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Design Analysis of Closed Vessel for Power Cartridge Testing

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Abstract. This paper discusses the design analysis of closed vessel (CV) for power cartridge application in water-jet disruptor. In this article, various design theories are presented in which the vessel is subjected to internal pressure. CV is a kind of pressure vessel utilized to evaluate the performance of power cartridge used for water-jet application. It is a test vessel which generates pressure - time profile by burning the propellant. Energy derived from burning of the propellant of power cartridge aids in neutralizing Improvised Devices (IED's). This energy creates high water-jet plume in the disruptor. In order to evaluate various performance parameters of the cartridge, CV design plays a vital role in the research and development activities, including, development, life trials, production, lot proof trials and life extension / life revision trials. CV is one of the methodologies / techniques from which energy generated is measured in terms of the maximum pressure (P_{max}) and the time to maximum pressure (TP_{max}).

This paper also discusses about various design aspects using the finite element method (FEM) and their comparative results with different design theories.

In the light of these theoretical, numerical, and experimental works, it was recommended that octahedral stress theory or van Mises theory should be used for vessel design. This satisfies the designer requirements. FEM analysis tool helps in reducing time & development cost.

Keywords: closed vessel, design analysis, disruptor, factor of safety, finite element method (FEM), internal pressure, power cartridge, stress, strain and water-jet disruptor

1. INTRODUCTION

A closed vessel (CV) is a pressure vessel or thick-walled container typically designed to hold fluid either gases or liquids at a pressure substantially different from the ambient pressure. General application of thick-walled cylinders includes, high pressure reactor vessels used in metallurgical operations, air compressor units, process plants, hydraulic tanks, pneumatic reservoirs, storage for gases like butane, LPG etc. The testing of power cartridge in vessel provides dependable and valuable means for evaluating the ballistic performance of different propellants. The vessel consists of cartridge holder, pressure transducer, body, and end plug. The test consists of burning of known mass of propellant in a fixed volume combustion chamber and recording the pressure-time (*P*-*t*) history via a pressure transducer [1]. Test equipment such as CV, designed to characterise the ballistic performance of propellants, must therefore be able to withstand increasingly firing pressures. The purpose of a cartridge holder is to retain the cartridge in the firing position. It also carries lead wire to make electrical connection. End plug is used for evacuation or release of the gases inside the vessel after propellant burning. CV design involves parameters such as temperature, maximum safe operating pressure, factor of safety and corrosion allowance. Consequently, vessel design, manufacture, and operation have been regulated by a designer. Power cartridges are typical initiators in which propellant combustion takes place for generating gases in short duration for performing various mechanical tasks [2]. The cartridge under study is filled with double base propellant. The shape of the propellant is spherical (called ball powder) [3]. On burning, they produce high temperature and high pressure gaseous products. The flame temperature of double base propellant is in between 2600 K to 3600 K [4]. The pressure vessels are designed with great care as bursting of it may cause loss to the property and life. Dhanaraj *et al.* [5] carried out pressure vessel design for static loading using ANSYS. The authors selected mild steel for the analysis. This paper presents, the complete CV design analysis of power cartridge for water-jet disruptor application using various theories. The objective of this paper was to design, fabricate, and test CV for evaluating the performance of power cartridge.

1.1. Background studies

Ramakrishnan [6-7] had explained the determination of various ballistics properties of propellant in CV using data acquisition system. Lokre *et al.* [8] concluded that the CV can be considered as a performance evaluation tool for the design, development and out-turn proof of crusher gauges and copper crushers. These studies have further opened up the possibility of calibrating ball copper-Mark 8 combinations in CV replacing gun firings. Song *et al.* [9] had proposed an ultrasonic full waveform analysis approach that can be efficiently used to identify the burning rates of a solid propellant in a wide range of pressure values. The authors also developed an ultrasonic measurement system in which the author proposed the approach to implement dedicated software. Dewangan and Panigrahi [10] had performed residual stress analysis of swage autofrettaged gun barrel via finite element (FE) method. Their study shows that the geometry should be designed thoroughly to determine the after effects. Bhetiwal *et al.* [11] studied that theoretical weight reduction of approximately 16 per cent is feasible with von Mises criterion as compared to Tresca criterion for autofrettaged gun barrel. Alegre *et al.* [12] reported that autofrettage technique is commonly used to produce compressive tangential residual stresses near the bore, which improve the fatigue life. The vessel material was stainless steel that shows a strong Bauschinger effect. Camilleri *et al.* [13] has investigated the effect of including a strain hardening material model on the calculated plastic load in pressure vessel design by analysis. Wilson and Skelton explained about the design of high pressure high cylinders using autofrettage pertaining to gun barrels [14]. There are two types of failures for the CV, the first mode of failure which occurs when deformation becomes excessive. The second type of failure is that due at higher magnitudes of pressure that may result into bursting leads to catastrophic failure [15]. A brief survey of a number of recent developments in pressure vessel design by the finite element analysis is discussed and assessed in light of code design requirements by Mackenzie *et al.* [16]. Snell and Jessop [17] describe the analysis by photo elastic methods of a model of a design for a full-bore closure for a reactor pressure vessel. The literature pertaining to CV was based on thick-walled pressure vessels.

