Advances in Science and Technology Research Journal
Volume 8, No. 22, June 2014, pp. 56–59
DOI: 10.12913/22998624.1105167

Research Article

DETERMINATION OF EFFECTIVE PROPERTIES OF FIBER-REINFORCED COMPOSITE LAMINATES

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Received: 2014.05.06
Accepted: 2014.05.20
Published: 2014.06.05

ABSTRACT
The determination of effective mechanical properties of multi-layer composite is presented in this paper. Computations based on finite element method predicting properties of inhomogeneous materials require solving huge tasks. More effective is Mori-Tanaka approach, typical for micromechanics problems. For regularly distributed fibers closed-forms for effective composite material properties are possible to derive. The results of homogenization are used in strength analysis of the composite pressure vessel.

Keywords: micromechanics, Mori-Tanaka, multi-layer composite.

INTRODUCTION

Composite materials are increasingly used in structural engineering. The composites consist of two or more phases, such as a matrix and reinforcement of much better mechanical properties. The most commonly used are matrix polymers, though it may also be a metal e.g. Mg, Al, or ceramics. Reinforcement in the form of powder or fibers improves mechanical properties of the composite such as strength, abrasion resistance, or resistance to high temperatures. Most composites exhibit anisotropy. Their properties are not the sum or average of properties of the individual components. Effective analysis of mechanical properties of the composite can be carried out using the finite element method. Figure 1 shows the FEM model of three-layer composite of regularly spaced fibers. The application of mechanical or kinematical load to the opposite surfaces of the model allows to find the effective Young's modulus, shear moduli and Poisson's ratios for various directions. Reliable results require solving very large problems of millions degrees of freedom. Computations become very sophisticated when elasto-plastic properties of the components and delamination of the composite are taken into account. In such cases, homogenization methods used in micromechanics are more efficient. Effective anisotropic material properties of the composite are based on the properties of the individual components, their volume fractions and spatial arrangement (directions). Among the homogenization approaches the Mori-Tanaka method is standard.

In the presented research the parallel fiber reinforced laminate is analyzed, for which the closed-form formulas to its effective linear elastic properties are derived. The goal of the study is to determine the anisotropic material properties of the composite material using micromechanics formulas and use them in the analysis of the thin-walled vessel under internal pressure. In the future research the composite materials with non-linear properties and complex structures will be investigated.

HOMOGENIZATION – ESHELBY AND MORI-TANAKA SOLUTIONS

Homogenization is the process of determination of the macroscopic properties of the material consisting of several phases i.e. the matrix and inclusions or inhomogeneities (fibers, microcracks, voids). As a result, there is no need to analyze the responses of the structure on micro or nano level which would require the time-consuming calculations. In micromechanics the concept of the representative volume element (RVE) is introduced. Its size depends on the scale level for which the response of the material to the external load is considered. For the three-layer composite shown in Figure 1, depending on the assumed scale the RVE can include: the volume comprising all three layers of the laminate, a single layer, or only the neighborhood of the single layer. Figure 2 shows the body *D* of the elastic modulus $\boldsymbol{C_0}$ which contains an ellipsoidal heterogeneity Ω of elastic modulus $\boldsymbol{C_1}$. In the homogenization process the effective stiffness $\boldsymbol{C}$ of the homogeneous body under identical boundary conditions is determined. Depending on the mutual ratio between the semi-axes of the ellipsoid, the following shapes of inclusions are possible to obtain: a sphere, a disk, or long fibers. Eshelby [1] was first to solve the problem with a single ellipsoidal inclusion ($\boldsymbol{C_0} = \boldsymbol{C_1}$). He assumed that the inclusion is initially cut out of the infinite body *D*, next subjected to deformations causes the occurrence of a uniform strain ε^*, and finally inserted back into the volume *D*. Strains occurring within inclusions depend on ε^* and so-called Eshelby tensor which for ellipsoidal inclusions is the function of the Poisson's ratio. Mori-Tanaka [2] extended Eshelby's solution into inhomogeneities ($\boldsymbol{C_0} \neq \boldsymbol{C_1}$). In the case of inhomogeneities, the eigenstrain ε^* is selected in such a way that the problem with the inclusion provides the same stress field as the problem with inhomogeneity. The problem of determining the properties of non-homogeneous materials has been extended by other researchers on the issues of multiple heterogeneity – cases when interactions between inhomogeneities are considered or neglected, the problems of inhomogeneity covered with additional layers, or structures composed almost exclusively of heterogeneities such as grains, etc. Their description is beyond the scope of this study.

