DOUBLY PERIODIC CRACKS IN THE ANISOTROPIC MEDIUM WITH THE ACCOUNT OF CONTACT OF THEIR FACES

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Abstract: The paper presents complex variable integral formulae and singular boundary integral equations for doubly periodic cracks in anisotropic elastic medium. It utilizes the numerical solution procedure, which accounts for the contact of crack faces and produce accurate results for SIF evaluation. It is shown that the account of contact effects significantly influence the SIF of doubly periodic curvilinear cracks both for isotropic and anisotropic materials.

Key words: Anisotropic, Doubly Periodic, Singular Integral Equation, Crack

1. INTRODUCTION

The doubly periodic problems are widely considered in scientific literature, since they are crucial for understanding of the effect of cracks interaction on stress state of defective solids (see Sawruk, 1981). Wang (2004) presented extremely accurate and efficient method for computing the interaction of a set or multiple sets of general doubly periodic cracks in elastic medium. Xiao and Jiang (2009) studied the orthotropic medium with doubly periodic cracks of unequal size under antiplane shear. Chen et al. (2003) have studied various multiple crack problems in elasticity. Xiao et al. (2011) obtained the closed-form solution for stress and electric displacement intensity factors and effective properties of piezoelectric materials with a doubly periodic set of conducting rigid line inclusions. Malits (2010) studied the doubly periodic arrays of rigid line inclusions in an elastic solid. Pasternak (2012) presented the general Somigliana integral identities and boundary integral equations for doubly periodic cracks in anisotropic magnetoelectroelastic medium.

However, equations of Pasternak (2012) contain both singular and hypersingular integrals, therefore, this paper is focused on the development of singular integral equation for doubly periodic cracks in anisotropic medium. Also the contact of crack faces is accounted for, thus, the paper presents new general approach that can be used both in theoretical and applied analysis, in particular, in rock mechanics.

2. OBTAINING OF SINGULAR INTEGRAL EQUATIONS BASED ON THE LEKHNITSKII FORMALISM

Consider a doubly periodic problem of elasticity for an infinite anisotropic plate, which representative volume element contains a set of cracks L_j $(j = 1, \ldots, J$. Assume that crack faces are symmetrically loaded with tractions (X_T, Y_T) and average stress $\langle\sigma_x\rangle$, $\langle\sigma_y\rangle$, $\langle\sigma_{xy}\rangle$ act in the medium.

2.1. Governing equations for anisotropic plates

Consider an arbitrary curve Γ that lays in the 2D domain D occupied by the plate, and assume its positive path. One can introduce the traction vector $\vec{S}_\Gamma$ in the tangential element of the curve Γ, which normal vector is placed at the right to the chosen positive path of the curve. The projections (X_Γ, Y_Γ) of the traction vector $\vec{S}_\Gamma$ and displacement (u, v) derivatives by the arc coordinate at the curve Γ can be evaluated based on the Lekhnitskii complex functions as (Bozhydarnyk, 1998; Grigolyuk & Filshtinskiy, 1994):

$$\begin{aligned} Y_\Gamma &= -2\mathrm{Re}[\Phi(z_1)z_1' + \Psi(z_2)z_2'] \\ X_\Gamma &= 2\mathrm{Re}[s_1\Phi(z_1)z_1' + s_2\Psi(z_2)z_2'] \\ u' &= 2\mathrm{Re}[p_1\Phi(z_1)z_1' + p_2\Psi(z_2)z_2'] \\ v' &= 2\mathrm{Re}[q_1\Phi(z_1)z_1' + q_2\Psi(z_2)z_2'] \end{aligned} \tag{1}$$

where: $z_j = x + s_j y$, $u' = du/ds$, $v' = dv/ds$, $z_j' = dx/ds + s_j dy/ds$, $j = 1,2$ ds is a differential of arc Γ; s_j are the complex roots (with positive imaginary part) of Lekhitskii characteristic equation (Bozhydarnyk, 1998; Grigolyuk & Filshtinskiy, 1994); $p_j = \alpha_{11}s_j^2 + \alpha_{12} - \alpha_{16}s_j$, $q_j = \alpha_{12}s_j + \alpha_{22}/s_j - \alpha_{26}$ and α_{ij} are elastic compliances (Bozhydarnyk, 1998; Grigolyuk and Filshtinskiy, 1994).