In this paper the authors have therefore only considered those papers which are of most interest to the designer.

1.2. Formulation of the problem optimization

Mostly spherical vessels are used in industrial applications. However, their manufacturing is critical. Cylindrical vessels are most popular as they are easy to manufacture. Consider an elastic, thick-walled, axially symmetrical pressure vessel. It consists of a cylindrical body with the radius r and the length L, which is closed at both sides. This vessel is loaded by the internal pressure P_i.

The pressure wall thickness was chosen in such a way that pressure transducer capable to mount on the vessel body and volume to sense the pressure.

1.3. Approaches to the problem

CV failures can be grouped into major categories, which describe why a vessel failure occurs. The designer must be aware with categories, types of failures, types of stresses and loadings. The probable reasons of the failures are given below:

- material - improper selection or defects in material,
- design - inaccurate or incorrect design methods, incorrect design data, inadequate shop testing,
- fabrication - poor quality control checks, insufficient or improper fabrication procedures including welding.

2. CLOSED VESSEL DESIGN CONCEPT

As far as design is concerned, the basic requirement of any design should be economic, a sound design practise which can be manufactured. The general objective is to relate the time to failure to the geometry of the component under the given loading conditions, the environment where it is operating and the material of construction (MoC) [18]. The design parameters for the current CV are:

- be manufactured using relatively cheap fabrication techniques to minimize cost,
- be kept light weight,
- reliable sealing,
- be easy to assemble for scientific teams with minimum tools and in harsh conditions,
- withstand the firing stresses and high pressure,
- provide for the required electronic connections for pressure measurement,
- to enhance the life by providing corrosion protection layer,
- temperatures which can vary with position and time, giving rise to steady and transient thermal stresses. This causes extreme variations in material properties.

The more important areas for detailed study to finalised the design:

- determination of stress distribution in the vessel body,
- design of threads for end plug,
- design of threads for cartridge holder,
- method of assembly and removal of end plug.

2.1. Design consideration and assumptions

If the wall thickness for the geometry of the vessel of the shell is greater than 1/10 of the diameter of the shell, then it is said to be a thick cylinder. Another criterion to classify the pressure vessels as a thick shell if the internal fluid pressure is greater than 1/6 of the allowable stress, then it is said to be a thick shell. The design of CV depends on material selection, its properties, operating temperature, pressure and working environment.

2.1.1. Design considerations

The following factors are considered for CV design:

- the cylinder is made up of homogenous and isotropic,
- factor of safety,
- firing stresses,
- safety consideration.

2.1.2. Assumptions

The following assumptions are made in the analysis of CV design:

- in case of thick cylinder, the stress across the wall thickness cannot be assumed to be uniform. Circumferential or hoop and radial stress is not uniformly distributed,
- axial stress is constant,
- young modulus in tension or compression is same,
- plane section perpendicular to the longitudinal axis of cylinder before loading remains plane after loading. This assumption is approximately true at fair distance from restrained ends of cylinder,
- generalised Hooks law is applicable.

2.2. Material selection

Various grades of steel with different composition of materials are available in market and being used in CV manufacturing. The material selection is based on the service condition and design requirements. The materials used in CV manufacturing must comply for a factor of safety and safety norms. The selection of vessel material shall take into account the suitability of the materials with the maximum working pressure and availability of a manufacturing process. The specification for CV is suitable for the higher temperature services.

2.3. Design pressure

The pressure generated inside the vessel by working fluid is called a design pressure. It is therefore recommended to design a vessel and its parts for the higher pressure than the working pressure. A design pressure higher than the working pressure with 10%, whichever is the greater, shall satisfy the design criterion. The pressure exerted by the fluid (combustion products of propellant) on the walls of vessel shall be also considered. The maximum allowable working pressure (MAWP) for a vessel is the permissible pressure in its normal operating position. For every element of the vessel, this pressure is based on calculations using nominal thicknesses.

2.4. Design temperature

Design temperature is the temperature at which those components can withstand or shall be maintained in various parts. However, there is a maximum design temperature and a minimum design temperature. For most of the vessels, it is the temperature that corresponds to the design pressure. Design temperature for vessels under external pressure shall not exceed the maximum temperatures.

2.5. Corrosion allowance

The CV walls are exposed to propellant gases. These gases are corrosive in nature and reacting to the internal surface of vessel. In designing CV, it is generally necessary to make provision for some reduction in the wall thickness to take place under operating conditions as a result of corrosion or erosion or both [19]. Corrosion allowance 1 to 2 mm is considered depending on the acidic nature of fluid. The gases generated on burning of the propellant are acidic in nature and corrode or erode due to moving gases.