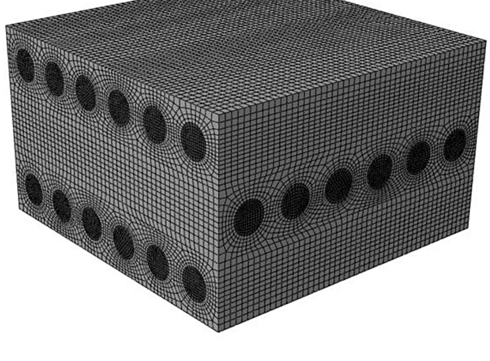

Fig. 1. FEM mesh for multi-layered composite

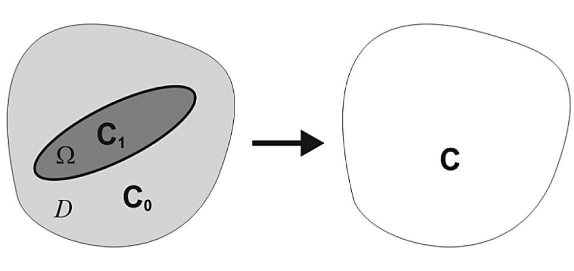

Fig. 2. Heterogeneous material (left) and equivalent homogeneous material (right)

DETERMINATION OF EFFECTIVE PROPERTIES OF THE FIBER-REINFORCED LAMINATES

Figure 3 shows the fiber-reinforced composite analyzed in this research. The volume fraction of the fiber is c_f and the volume fraction of the matrix is c_m:

$$c_f + c_m = 1 \,. \tag{1}$$

Young's moduli of composite fibers and matrix are E_f i E_m, and Poisson's rations are n_f i n_m, respectively. The effective Young's modulus and Poisson's ratio may be estimated [3] by the Reuss lower bounds (E^R, ν^R) or Voigt upper bounds (E^V, ν^V). Assuming that the Poisson's ratios of the fibers and the matrix are the same $\nu_f = \nu_m = \nu$ we obtain:

$$E^V = c_f E_f + c_m E_m, \quad \nu^V = \nu \tag{2}$$

$$E^R = \frac{E_f E_m}{c_f E_m + c_m E_m}, \quad \nu^R = \frac{c_f \nu_f E_m + c_m \nu_m E_f}{c_m E_f + c_f E_m} \tag{3}$$

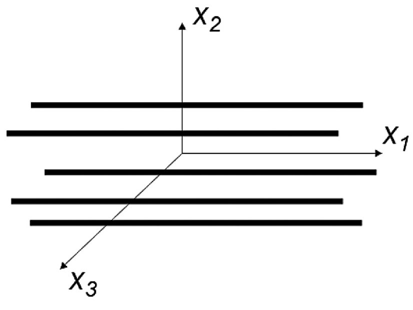

Fig. 3. Model of unidirectional fiber-reinforced composite

It can be noticed that the upper Voigt estimate gives a better approximation in the fibers direction, while the lower Reuss estimate gives a better approximation in a direction perpendicular to the fibers. Because of simplicity, equations (2) and (3) are widely used in engineering practice to determine the effective longitudinal and transverse Young's modules of the composite. More precise estimates can be obtained using methods typical for micromechanics based on the Eshelby and Mori-Tanaka solutions. For long fibers of circular cross-sections it is possible to derive the Eshelby tensor analytically, which leads to the following formulas for the longitudinal E_L and transverse E_T effective Young's moduli:

Due to the complex expression for E_T the approximation (5) is often used, which is a very good estimate for most real materials. The expressions for effective shear moduli of the composite can be found in [3].

BENCHMARK TEST – COMPOSITE VESSEL UNDER INTERNAL PRESSURE

Equations (3) – (5) allow for the calculation of effective material data of the composite. These expressions were used to determine the input (material) data for the test problem. The pressure vessel made of the composite material (Figure 4)

$$E_L = \left[\eta c + (1-c)\right]E_m + \frac{2c(1-c)(\nu_f - \nu_m)^2 \eta E_m}{(1-c)(1+\nu_f)(1-2\nu_f) + \eta(1+\nu_m)\left[1 + c(1-2\nu_m)\right]} \tag{4}$$

$$E_T \approx \frac{2(1-c) + \eta(1+2c)}{\eta(1-c) + (2+c)} E_m . \tag{5}$$

where: $c = c_f$, $\eta = \dfrac{E_f}{E_m}$.