Assume that the functions u', v', X_Γ, Y_Γ in Eq. (1) are known at Γ. Then according to Maksymovych (2009) one can obtain:

$$\begin{aligned} \Phi(z_1) &= \frac{-v' + s_1u' + p_1X_\Gamma + q_1Y_\Gamma}{\Delta_1 z_1'} \\ \Psi(z_2) &= \frac{-v' + s_2u' + p_2X_\Gamma + q_2Y_\Gamma}{\Delta_2 z_2'} \end{aligned} \tag{2}$$

where:
$\Delta_1 = \alpha_{11}(s_1 - s_2)(s_1 - \overline{s_1})(s_1 - \overline{s_2})$,
$\Delta_2 = \alpha_{11}(s_2 - s_1)(s_2 - \overline{s_1})(s_2 - \overline{s_2})$.

2.2. Conditions for the complex functions at the boundary contours

Assume that the projections (X_Γ, Y_Γ) of traction vector and the moment M_L about the origin of all forces applied to the contour L are known. Then one can obtain the following conditions:

$$\int_{L_1} \Phi(z_1)dz_1 = -\frac{p_1 X_L + q_1 Y_L}{\Delta_1}$$
$$\int_{L_2} \Psi(z_2)dz_2 = -\frac{p_2 X_L + q_2 Y_L}{\Delta_2} \tag{3}$$
$$\mathrm{Re}\left[\int_{L_1} \Phi(z_1)dz_1 + \int_{L_2} \Phi(z_2)dz_2\right] = -M_L/2$$

where L_j are the curves in the coordinate systems (x_j, y_j), which are the mappings of the curve L with the affine transformations $x_j = x + \mathrm{Re}(s_j)y$, $y_j = \mathrm{Im}(s_j)y$.

2.3. Integral equations for displacement discontinuities in an infinite cracked plate

The complex variable integral equations for cracked anisotropic plates, in general, are written for the discontinuities of complex functions at cracks, which do not have direct physical meaning. At the same time, complex variable integral equations for isotropic plates are written for the displacement discontinuities, which significantly simplify the study and solution of fracture mechanics problems. In particular, these equations allow considering the problems for cracks with contacting faces. Therefore, this section develops this approach for cracked anisotropic plates.

Assume that the tractions applied to the crack faces are symmetric with respect to a crack. Denoting the displacement discontinuities $[u]$, $[v]$ with g_1, g_2, respectively, based on Eq. (2) one obtains the formulae for discontinuities of complex potentials at the crack:

$$[\Phi(z_1)] = \frac{-g_2' + s_1 g_1'}{\Delta_1 z_1'},\ [\Psi(z_2)] = \frac{-g_2' + s_2 g_1'}{\Delta_2 z_2'}$$

Then the following integral formulae can be obtained for an anisotropic plate containing a set of cracks (Bozhydarnyk, 1998; Maksymovych, 2009):

$$\Phi(z_1) = \int_L \frac{G_1'}{t_1 - z_1} ds + \Phi_s(z_1)$$
$$\Psi(z_2) = \int_L \frac{G_2'}{t_2 - z_2} ds + \Psi(z_2) \tag{4}$$

where $G_1' = A_1 g_1' + A_2 g_2'$, $G_2' = B_1 g_1' + B_2 g_2'$ and $A_1 = \frac{i s_1}{2\pi\Delta_1}$ $A_2 = \frac{i}{2\pi\Delta_1}$, $B_1 = \frac{i s_2}{2\pi\Delta_2}$, $B_2 = \frac{i}{2\pi\Delta_2}$

It should be mentioned that for internal cracks the following crack tip displacement continuity conditions hold:

$$\int_L G_i'(s)ds = 0 \quad (i = 1,2) \tag{5}$$

Now consider the doubly periodic lattice defined by the periods ω_1 and ω_2. The first period is assumed to be real-valued, and the second one is complex-valued. Then the periods in the mathematical planes z_k are denoted as $\omega_i^{(k)}$ $(i = 1,2)$, moreover $\omega_1^{(k)} = \omega_2$ and $\omega_2^{(k)} = x_\omega + s_k y_\omega$, where $x_\omega + i y_\omega = \omega_2$ and $y_\omega > 0$.