3. SIMPLE ELASTIC CYLINDER DESIGN

3.1. Estimation of dimensions of the CV

Selection of volume of the vessel is based on propellant mass of to sense that volume of the gas produced. Based upon rich experienced, a volume of 150 cc (approx) is selected for safe measurement of pressure.
Volume of vessel (V) is expressed as :

$$V = \text{area} \times \text{length} \tag{1}$$

Generally, it is assumed that the length l = 3 × the diameter d, to avoid collapsing of generated wave and reflected wave. The internal diameter is selected as 40 mm. From Eq. (1) the length is estimated as 120 mm.

In this article, it is a strong cylindrical chamber having 40-mm internal diameter, 80-mm external diameter, and the length of 120 mm. It is made in three pieces, one hollow cylinder, one cartridge holder, and one end plug.

3.2. Estimation of the factor of safety

The factor of safety and the design factor are commonly used synonymously to indicate the ratio of maximum stress to working stress. Due to uncertainties and loading conditions, the factor of safety is introduced in the design to ensure that it will satisfactorily perform its intended function. This is also called as 'factor of ignorance'. The maximum stress is generally taken as yield stress. It depends upon following factors:

- variations in material properties, chemical composition,
- type of loading, i.e., static or dynamic,
- parameters related to a manufacturing process such as forming or machining process surface roughness, internal stresses, sharp corners, and stress raisers,
- effect of heat treatment upon the physical properties of the material,
- effect of wear upon the functions and life of the machine member,
- effect of environment in which the component or device is expected to operate such as temperature, time, humidity, and pressure,
- specific requirement of life and reliability,
- overall concerned for human safety.

As per experiment, 0.2% PS (Proof Stress) of steel material is 600 MPa

$$Factor\ of\ \ safety\ \ (N) = \frac{Maximum\ stress}{Working\ \ Stress} = \frac{600}{100} = 6 \tag{2}$$

As the working stress or equivalence stress [100 MPa determined by Eq. (10) in section 3.7] is less than proof stress of material, the design is safe. Further, the closed vessel is subjected to repeated firing in the development as well as for proving various lots in a production stage. It is the reason that high factor of safety is assigned.

3.3. Margin of safety

Margin of safety is calculated as :

Margin of Safety = Factor of safety (N) – 1, therefore the value of Margin of Safety = 6-1 =5

3.4. Estimation of thickness (*t*)

A thickness of the closed vessel can be determined based on a thick-cylinder theory where $d < 10\ t$. The thickness t can be determined by using the Lames equation for both closed ends. The internal pressure P_i is 30 MPa. Substituting all the values in Eq. (3), we get t

$$t = \frac{D_i}{2}\left\{\left[\sqrt{\frac{\sigma + (1-2\mu)\,P_i}{\sigma - (1+\mu)\,P_i}}\right] - 1\right\} \tag{3}$$

where, μ is the Poisson's ratio, property of the material of construction (steel) = 0.29

$$t = \frac{40}{2}\left\{\left[\sqrt{\frac{100 + (1-2\times 0.29)\times 30}{100 - (1+0.29)\times 30}}\right] - 1\right\} \tag{4}$$

and t = 7.1 mm

Considering practical difficulties of mounting the gauge adaptor to the vessel body with the pressure transducer, loading and removal of the cartridge holder and end plug to the vessel, a suitable 20 mm thickness is chosen. Further, the pressure vessel was subjected to repeated firing trials and required to withstand the fluctuations in firing stresses.

The thread length of the pressure transducer is 12.1 mm (37.65-25.55) as illustrated in Fig. 1.

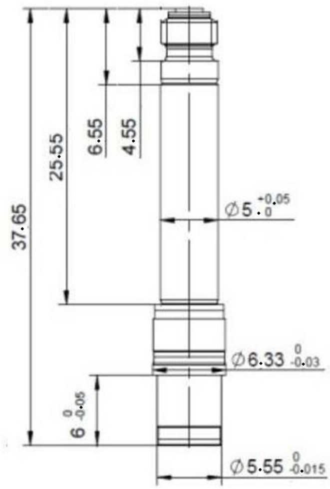

Fig. 1. Pressure transducer

To mount the pressure transducer on the body, more thread length is required which is not sufficient considering the thickness of the vessel as estimated 7.1 mm above. Alternatively, the outer diameter of the vessel = inner diameter + 2 t = 40 + 2 × 20 = 80 mm

3.5. Estimation of thread length

Assume that shearing strength is 50% of the tensile strength and the factor of safety is considered as 2. Thread length for M 50 × 1.5 - 6H can be found out as -

r = major radius of the thread
depth of the thread = 0.6 × pitch = 0.09 cm
d = minor diameter of the thread = 5 – 0.09 × 2 = 4.82 cm
s = shear stress = 0.5 × σ_t = 0.5 × 800 = 400 MPa

Length of thread = 0.4 × core diameter of thread d_c
= 0.4 × 5 = 2 cm or 20 mm

For proper sealing, thread length selected more than 20 mm, i.e., 34 mm.

