzchni rozwijalnych*” (On development of developable surfaces).

Eighth part – „*O własnościach ogólnych rozległości graficznych*” (On general properties of graphic extensiveness).

The contents of separate chapters consist of general analytical and practical parts where individual spatial problems are presented and based on examples of existing problem solutions. We present this pattern according to the composition of chapter third “On curved lines, tangent and normal to them “.

The analytical part contains two subsections:

1. “*I Opisania poprzednicze*” (Preceding descriptions) – they concern definitions of curved, tangent and normal lines along with the method of their representation on the Monge projection planes.

2. “*II Rodzenie się krzywych*” (Creation of curves) – it concerns the method of making curves on a plane and in space – movement of a point toward two given points, movement of a point toward three points in space and lines normal to a given curve which by means of their mutual intersections generate a curve.

The practical part contains three subsections within which both graphical and descriptive illustrations of selected geometrical problems have been presented:

3. “*III Zagadnienia tyżące się linii krzywych*” (Problems concerning curved lines).

4. „*IV O liniach stycznych normalnych linii krzywych płaskich*” (On lines tangent to lines normal to plane curved lines).

5. „*V O liniach stycznych i płaszczyznach normalnych linii krzywych podwójnie krzywych*” (On lines tangent and planes normal to curved lines double – curved).

3 Practical Applications of Descriptive Geometry comprised in the 2nd volume of the work, „*Zastosownia Geometrii Wykreślnej*” Journal I - edited in Cracow in 1839.

„*Zastosowanie pierwsze* (First Application): *Rozmaite zagadnienia geometryczne tyżące się: I. Geometrii teoretycznej. II. Kąta bryłowego z trzech ścian złożonego czyli Trygonometry*

kulistej. III. Geometrii praktycznej” (Various geometrical problems concerning: I. Theoretical Geometry. II. Solid angle composed of three faces, which is spherical trigonometry. III. Practical Geometry).

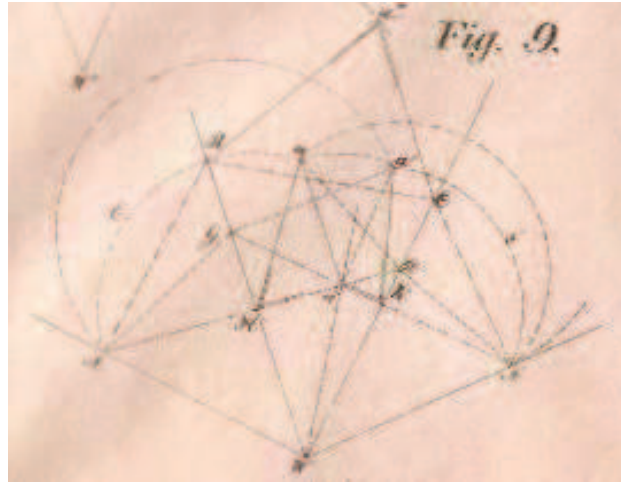


Figure 3: Various geometrical problems. (First Application) Given three - face angles in a solid triple - face angle. Find three slope angles

„Zastosowanie drugie (Second Application): *Teoryia cieniów. I. Opisania poprzednicze. II. Cienie ciał oświetlonych przez promienie światła do siebie równoległe. III. Cienie ciał oświetlonych przez punkt świecący. IV. Cienie ciał oświetlonych przez ciało świecące*” (Theory of Shades. I. Preceding descriptons. II. Shades of the bodies lit by beams of light parallel to each other. III. Shades of the bodies lit by a shining point. IV. Shades of the bodies lit by a shining body).