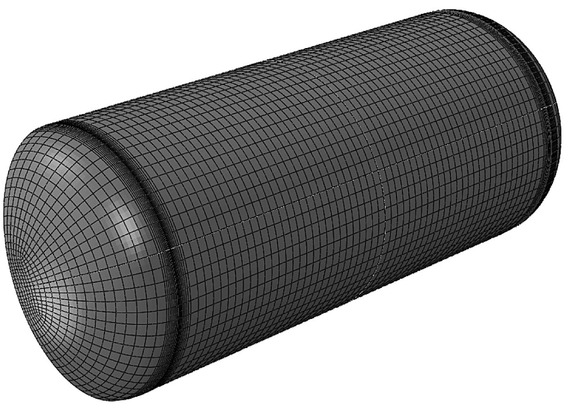

Fig. 4. Model of cylindrical vessel under internal pressure

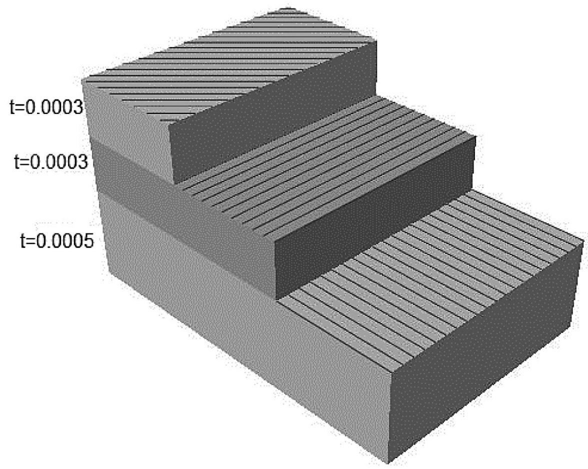

Fig. 5. Composite layers and thicknesses [mm]

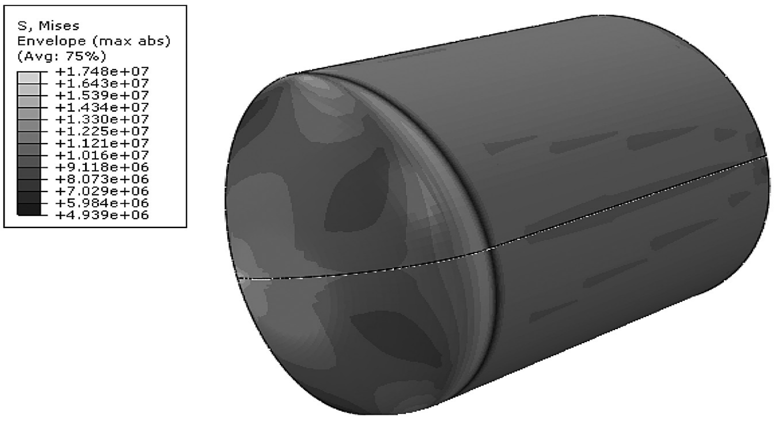

Fig. 6. Maximum effective stress [Pa] in pressure vessel

is considered as a benchmark test. Due to multiple symmetry only 1/8 of the vessel is analyzed. On the planes of symmetry appropriate boundary conditions are applied. It is assumed that the tank is made of a composite including the base (Nomex material) and two layers of glass fiber reinforcements. Figure 5 shows the individual composite layers: the core of 0.5 mm thickness and a glass fiber layers of 0.3 mm thicknesses (fiber directions are 90 and 45 degrees). It is assumed that the applied internal pressure is 0.05 MPa. Dense FEM mesh is generated at the connections between the cylindrical part of the tank and its bottoms, where very large stress gradients occur. Numerical calculations are performed by the commercial ABAQUS program. ABAQUS enables to display the stress distributions separately for each of the composite layers, the stress in the outer layers only, or the extreme stress in all layers of the composite (option envelope). Figure 6 presents the distributions of maximum Huber-Mises stress in the tank (the envelope option). As one can see due to the anisotropy of the composite the stress distribution is not axisymmetric.

FINAL REMARKS

The calculation of the effective material properties of the composite with a regular pattern of parallel fibers is presented in this paper. Effective material data are determined analytically by the use of classical micromechanics approaches. In the case of irregular composite structures they should be derived numerically – for this purpose the commercial software Digimat can be used. As a benchmark test the thin-walled tank under internal pressure is analyzed. Obtained stress distribution varies in the subsequent layers of the composite, and stress contours are not axisymmetric.

Formulas for the longitudinal and transverse Young's modulus can be also used for solving reverse engineering problems in which for assumed material properties the thickness of composite layers as well as the volume fractions of the matrix and fibers should be determined. Methodology presented in this paper can be also successfully applied to the analysis of metal composites.

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