3.6. Checking of shear stress of threads

Shear force experience by Copper foil = Pressure × Core area of thread

$$P_i \times A = P_i \frac{\pi d_c^2}{4} \quad (5)$$

The shear area = π × d_c × thickness (t)× number of threads (n)

$$\text{Shear stress} = \frac{ShearForce}{\text{Shear Area}} \quad (6)$$

For d_c=50 mm or 5 cm, t = 0.2 cm = 2 mm, n = 17 and P_i= 30 MPa
Shear stress = 11.3 MPa < 400 MPa

As the shear stress is less than material property, therefore design is safe.

3.7. Pressure stresses

Consider the cylinder is made up of steel having the thickness t and the diameter d subjected to the internal pressure P_i. In thick-wall cylinder, the stress over the section of the walls cannot be assumed to be uniformly distributed. They develop both circumferential or the hoop stress σ_t and the radial stresses σ_r with values which are dependent upon the radius of the element under consideration. σ_t is greatest at the inner surface of the cylinder.

This stress is always greater than the internal pressure and approaches this value as r_o increases so that it can never be reduced below P_i irrespective of the amount of material added on the outside. The variations of the stresses in the cylinder together with a plot of the shear stress τ distribution [in red colour] are shown in Fig. 2. The radial stress is maximum at the inner surface and zero at the outer surface of the shell. The axial stress σ_z is constant throughout the radius. The 30 MPa internal pressure is generated inside the vessel. Using the analysis of Lame's and Clapeyron [20] it can be shown that the stresses in the elastic closed-ended cylinder subjected to an internal pressure are -

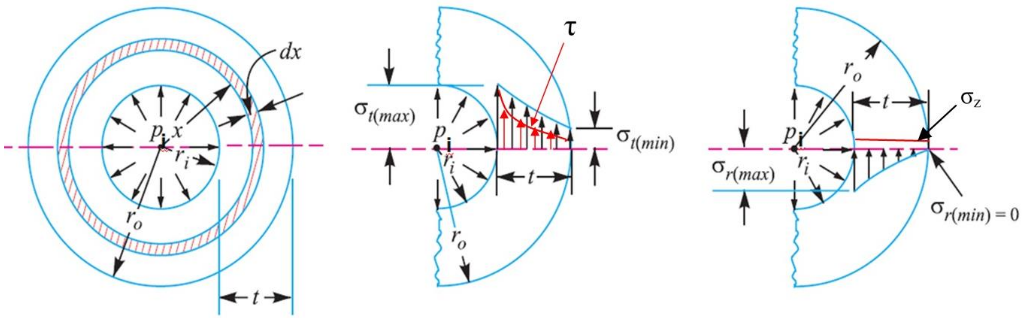

Fig. 2. Variations of stresses in the thick cylinder

$$\sigma_t = Hoop\,Stress = \frac{P_i\,r_i^{\,2}}{r_o^{\,2} - r_i^{\,2}}\left[1 + \frac{r_o^{\,2}}{r_i^{\,2}}\right] \tag{7}$$

$$\sigma_z = Longitudin\,al\;\;Stress = \frac{P_i\,r_i^{\,2}}{r_o^{\,2} - r_i^{\,2}} \tag{8}$$

$$\sigma_r = Radial\,stress = \frac{P_i\,r_i^{\,2}}{r_o^{\,2} - r_i^{\,2}}\left[1 - \frac{r_o^{\,2}}{r_i^{\,2}}\right] \tag{9}$$

The allowable tensile stress σ at the elastic limit can be determined as –

$$\sigma = \left[\sqrt{(\sigma_t - \sigma_z)^2 + (\sigma_z - \sigma_r)^2 + (\sigma_t - \sigma_r)^2}\;\right] \tag{10}$$

As the axial stress is the mean of all three stresses, the stress system in the cylinder wall may be reduced to one of simple shear of value.

$$\tau = Shear\,Stress = \frac{P_i}{K^2 - 1}\left[\frac{r_o^{\ 2}}{r^2}\right] \tag{11}$$

where K = overall diameter ratio

$$K = \frac{D_o}{D_i} \ or \ \frac{r_o}{r_i} \tag{12}$$

It is observed that in CV design to work in terms of the shear stress distribution and it is evident from Fig. 2 that an elastic cylinder is much more highly stressed at the bore than at the outside diameter. The available strength of the material in the outer region of the vessel is thus very poorly utilized. The estimation of strains and thermal stresses are given at paragraphs 3.8, respectively.

3.8. Estimation of strains

The Hoop strain ε_θ [21] is calculated by the following equation

$$\varepsilon_\theta = \frac{1}{E}\left[\sigma_\theta - \mu\,(\sigma_r + \sigma_z)\right] \tag{13}$$

where E is the Young modulus of steel material = 200 GPa, μ is the Poisson's ratio = 0.29.