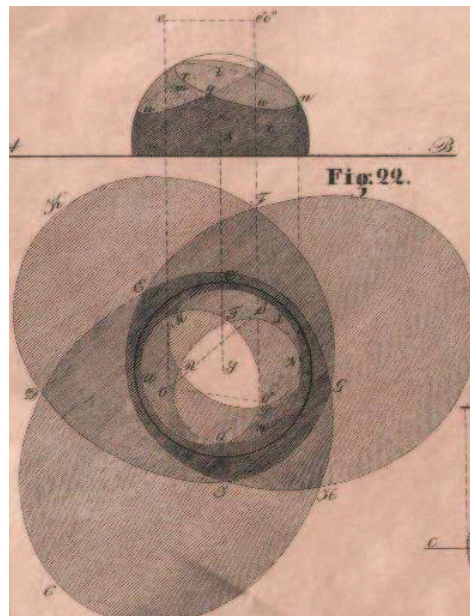


Figure 4: Theory of Shades. (Second Application)

Journal II - unpublished, concerning the theory of optics (according to the editor it was to be published in March 1839).

Construction of a shade and half-shade of ellipsoid of revolution lit by shining sphere
„Zastosowanie trzecie (Third Application): *I. Opisania poprzednicze. II. Punkta błyszczące*

(point brillant). III. Obrazy błyszczące (image brillant). IV. Widma świetlne odbite (spectre lumineux refracte). V. Widma świetlne rzucone (spectre lumineux porte). VI. Obrazy odbite lub złamane (image refractee et reflechie). VII. Przekształcenia (anamorphose). VIII. Sposób widzenia wielkości i kształty pozorne (apparent)” (I. Preceding descriptions. II. Glistened points. III. Glistened images. IV. Reflected light spectrums. V. Cast light spectrums. VI. Reflected or refracted images. VII. Transformations. VIII. The way of vision of magnitudes and apparent shapes).

Journal III – unpublished, concerning the theory of perspective (according to the editor it was to appear in three months after Journal II).

„Zastosowanie czwarte (Fourth Application): *Perspektywa. I. Obwody ciał pozorne (contour apparent). II. Perspektywa ciał. III. Perspektywa ciekawa. IV. Perspektywa kulna*” (Perspective. I. Apparent parameters of the bodies. II. Perspective of the bodies. III. Interesting perspective. IV. Spherical perspective).

Journal IV – unpublished, concerning the theory of light – the solar line (according to the editor it was to appear in three month after Journal III).

„Zastosowanie piąte (Fifth Application): *Kreślenie zegarów słonecznych (kompasów) (Gnomonika). I. Opisanie poprzednicze. II. Cztery przypadki kreślenia kompasów*” (Drawing of sundials (compasses) (Gnomonics). I. Preceding descriptions. II. Four cases of compasses drawing).

Journal V – unpublished, concerning the theory of shaping of earthwork embankments (according to the editor it was to appear in three months after Journal IV).

„Zastosowanie szóste (Sixth Application): *Nauka szanowania (foryfikacja). I. Opisanie poprzednicze. II. Zagadnienia dotyczące się zwysoczenia (defilement). III. Wyznaczenie płaszczyzn okolicy. IV. O osłonięciu w ogólności. V. Dzieje tej nauki w rozkładzie na szkoły*” (The science of fortification building. I. Preceding descriptions. II. Defilement problems. III. Determination of vicinity planes. IV. About shielding in general. V. History of this science in schedule on schools).

Journal VI – unpublished, concerning Descriptive Geometry applications to construction industry.

„Zastosowanie siódme (Seventh Application): *Stereotomia właściwa. – A) Kamieniarstwo (coupe des pierres). I. Opisanie poprzednicze. I. Różne gatunki sklepień. B) Ciesiołka (charpenterie). I. Opisanie poprzednicze. II. Rozmaita budowa dachów i szczególnych tychże części, jako to: grzbietów prostych, ukośnych, krokiew prostych, złożonych, podkopów, dymników, podpasków – tudzież obłąków schodowych, mostów i różnych cembrowań*” (Real stereotomy – A) Masonry. I. Preceding descriptions. II. Various types of vaults. B) Carpentry. I. Preceding descriptions. II. Various structure of roofs and their particular parts, namely: straight and slanting ridges, straight and complex rafters, tunnels, roof vents, girds – as well as staircases, bridges and various casings).

As the last element of the work “*Zastosowania Geometrii Wykreślnej*” was to be “*Słowniczek techniczny*” (Technical Dictionary) the plan and scope of which resulted from earlier presented Journals.