According to Grigolyuk & Filshtinskiy (1994), Sawruk (1981), the integral formulae (4) for the doubly periodic problems can be replaced with:

$$\Phi(z_1) = \int_L G_1'(s)\xi(t_1 - z_1)ds + A_S$$
$$\Psi(z_2) = \int_L G_2'(s)\xi(t_2 - z_2)ds + B_S \tag{6}$$

where $\xi(z_k) = \xi\left(z_k | \omega_1^{(k)}, \omega_2^{(k)}\right)$ is a Weierstrass zeta function for the periods $\omega_1^{(k)}$ and 2; and A_S, B_S are unknown constants to be determined.

Using the conditions (5) and the property $\xi\left(z_k + \omega_\mathrm{n}^{(k)}\right) = \xi(z_k) + \delta_\mathrm{n}^{(k)}$ it is easy to show that the complex potentials (6) are periodic (thus, the stresses and strains calculated based on these potentials are periodic too). Here $\delta_\mathrm{n}^{(k)} = 2\xi\left(\omega_\mathrm{n}^{(k)}/2\right)$. At the same time, the potentials (6) (and the same as (4)) are dependent on the displacement discontinuities $g_1 + i g_2$ at the curve L. Therefore, (6) presents the solution of doubly periodic problem for cracked domain.

For determination of the constants A_S and B one should first determine the traction vector acting at the lines parallel to the main periods (Grigolyuk & Filshtinskiy, 1994). The projections of the resultant vector of tractions acting at the arbitrary curve AB can be determined with the following equation:

$$Y_{AB} = -2\mathrm{Re}[\varphi(z_1) + \psi(z_2)]_{AB}$$
$$X_{AB} = 2\mathrm{Re}[s_1\varphi(z_1) + \psi\Psi(z_2)]_{AB}$$

First assume that AB is a line parallel to the Ox axis. Then accounting for (6), one obtains that:

$$\varphi(z_1)_{AB} = \int_{z_1}^{z_1+\omega_1^{(1)}} \Phi(z_1)dz_1$$
$$= \int_L G_1'(s)\left(\int_{z_1}^{z_1+\omega_1^{(1)}} \xi(t_1 - z_1)dz_1\right)ds + A_S\,\omega_1^{(1)}$$

According to Sawruk (1981):

$$\int_{z_1}^{z_1+\omega_1^{(1)}} \xi(t_1 - z_1)dz_1 = \delta_1^{(1)}(t_1 - z_1) + const.,$$

therefore:

$$\varphi(z_1)_{AB} = \delta_1^{(1)} \int_L G_1'(s)(t_1 - z_1)ds + A_S\,\omega_1^{(1)}$$
$$= \delta_1^{(1)} \int_L G_1'(s)t_1 ds + A_S\,\omega_1^{(1)}$$

The same concerns the function $\psi(z_2)$:

$$\psi(z_2)_{AB} = \delta_1^{(2)} \int_L G_2'(s)t_2 ds + B_S\,\omega_1^{(2)}$$

Thus, the following relations are obtained for the bottom edge of the representative volume element:

$$
\begin{aligned}
Y_1 &= -2\mathrm{Re}\big(\delta_1^{(1)}J_1 + \delta_1^{(2)}J_2 + A_S\omega_1^{(1)} + B_S\omega_1^{(2)}\big)\\
X_1 &= 2\mathrm{Re}\big(s_1\delta_1^{(1)}J_1 + s_2\delta_1^{(2)}J_2 + A_S s_1\omega_1^{(1)} + B_S s_2\omega_1^{(2)}\big)
\end{aligned}
\tag{7}
$$

where $J_k = \int_L G_k'(s)t_k ds$.