The radial strain ε_r is calculated by the following equation

$$\varepsilon_r = \frac{1}{E}\left[\sigma_r - \mu\,(\sigma_\theta + \sigma_z)\right] \tag{14}$$

The longitudinal strain ε_z is calculated by the following equation

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \mu\,(\sigma_\theta + \sigma_r)\right] \tag{15}$$

3.8.1. Thermal stresses

When propellant undergoes burning, they produce high temperature and high pressure gaseous products. The flame temperature of double base propellant lies in between 2600 K to 3600 K. The inner walls of vessel are exposed to high temperature for mille second phenomena.

The thermal stresses in a thick-walled monobloc pressure cylinder with a constant temperature difference across the wall, considered by Timoshenko and Goodier [22]. For a long unrestrained cylinder, the thermal stress are

$$\sigma_t = \frac{\alpha E T}{2(1-\mu)\ln K}\left[-\ln\frac{r_o}{r} - \left(\frac{1}{K^2-1}\right)\left(1-\frac{r_o^{\,2}}{r^2}\right)\ln K\right] \tag{16}$$

$$\sigma_\theta = \frac{\alpha E T}{2(1-\mu)\ln K}\left[1-\ln\frac{r_o}{r}\left(\frac{1}{K^2-1}\right)\left(1+\frac{r_o^{\,2}}{r^2}\right)\ln K\right] \tag{17}$$

$$\sigma_z = \frac{\alpha E T}{2(1-\mu)\ln K}\left[1-2\ln\frac{r_o}{r} - \frac{2}{K^2-1}\ln K\right] \tag{18}$$

where α is the thermal coefficient of expansion and T is the temperature difference across the cylinder wall. Figure 3 illustrates the general representation of non-dimensional thermal stress distributions vs. radius for a thick cylinder of diameter ratio two.

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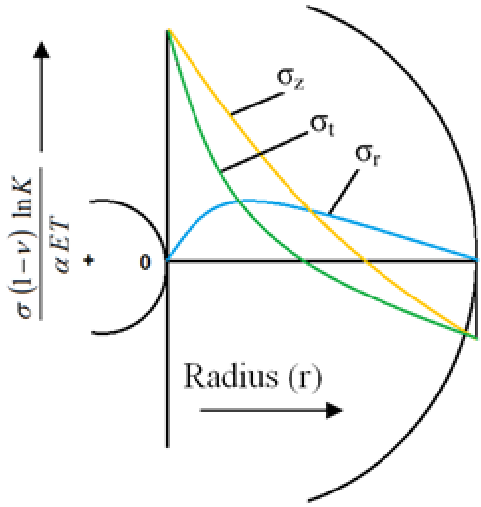


Fig. 3. Variations of thermal stresses in the thick cylinder

To determine the yield pressure for a vessel with a temperature gradient across the wall, the elastic stress distribution is shown in Fig. 2 must be combined with the thermal stress distribution in Fig. 3. It can be seen that if the heat flow is outward *(T* negative), the effect is to reduce the bore hoop stress with a consequent strengthening effect on the vessel. If the heat flow is inward, then the bore hoop stress is increased and the allowable working pressure of the vessel reduced.

In a more rigorous examination of the effect of thermal stresses on the working pressure it is necessary to take account not only of the reduction of the yield strength but also the reduction in the elastic modulus with rising temperature.

3.9. Estimation of increase in internal and external radii

The change in internal & external radii for closed ended cylinder can be found out using following equations [21]:

(a) Increase in the internal radius can be found out as

$$= \frac{P_i R_i}{E}\left[\frac{(K^2+1)+\mu(K^2-2)}{(K^2-1)}\right] \tag{19}$$

where, K = ratio of the outside diameter to the inside diameter;
substituting all the values in Eq. (19) = 55.8×10^{-4} mm

(b) Increase in the external radius can be found out as

$$= \frac{P_i R_i}{E}\left[\frac{(2-\mu)}{(K^2-1)}\right] \tag{20}$$

substituting all the values in Eq. (20) = 17.1×10^{-4} mm

4. MATERIAL AND METHODS

4.1. Material of construction (MOC) of vessel

Steel with adequate strength with corrosion and high temperature resistance is selected as MOC in the design and manufactured. Tables 1 and 2 illustrates the chemical and mechanical properties of carbon steel for high temperature service.

Table 1. Chemical composition

Elements	Percentage (%)
C	0.43
Si	0.31
Mn	0.58
P	0.027
S	0.006
Cr	0.94
Mo	0.33
Ni	1.43
Cu	0.09

Table 2. Mechanical properties

Mechanical Properties	Accepted values
Yield stress in tension	350 MPa
Young modulus	207×10^{6} kPa
Poisson's ratio	0.25
UTS	800 MPa (Min)
0.2% PS	600 MPa (Min)
% elongation	16 (Min)
Density	7.8 g/cc
Hardness on finish product	360 to 380 HV at 100g

4.2. Propellant composition

The double base propellant is used in CV firing. Its chemical composition and physical properties are given in Tables 3 and 4, respectively. The image of propellant used for testing in the cartridge is shown in Fig. 4.