4 Methodology of the Descriptive Geometry lecture according to F. Sapalski

Franciszek Sapalski as early as in the preface to his textbook introduces and explains the philosophy and the method of teaching Descriptive Geometry. It is noteworthy that for the first time in a printed work the name “Descriptive Geometry” is used as a separate field of science. Also for the first time Sapalski realizes the essence of professional training in solving

spatial problems. He pays special attention to the combination of theory and practice that is to the application of Descriptive Geometry “because it not only trains and develops the young man’s mind as each part of Mathematics does but also gives methods well adapted to the needs and national conveniences which are the arts, artistry, craft and other various inventions” („*bo nie tylko ćwiczy i wydoskonala umysł młodzieńca, tak jak każda część Matematyki; ale podaje sposoby przystosowane do potrzeb i wygod krajowych iakimi są sztuki, kunszta, rzemiosła i inne rozliczne wynalazki*”). The author clearly defines methodological order which he intends to use in his work and explicitly determines the method of teaching Descriptive Geometry starting from general and theoretical knowledge to practical applications. The guiding idea is clear and legible; to separate the analytical part from the synthetic one. General knowledge is comprised in the 1st volume of the textbook entitled „*Geometria Wykreślna z zastosowaniem do perspektywy, cieniów kamieniarstwa, ciosiołki i innych konstrukcyi, wypracowana dla użytku szkoły woyskowej aplikacyyney*” and issued in 1822 in Warsaw. Practical knowledge, in other words applications, was to be presented in the 2nd volume in the form of journals. In turn, the journals were to be arranged according to the trade outline. They were also supposed to present selected and most difficult, according to the author, practical problems solved on the basis of the Descriptive Geometry principles. Unfortunately, the premature death of F. Sapalski prevented the above – said ideas from realization. As early as in 1839 after the author’s death, the work „*Zastosowania Geometrii Wykreślnej*” Journal I based on his notes, was issued. It is worth mentioning that Sapalski formulated some requirements that must be met in order to comprehend the essence of his lecture in Descriptive Geometry. The knowledge of geometry, algebra, the basics of analytical geometry and mechanics as well as drawing skills as *sine qua non* make it possible to get to know consciously theoretical principles and practical applications of Descriptive Geometry. The particular issues shown in the 1st volume constitute a development of theoretical problems and contain concrete exercises given in the empirical form; theoretical solution, structural drawing and conclusions where additional geometrical elements extending structural range of the presented solution were proposed..

As the conclusion of each chapter is its contents, interesting because the author refers in it to applications of selected problems and submits the theories of the time as well as the methods of their presentation to critical analysis.

5 Applications of Descriptive Geometry–Journal I (*Zastosowania Geometrii Wykreślnej*)

In the only journal issued as an element of the 2nd volume of „*Geometria Wykreślna*” (Descriptive Geometry) F.Sapalski presents practical applications of the Monge projection theory. Worthy noticing is the fact that in chapter „*Zastosowanie pierwsze rozmaite zagadnienia geometryczne*”, pkt I „*Zagadnienia geometryi teoretycznej*”(First Application – various geometrical problems, p.I - Problems of theoretical geometry) the author – using problem (N.1-N.11) as an example – solves metric issues: distances and angles between basic elements of space in both general and special positions. In problems 13 - 19 he takes up theory of tangency of lines and spheres.

In chapter II: „*Zagadnienia tyczące się kąta bryłowego z trzech ścian złożonego czyli Trygonometrii kulistej*” (Problems concerning a solid angle composed of three faces, that is spherical trigonometry) Sapalski shows examples of solutions (N.22 – N.29) of geometrical problems (angles between planes) appearing in the system of three planes in general positions. This is the only problem concerning the theory of polyhedrons presented in the Sapalski’s work In chapter III. „*Zagadnienia tyczące się Geometrii praktycznej*”(Problems concerning