Applying the same procedure for the right edge of the representative volume element one obtains:

$$
\begin{aligned}
Y_1 &= -2\mathrm{Re}\big(\delta_2^{(1)}J_1 + \delta_2^{(2)}J_2 + A_S\omega_2^{(1)} + B_S\omega_2^{(2)}\big)\\
X_1 &= 2\mathrm{Re}\big(s_1\delta_2^{(1)}J_1 + s_2\delta_2^{(2)}J_2 + A_S s_1\omega_2^{(1)} + B_S s_2\omega_2^{(2)}\big)
\end{aligned}
\tag{8}
$$

The right hand sides of Eqs. (7) and (8) are known and equal:

$$Y_1 = -\omega_1\langle\sigma_y\rangle$$

$$X_1 = -\omega_1\langle\tau_{xy}\rangle$$

$$Y_2 = |\omega_2|\big(\langle\tau_{xy}\rangle > cos\alpha - \langle\sigma_y\rangle sin\alpha\big)$$

$$X_2 = |\omega_2|\big(\langle\sigma_x\rangle > cos\alpha - \langle\tau_{xy}\rangle sin\alpha\big)$$

where α is an angle between the second period ω_2 and the Oy axis.

Eqs. (7) and (8) are considered as a linear algebraic equations system for determination of the unknown constants A_S and B_S. First consider Eq. (7) and the first equation in (8):

$$-2\mathrm{Re}\big((A_S + B_S)\omega_1 + \delta_1^{(1)}J_1 + \delta_1^{(2)}J_2\big) = Y_1$$

$$2\mathrm{Re}\big(\omega_1(s_1A_S + s_2B_S)\omega_1 + s_1\delta_1^{(1)}J_1 + s_2\delta_1^{(2)}J_2\big) = X_1$$

$$
\begin{aligned}
-2\mathrm{Re}\big(x_\omega(A_S + B_S) + y_\omega(s_1A_S + s_2B_S)\big)&\\
- 2\mathrm{Re}\big(\delta_2^{(1)}J_1 + \delta_2^{(2)}J_2\big) &= Y_2
\end{aligned}
$$

These equations result in the condition:

$$
\begin{aligned}
2\mathrm{Re}\big[&\big(\delta_1^{(1)}\omega_2^{(1)} - \delta_2^{(1)}\omega_1^{(1)}\big)J_1\\
&+ \big(\delta_1^{(2)}\omega_2^{(2)} - \delta_2^{(2)}\omega_1^{(2)}\big)J_2\big]\\
&= x_\omega X_1 + y_\omega X_1 + \omega_1 Y_2 = 0
\end{aligned}
$$

Accounting for $\delta_1^{(k)}\omega_2^{(k)} - \delta_2^{(k)}\omega_1^{(k)} = 2\pi i$ one obtains that $4\pi\mathrm{Re}i(J_1 + J_2) = 0$.

The relation:

$$2\mathrm{Re}\left(\frac{s_1^j}{\Delta_1} + \frac{s_2^j}{\Delta_2}\right) = 0 \quad \text{for } j = 0,1,2$$

shows that this condition is satisfied identically.

Thus, for determination of the unknown constants we have only three equations (Eq. (7) and the second equation in (8)). The solution of this system is sought in the following form:

$$A_S = -\frac{\delta_1^{(1)}}{\omega_1}J_1 + \Phi_\infty + A,\ B_S = -\frac{\delta_1^{(2)}}{\omega_1}J_2 + \Psi_\infty + B$$

where Φ_∞ and Ψ_∞ are complex potentials corresponding to the stress state of uncracked plate under the load $\langle\sigma_x\rangle$, $\langle\sigma_y\rangle$, $\langle\tau_{xy}\rangle$ applied at infinity.