Table 3. Chemical composition

Chemical composition	
Component	Composition (wt %)
Nitro-cellulose (NC)	85.3 Nitration (13% N)
Nitro-glycerine (NG)	10.05
Dibutyl phthalate (DBP)	3
Diphenyl amine (DPA)	0.95
Calcium carbonate	0.5
Graphite	0.2

Table 4. Physical properties

Propellant shape	Spherical grains
Density	1.55 g / cc
Calorific value	880
Average diameter	0.483 mm
Mass of propellant	3 g
Average Web	0.40 mm

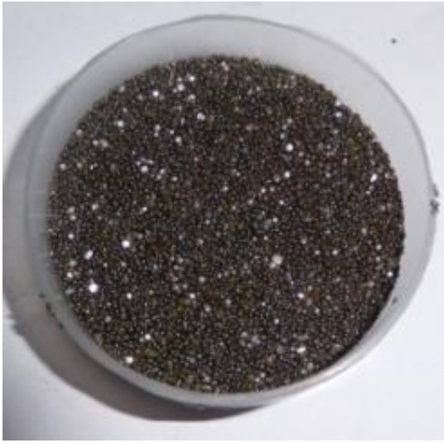


Fig. 4. Image of double base propellant

4.3. Closed vessel (CV) experiments

A CV is cylindrical in shape. It consists of cylindrical body, closing plug at one end and cartridge holder at another end. A suitable gauge adapter is screwed in the vessel body into which a pressure transducer is fitted. Sketch of CV and its model are shown in Fig. 5 and Fig. 6, respectively.

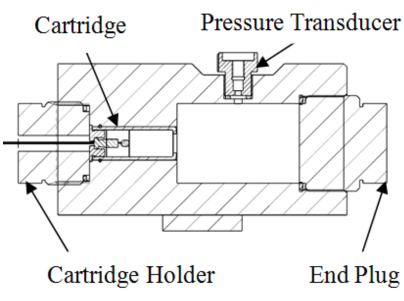


Fig. 5. Sketch of (CV)

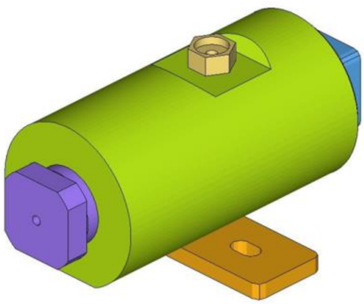


Fig. 6. Model of CV

5. THEORIES OF FAILURE

The following theories of failures are applied to present design [23].

5.1. Maximum principle or normal stress theory (Rankine's theory)

According to this theory, the failure or yielding begins at a point in a member when the maximum principal or normal stress in member reaches the limiting strength of the material in a simple tension test.

The maximum normal stress = σ_t at $r = r_i$

$$= \frac{P_i}{r_o^2 - r_i^2}\left[\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right] \tag{21}$$

$$= \frac{P_i}{r_o^2 - r_i^2}\left[\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right] = \frac{\sigma_y}{N} \tag{22}$$

Here N is the factor of safety

5.2. Maximum shear stress theory (Guest's or Tresca's theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum shear stress in a uni-axial stress system reaches a value equal to the shear stress at a yield point under general state of loading reaches as determined in a simple tension test.

$$= \frac{1}{2}\left(\sigma_t - \sigma_r\right) \tag{23}$$

at $r = r_i$

$$= \frac{P_i}{2}\left[\frac{r_o^2}{r_o^2 - r_i^2}\right] = \frac{\sigma_y}{2N} \tag{24}$$

5.3. Maximum strain theory (Saint Venant's theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal (or normal) strain in a uni-axial stress system reaches the limiting value of strain (*i.e.* strain at a yield point) under general state of loading reaches as determined from a simple tensile test.

The maximum principal (or normal) strain in a uni-axial stress system is given by

The maximum strain $= \frac{1}{E}\left(\sigma_t - \mu\sigma_r\right)$ at $r = r_i$

$$= \frac{P_i\, r_i^{\,2}}{r_o^{\,2} - r_i^{\,2}}\left[\left(1 + \frac{r_o^{\,2}}{r_i^{\,2}}\right) - \mu\left(1 - \frac{r_o^{\,2}}{r_i^{\,2}}\right)\right] \tag{25}$$

$$= \frac{P_i}{E\left(r_o^{\,2} - r_i^{\,2}\right)}\left[\left(r_o^{\,2} + r_i^{\,2}\right) - \mu\left(r_o^{\,2} - r_i^{\,2}\right)\right] = \frac{\sigma_y}{N\,E} \tag{26}$$