descriptive geometry) the author introduces elements of the vision theory and takes up the task of finding characteristic points in the terrain so that, not leaving the post, to find location of the point where we are and its uplift above other points marked on the card (N.30 - N.33). The issues combine the geodetic problems (topography) and the recording of spatial positions of separate objects according to the Monge projection method. „*Geometrii wykreślnej zastosowanie drugie*” (The second application of descriptive geometry) presents the theory of shades depending on the types of lighting. Parallel lighting is represented in problems N. 18-N.37 and based on the examples of the shades outlining from engineering (bridges), architectural (e.g. attic base, Doric capital) and machine (shade on the” three-prismatic screw”) objects. The issues are backed up with theoretical considerations (N.1 – N.17) presented in subsection „*Opisanie poprzednicze*” {Preceding descriptions), where the author, basing on the physical properties of light, presents and explains the theory of objects illumination and systematizes definitions as well as, for the first time in Poland, introduces geometrical nomenclature concerning the shade construction.

6 Conclusions

In the works „*Geometria Wykreślna z zastosowaniem do perspektywy, cieniów, kamieniarstwa, ciesiołki i innych konstrukcyi, wypracowana dla użytku szkoły wojskowej aplikacyjney*”³ and „*Zastosowań geometrii wykreślnej*”⁴ by F.Sapalski, the theory and practice – therefore examples of applications in the engineering problems – have been presented. F. Sapalski was comprehensively educated in the fields of mathematics, physics, chemistry, optics, astronomy and mechanics. Having acquainted with Descriptive Geometry on the Hachett lectures he broadened his knowledge by practically touring industrial plants in southern France. His analytical and practical studies resulted in the first Polish academic publication on Descriptive Geometry. In the 1st volume the author elaborates theoretical problems of recording points, lines and planes in the Monge projection method as well as parallelism and perpendicularity of the basic elements of space. In addition he introduces a definition of rabatment in order to find true magnitudes of angles between elements of space i.e. between straight lines and planes. It is worth noticing that there is no definition of the distances between the elements of space. This problem, among others, bound up with the construction of polyhedrons is omitted in the textbook content. The 2nd volume was supposed to contain applications of selected geometrical problems to the engineering practice. Unfortunately, only journal 1 out of the work composed of 7 journals, was published. The presented applications are shown in the form of real exercises together with the description of their solution methods. They reflect philosophy of the practicing military and artillery engineer and clearly show selected issues. Notice should be taken to the nomenclature related to Descriptive Geometry and introduced for the first time in Poland by F. Sapalski. For the contemporary reader the work written by Sapalski is very interesting because the presented geometrical problems are seldom considered in the current Descriptive Geometry textbooks.

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³ Descriptive Geometry with Application to Perspective, Shadows, Masonry, Carpentry and Other Constructions, Worked for the Use of Military Application School

⁴ Applications of Descriptive Geometry

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**PROBLEMATYKA GEOMETRII WYKREŚLNEJ W ZAWARTĄ
W PRACY F.SAPALSKIEGO „GEOMETRIA WYKREŚLNA
Z ZASTOSOWANIEM DO PERSPEKTYWY, CIENIÓW,
KAMIENIARSTWA, CIESIOŁKI I INNYCH KONSTRUKCYY,
WYPRACOWANA DLA UŻYTKU SZKOŁY WOYSKOWEY
APLIKACYJNEY” ORAZ „ZASTOSOWAŃ GEOMETRYI
WYKREŚLNEJ”**

W publikacji przedstawiamy teorię i praktykę zastosowań geometrii wykreślnej zawartą w pierwszym podręczniku geometrii wykreślnej wydrukowanym na ziemiach polskich w 1822 roku w Warszawie (tom I) oraz w Krakowie (tom II, zeszyt 1). Zawartość merytoryczna prezentowanego podręcznika świadczy o światowym poziomie ówczesnej dydaktyki przedmiotu geometria wykreślna na ziemiach polskich. Ocena i analiza merytoryczna zawiera podstawowe odniesienia do ówczesnej metodyki nauczania przedmiotu geometria wykreślna oraz do formy zapisu konstrukcji trójwymiarowej na rysunku płaskim.