Then for determination of the constants A and B one obtains the following system:

$$2\mathrm{Re}(A + B) = 0,\ 2\mathrm{Re}(s_1A + s_2B) = 0$$

$$2\mathrm{Re}(As_1^2 + Bs_2^2) = -\frac{4\pi}{y_\omega\omega_1}\mathrm{Re}i(s_1J_1 + s_2J_2)$$

Solving the latter one can obtain the expressions for the unknown constants up to the values which do not influence the stress field:

$$A = -\frac{1}{y_\omega\omega_1\Delta_1}\int_L g_1't_1ds,\ B = -\frac{1}{y_\omega\omega_1\Delta_2}\int_L g_1't_2ds$$

For convenience one can present the Weierstrass zeta function in the form (Sulym, 2007):

$$\xi(z) = \frac{\delta_1}{\omega_1}z + S(z;\ \omega_1, \omega_2)$$

where:

$$
\begin{aligned}
S(z;\ \omega_1,\omega_2) = \frac{\pi}{\omega_1}\Bigg\{&\mathrm{ctg}\frac{\pi z}{\omega_1} + \sum_{n=1}^{N}\mathrm{ctg}\left(\frac{\pi z}{\omega_1} + n\pi\frac{\omega_2}{\omega_1}\right)\\
&+ \sum_{n=1}^{N}\mathrm{ctg}\left(\frac{\pi z}{\omega_1} - n\pi\frac{\omega_2}{\omega_1}\right)\Bigg\}
\end{aligned}
$$

for $N \to \infty$.

These relations allow to rewrite (6) as:

$$
\begin{aligned}
\Phi(z_1) &= \int_L [g_1'\Phi_1(t_1 - z_1) + g_2'\Phi_2(t_1 - z_1]ds + \Phi_\infty\\
\Psi(z_1) &= \int_L [g_1'\Psi_1(t_2 - z_2) + g_2'\Psi_2(t_2 - z_2]ds + \Psi_\infty
\end{aligned}
\tag{9}
$$

where $\Phi_1(z) = A_1\left[S_1(z) + \frac{\gamma}{s_1}z\right]$, $\Psi_1(z) = B_1\left[S_z(z) + \frac{\gamma}{s_2}z\right]$
$\Phi_2(z) = A_1S_1(z)$, $\Psi_2(z) = B_2S_2(z)$, $S_k(z) = S\big(z;\ \omega_1, \omega_2^{(k)}\big)$
$\gamma = \frac{2\pi i}{y_\omega\omega_1}$.

The kernels of integral formulae (9) are written in the form of sums of the kernels for singly periodic problems and contain additional terms, which has a multiplier γ. It should be mentioned, that in the literature one can found the attempts to derive the analogous formulae with direct summation. However, now it is obvious that these approaches are incorrect. The reader is referred to Pasternak (2012), where for the first time the mathematically strict and correct approach of direct summation for anisotropic magnetoelectroelastic material with doubly periodic sets of defect was presented.

3. NUMERICAL SOLUTION OF THE INTEGRAL EQUATIONS FOR PARTICULAR PROBLEMS

Integral formulae (9) have the same structure as those obtained for other problems of elasticity for cracked anisotropic plates obtained by Bozhydarnyk (1998) and Maksymovych (2009). Therefore, numerical determination of the displacement discontinuities incorporated in these formulae can be determined within the algorithm proposed by Maksymovych (2009). For evaluation of the kernels of the integral equations it is convenient to use the following relation:

$$
\begin{aligned}
&S(z;\ \omega_1,\omega_2) = \lambda_1\{\mathrm{ctg}\lambda_1 z\\
&+4\sin(2\lambda_1 z)\sum_{n=1}^{\infty}\frac{\lambda^n}{(1-\lambda^n e^{2i\lambda_1 z})(1-\lambda^n e^{-2i\lambda_1 z})}\}
\end{aligned}
$$

where $\lambda_1 = \frac{\pi}{\omega_1}$, $\lambda = \exp\left(2\pi i \frac{\omega_2}{\omega_1}\right)$ and $|\lambda| < 1$.

This series converges fast. In particular, for isotropic material with a square lattice it is enough to leave only two terms in the sum.