5.4. Octahedral shearing stress or van Mises theory of failure

According to this theory, the failure or yielding occurs at a point in a member when the value octahedral shearing stress at a point under a general state of loading reaches the octahedral shearing stress at a yield point under uni-axial state of loading as determined from a simple tension test.

$$\tau_{oct} = \frac{1}{3}\left[\sigma_t^{\,2} + \sigma_r^{\,2} + \left(\sigma_r - \sigma_t\right)^2\right]^{1/2}$$

at $r = r_i$

$$= \frac{1}{3}\left[2\left(\sigma_r - \sigma_t\right)^2 + 2\,\sigma_t\;\sigma_r\;\right]^{1/2}$$

$$= \frac{\sqrt{2}}{3}\left[\left(-P_i - \frac{P_i\left(r_o^{\,2} + r_i^{\,2}\right)}{\left(r_o^{\,2} - r_i^{\,2}\right)}\right) - \frac{P_i^2\left(r_o^{\,2} + r_i^{\,2}\right)}{\left(r_o^{\,2} - r_i^{\,2}\right)}\right]^{\frac{1}{2}} = \frac{\sqrt{2}\,\sigma_y}{3\,N} \tag{27}$$

$$= \frac{\sqrt{2}\,P_i}{3}\left[\left(\frac{4r_o^{\,2}}{\left(r_o^{\,2} - r_i^{\,2}\right)^2}\right) - \frac{\left(r_o^{\,2} + r_i^{\,2}\right)}{\left(r_o^{\,2} - r_i^{\,2}\right)}\right]^{\frac{1}{2}} = \frac{\sqrt{2}\,\sigma_y}{3\,N}$$

5.5. Energy distortion theory

According to this theory, the failure or yielding occurs at a point in a member when the distortion strain energy (also called shear strain energy) per unit volume in a uni-axial stress system reaches the limiting distortion energy (*i.e*. distortion energy at a yield point) per unit volume as determined from a simple tension test. This will give the values identical to that obtained based on octahedral shear stress theory.

6. RESULTS AND DISCUSSION

6.1. Analysis of stress strength results of CV

Geometry clean-up was performed using 3D CAD Model. Material properties and boundary conditions were applied for the mentioned load cases to obtain the results. The mesh geometry is shown in Fig. 7. Various stresses and displacement for CV is depicted in Fig. 8. Hoop, radial, and axial stresses are estimated as 50 MPa, -30 MPa, and 10 MPa, respectively, as explained in Section 3.7. The equivalent stress is worked out as 100 MPa. The factor of safety is 6.

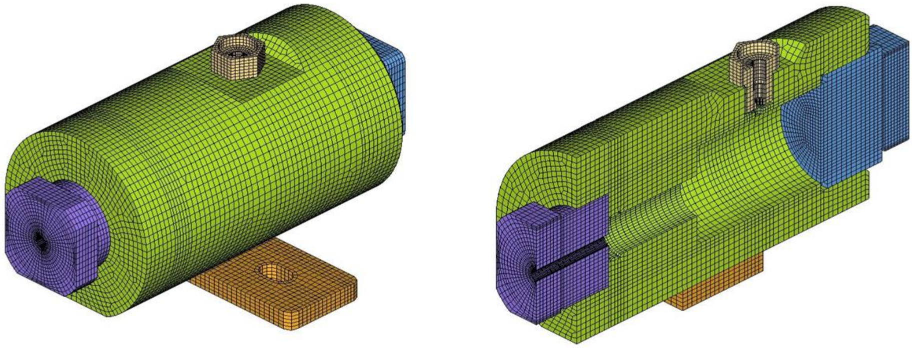


Fig. 7. Mesh geometry for closed body and sectional view

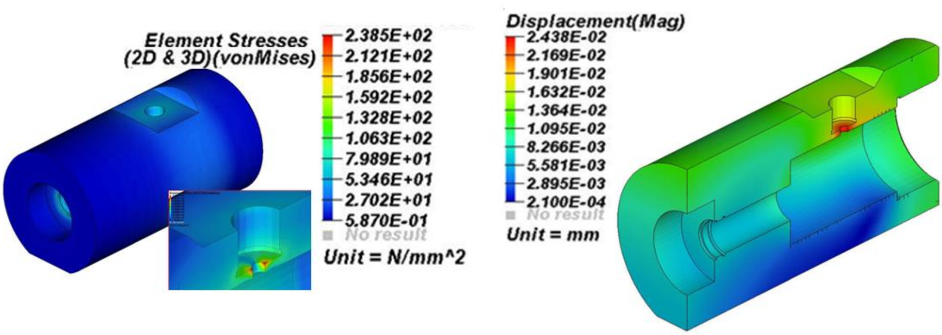


Fig. 8. Stresses and displacements

The number of defined elements, nodes and DOF per node is given in Table 5.

Table 5. Parameters used in analysis

Sl. No.	Parameter	Total No.
1	Number of defined elements	329026
2	Number of defined nodes	13193
3	Maximum DOF per node	6
4	Maximum van Mises stress	238.5 MPa
5	Maximum displacement	0.02438 mm

6.2. Determination of working pressure

According to different theories of failures, explained in Section 5, the working pressure is determined and given in Table 6.

Table 6. Theories of failure and working pressure

Sl. No.	Theories of failures	Working pressure (MPa)
1	Maximum Normal Stress Theory	84
2	Maximum shear stress theory	52.5
3	Maximum strain Theory	98.82
4	Octahedral shear stress or von Mises theory	60.08
5	Energy distortion theory	60.08

It is revealed from the above table that von Mises theory is most popularly used for elastic failure as its results practically matches with the experiments. The other theories of failures are not being used as the experimental results that do not match.

6.3. Determination of thermal and pressure stress

According to different theories of failures, explained in Section 5, the thermal and pressure stresses are determined and given in Table 7.

Table 7. Thermal and pressure stress

Sl. No.	Stress	Thermal stress (MPa)	Pressure stress (MPa)
1	Hoop stress	738.38	50
2	Radial stress	-24.70	-30
3	Axial stress	-3024.88	10
4	von Mises theory	--	40

It is revealed from the above table and Fig. 2 that radial thermal stress follows the parabola path and maximum at a radius away from the inner radius. It is minimum at the inner and outer radius. Hoop and axial thermal stresses is maximum at the inner radius and minimum at the outer radius.

6.4. Determination of various strains

Various strains, generated in CV, are determined as explained in section 3.8 and in Table 8.

Table 8. Thermal and pressure stress

Sl. No.	Strain	Estimated values
1	Hoop strain	2.79×10^{-4}
2	Radial strain	-2.37×10^{-4}
3	Axial strain	0.21×10^{-4}

6.5. Metallographic results

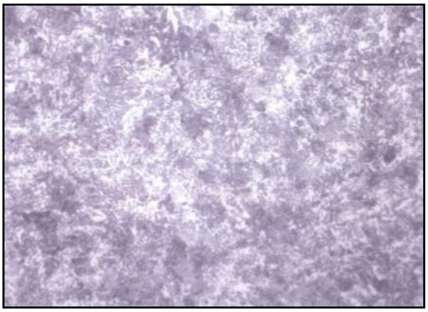
The chemical analyses for both the samples were carried out to verify the material using optical emission spectroscopy. The results of both the samples is given in Table 9.

Table 9. Results of chemical analysis of samples

	Elements present with % composition									
	C	Si	Mn	P	S	Cr	Mo	Ni	Cu	Base
Raw material	0.43	0.31	0.58	0.027	0.006	0.94	0.33	1.43	0.09	Fe
After firing	0.39	0.25	0.59	0.012	0.016	1.28	1.36	1.36	0.29	Fe

It is observed from the chemical analysis that the compositions of both the samples are within the range of the standard specifications.

Figure 9 depicts the microstructure obtained for the raw material sample before and the after firing material sample at 500X.



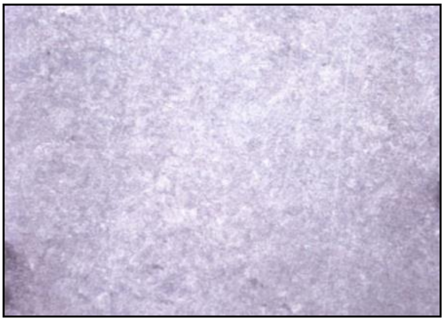


Fig. 9. Before firing (left) and after firing (right)

The microstructure of the raw material sample was found to be coarse pearlite which is usually obtained in hot rolled condition. There is an effect of temperature on the structure of the material though very less after firing, finer pearlite is observed in the microstructure of the after firing sample. Vickers hardness tested for both the samples is given in Table 10.

Table 10. Results of hardness testing of samples

Condition of material	Hardness
Raw material	186-196 HV
After firing sample	416-432 HV

From the hardness test results, the hardness of the raw material in ‘R’ condition has hardness in the range of the standard specifications. After manufacturing, the component is heat treated with the required hardness of 380 BHN or 400 HV. The hardness of after firing sample was found to be slightly increased.

7. CONCLUSIONS

The intent of this paper was to design, fabricate, and test a CV. FEM approach to determine various deformations, strains, and stresses is discussed. The prototype significantly reduces cost and weight compared to expensive composite materials or heavy metal structures. Various theories of failure is also covered in this manuscript.

The magnitude of the stress in the material is a dominant factor in the design of a closed vessel. The dimensions of the vessel have been derived as explained in section 3.

In the light of this theoretical and experimental work, it was realized and recommended that Octahedral stress theory should be used for vessel design. Its results are similar to van Mises theory. This will satisfy the designer requirements. However, after detailed analysis the major conclusions are as below:

- the design of pressure vessel (i.e. CV) is initialized with the specification requirements in terms of standard technical specifications along with numerous requirements,
- this type of analytical approach for design greatly reduces the development time and cost for pressure vessel.

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