3.1. Verification of the approach

For verification of the proposed approach consider a doubly periodic set of line cracks of length $2l$ inclined at $30°$ to the Ox axis. The centers of the cracks form the equilateral triangular lattice. The only nonzero average stress is $\langle\sigma_y\rangle = p$. The material of the plate is highly anisotropic fiberglass CF1, whose properties are given by Maksymovych (2009). Table 1 compares the normalized stress intensity factors (SIF) $F_{\mathrm{I,II}} = K_{\mathrm{I,II}}/\left(p\sqrt{\pi l}\right)$ obtained with the proposed approach and by the boundary element method (BEM) developed by Pasternak (2012) with a crack face meshed with only 20 elements. Good agreement of the results is observed, which testifies the validity and efficiency of the developed approach.

Tab. 1. Verification of the approach

$\lambda_1 = \frac{2l}{\omega_1}$	F_{I} present	F_{I} BEM	Deviation %	F_{II} present	F_{II} BEM	Deviation %
0.05	0.739	0.739	0.064	0.426	0.426	0.032
0.10	0.714	0.714	0.029	0.411	0.411	0.070
0.20	0.687	0.687	0.055	0.394	0.394	0.118
0.30	0.733	0.733	0.068	0.421	0.421	0.103
0.40	0.897	0.897	0.034	0.514	0.514	0.069
0.50	1.344	1.344	0.014	0.768	0.768	0.004
0.60	2.355	2.351	0.191	1.280	1.277	0.224
0.70	2.560	2.549	0.412	1.281	1.276	0.368
0.80	2.579	2.565	0.541	1.196	1.190	0.502
0.90	2.586	2.568	0.691	1.112	1.105	0.658
0.95	2.597	2.578	0.713	1.078	1.071	0.671

3.2. Doubly periodic curved cracks with contacting faces

Consider a doubly periodic curved cracks, whose shape is defined by the parabola equation $y = k\left(\frac{x^2}{l^2} - 1\right)$ for $-l \le x \le l$. Here we account for the possible contact of crack faces using the algorithm developed by Maksymovych (2009). The only nonzero average stress is $\langle\sigma_y\rangle = p$. The normalized stress intensity factors $F_{\mathrm{I,II}} = K_{\mathrm{I,II}}/\left(p\sqrt{\pi l}\right)$ for the left (A) and right (B) tip of the parabola cracks with $k = 1$ in the isotropic material are presented in Tab. 2.

The calculations held show that the contact of crack faces occurs near the right tip of the crack approximately at a one third of its length. The table also shows significant influence of the account for the crack faces contact on the calculated values of SIF. Following table also shows the results of the problem with the same geometry, however, the material of the medium is anisotropic fiberglass CF1 (CF190 corresponds to the same material with the principal anisotropy axes rotated at a right angle).

For all considered particular problems the crack faces contact occurs near the right tip, excepting material CF190 and $\frac{\omega_1}{k} = 2.5$, where the faces contact at the right of the crack center. Tab. 3 shows that for the contact of crack faces the SIF K_{IB} for an anisotropic material (in contrast with the isotropic one) differs from zero, moreover, for the CF190 material SIF is significantly big. To testify this phenomenon, consider the crack tip normal displacement discontinuities. According to Bozhydarnyk (1998) they equal:

$$[u_n] = 4a_{11}\frac{\sqrt{r}}{\sqrt{2\pi}}\left(u_{11}K_{\mathrm{I}} + u_{12}K_{\mathrm{II}}\right) \tag{10}$$

where r is a distance to the tip; $u_{11} = -\mathrm{Re}[i(s_1 - \bar{s}_2)g_2\bar{g}_1]$, $u_{12} = \mathrm{Re}[i(s_1 - \bar{s}_2)d_2\bar{g}_1]$, $d_j = \cos\varphi + s_j\sin\varphi$, $g_j = \sin\varphi - s_j\cos\varphi$, and φ is an angle between the tangent to the crack at its tip and Ox axis.

It should be mentioned that for an orthotropic material, for which $s_j = i\beta_j$, the following relations hold:

$$u_{11} = (\beta_1 + \beta_2)\cos^2\varphi(k^2 + \beta_1\beta_2),$$
$$u_{12} = -(\beta_1 + \beta_2)\cos^2\varphi k(1 - \beta_1\beta_2).$$

Tab. 2. Doubly periodic parabola cracks in the isotropic medium

$\frac{\omega_1}{l}$	$F_{\mathrm{I}A}$	$F_{\mathrm{II}A}$	$F_{\mathrm{I}B}$	$F_{\mathrm{II}B}$	$F_{\mathrm{I}A}$	$F_{\mathrm{II}A}$	$F_{\mathrm{I}B}$	$F_{\mathrm{II}B}$
	Not accounting for crack faces contact				Accounting for crack faces contact			
2.5	1.782	0.880	-1.782	0.880	1.684	0.434	0	0.941
3.0	1.394	0.709	-1.394	0.709	1.340	0.412	0	0.783
3.5	1.249	0.638	-1.250	0.638	1.207	0.395	0	0.717
4.0	1.174	0.599	-1.175	0.599	1.137	0.384	0	0.681
4.5	1.129	0.575	-1.129	0.575	1.095	0.376	0	0.659
5.0	1.098	0.558	-1.098	0.558	1.066	0.371	0	0.644
5.5	1.077	0.547	-1.077	0.547	1.046	0.366	0	0.633
6.0	1.061	0.538	-1.061	0.538	1.032	0.363	0	0.626
6.5	1.048	0.532	-1.049	0.532	1.021	0.360	0	0.620
7.0	1.039	0.527	-1.039	0.527	1.012	0.358	0	0.615

Tab. 3. Doubly periodic parabola cracks in the anisotropic medium

$\frac{\omega_1}{l}$	$F_{\mathrm{I}A}$	$F_{\mathrm{II}A}$	$F_{\mathrm{I}B}$	$F_{\mathrm{II}B}$	$F_{\mathrm{I}A}$	$F_{\mathrm{II}A}$	$F_{\mathrm{I}B}$	$F_{\mathrm{II}B}$	$\frac{F_{\mathrm{I}B}}{F_{\mathrm{II}B}}$
	CF1				CF190				
2.5	1.028	-0.604	0.162	0.393	1.633	-0.495	-0.525	0.605	-0.867
3.0	0.880	-0.394	0.065	0.157	1.399	-0.372	-0.560	0.521	-1.075
3.5	0.826	-0.326	0.025	0.061	1.303	-0.317	-0.533	0.495	-1.075
4.0	0.799	-0.298	0.003	0.007	1.253	-0.283	-0.520	0.484	-1.075
4.5	0.784	-0.286	-0.012	-0.028	1.221	-0.260	-0.514	0.478	-1.075
5.0	0.775	-0.281	-0.021	-0.052	1.201	-0.243	-0.510	0.474	-1.075
5.5	0.769	-0.280	-0.028	-0.069	1.187	-0.230	-0.507	0.472	-1.075
6.0	0.766	-0.282	-0.033	-0.081	1.177	-0.219	-0.505	0.470	-1.075
6.5	0.764	-0.284	-0.037	-0.089	1.170	-0.211	-0.503	0.468	-1.075
7.0	0.763	-0.288	-0.039	-0.095	1.165	-0.203	-0.501	0.466	-1.075

From Eq. (10) it follows that under the crack faces contact at the tip the following condition hold:

$$\frac{K_{\mathrm{I}}}{K_{\mathrm{II}}} = -\frac{u_{12}}{u_{11}} \tag{11}$$

For the CF190 material $k = 2$, and thus, $u_{12}/u_{11} = 1.0751.$ One can see that the ration of SIFs presented in the last column of Tab. 3 is in good agreement with this estimation, thus, the crack faces contact condition (11) holds with high accuracy.

4. CONCLUSION

The paper derives compact and easy to use singular integral equations for doubly periodic cracks in an anisotropic medium. The study of the influence of anisotropy and contact of crack faces showed its significance in the calculated values of stress intensity factors. Also it is proved the mode I SIF can be nonzero for an anisotropic material with crack, which faces are in contact.

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