

# Sets of Waveform and Mismatched Filter Pairs for Clutter Suppression in Marine Radar Application

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**ABSTRACT:** Sets of waveform and mismatched filter pairs are used. On the contrary with Golays matched waveform filter pair the mismatched waveform filter pair does exist for all  $N$  (number pulses in waveform). Using corresponding shapes of filter good Doppler tolerance may be provided. This property together with a good range side-lobes level suppression makes it's attractable for use in marine radar.

## 1 INTRODUCTION

Nowadays the necessity of marine radar design with using pulse compression waveforms exists, allowing to reduce significantly the peak power of radiation and thus to improve the working conditions of seafarers and electromagnetic compatibility with other vessel's devices. There are two main tasks of radar: detecting a target and determining its range. Fairly early range has expanded to include direction to the target and radial velocity between the radar and the target. Application of pulse compression waveforms gives the possibility remove many of constraints, providing executive new tasks: increasing the range of radar operation under limited transmitter peak power; improving detectability small-size targets on the background sea surface by mean increasing Doppler selectivity. Extraction of signals from interfering reflections is very important for such kind of radar waveform and filter design. The quality of such extraction significantly depends from range-velocity distribution of interfering reflections [1,2,3]. The problem of the mismatched filter and waveform design that maximizes the signal-to-noise-plus-clutter ratio at the receiver filter output has been formulated and addressed in [4], [5], [6], [7], [8], [9], [10], [11].

Mismatched filtering may causes degradations in signal-to-white noise ratio. In [10] the method of a filter optimization which maximizes the signal-to-noise ratio under additional quadratic constraints was developed. In [12] the methods of joint optimization signal and filter for interfering reflections suppression under additional constraints on range resolving performance, signal-to-noise ratio loses and given amplitude modulation of signal with different limitations on the memory and the width of the pass band of the filter were developed.

The signals and filters design technique presented in [13] is extension of methods [12] to the case of waveforms and filters sets with group-complementary properties, which are optimized simultaneously. In this work we consider discrete signals and the optimizing discrete filters for cases

electronically scanning antenna and mechanical scanning rotating antenna. The method of filter optimization, considering Doppler shift of signal for both cases, is suggested.

We consider discrete signals and the optimizing discrete filters with complex envelopes [10, 11]:

$$S(t) = \sum_{p=1}^P \sum_{n=0}^{N-1} S_{pn} u(t - (p-1)T - nT_0) \quad (1)$$

$$W(t) = \sum_{p=1}^P \sum_{m=0}^{M-1} (W_{pm} u(t - (p-1)T - mq_f T_0)) \quad (2)$$

where

$$u(t - mZ) = \begin{cases} 1, & (m-1)Z \leq t \leq mZ \\ 0, & \text{for other values of } t \end{cases};$$

$T_0$  – is elementary pulse  $u(t)$  duration;  $T$  – repetition period of signals;  $q_f = \Delta F_S / \Delta F_W$  – is a parameter which characterizes the pass-band of the filter  $\Delta F_W$  in respect to the spectral width of the signal  $\Delta F_S$ ;

$S_{pn}$ ;  $W_{pm}$  – are complex amplitudes and weighting coefficients of waveform  $S_p$  and mismatched filter  $W_p$  pairs.

$P$  – number of waveform and mismatched filter pairs in set.

Considering optimization reduces to a choice of the signal  $S(t)$  and of the filter  $W(t)$  which maximizes the ratio [9]:

$$\sigma = \frac{A^2 |X_{SW}(0,0)|^2}{\nu \int_{-\infty}^{+\infty} |W(t)|^2 dt + \sigma_0 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \sigma_{\xi}(\tau, f) |X_{SW}(\tau, f)|^2 d\tau df} \quad (3)$$

where:

$$X_{SW}(\tau, f) = \int_{-\infty}^{+\infty} W^*(t) S(t - \tau) e^{i2\pi f t} dt \quad (4)$$

is the Cross ambiguity function of signal (1) and filter (2);  $\sigma_{\xi}(\tau, f) = \sigma(\tau, f) + \xi \delta(f)$ ;  $\sigma(\tau, f)$  is the range – velocity distribution of the interfering reflections;  $\delta(f)$  is delta-function;  $A, \sigma_0$  are parameters which characterizes the reflecting properties of the target and the interfering reflections;  $\nu, \xi$  are the parameters for controlling signal-to-noise ratio losses and range resolving properties correspondingly. More over, for the considered below problem of multiple joint waveforms and filters optimization, by a proper parameter selection, the ideal correlation properties (no range side-lobes) for the sum of cross-correlation functions (complementary property) may be enforced.

## 2 FILTER OPTIMIZATION

We consider the case when,

$$NT_0 + Mq_f T_0 \leq T \quad (\text{if } q_f = 1, M = N, \text{ so } 2NT_0 \leq T) \quad (5)$$

In the case (5) expression (4) can be written as follows (see [13], [14],[15],[16])

$$X_{SW}(\tau, f) = \sum_{p=1}^P X_{S_p W_p}(\tau, f) e^{i2\pi f (p-1)T} \quad (6)$$

Formula (6) simplifies the task of maximizing (3). So we have got expression (3) for the sets of signals and filters at the same form as we have for the single signal and filter and we can use iteration process of joint optimization signal and filter which was described in [12]. The iteration process is follows: at first for the given sets of signals we are getting optimal sets of filters, then for this sets of filters we are getting optimal sets of signals and so on. The convergence of the iteration process was proved in [17].

Ratio (3) for signal (1) and filter (2) with using (6) may be rewrite in the matrix form

$$\sigma = \frac{A^2 |W * S|^2}{W * [\nu I + \sigma_0 D_{\theta_{\xi}}] W} \quad (7)$$

where  $S, W$  are the vectors of complex amplitude of signals set and filters set;  $I$  – identity matrix;  $D_{\theta_{\xi}}$  is correlation matrix of interfering reflections with range-velocity distribution  $\sigma_{\xi}(\tau, f)$ .

At the first step of iteration process we choose any initial set of signals vector  $S^0$  and find for it optimum vector sets of filters  $W^0$  according to expression [12]

$$W^{(0)} = \left[ \nu I + \sigma_0 D_{\theta_{\xi}}^{S^{(0)}} \right]^{-1} S^{(0)} \quad (8)$$

At the next step for the  $W^{(0)}$  we find  $S^{(1)}$  [12]

$$S^{(1)} = \left[ \nu I + \sigma_0 D_{\theta_{\xi}}^{W^{(0)}} \right]^{-1} W^{(0)} \quad (9)$$

For the given amplitude of the signal at this step we choose only phases of the signal according to algorithm which was described in [12]. In this work only first step of optimization will be considered.

If consider the case  $M=N$  and  $q_f = 1$  in (7), (8), (9) then dimension of set of signals vector and dimension of set of filters vector are the same and equal  $PN$ , dimension of matrix is  $PN \times PN$ .

## 3 NUMERICAL RESULTS

We consider the case only filter optimization according to (8). As an example we calculate  $W^{(0)}$  from (8) for the case  $N=M=3, P=2, \sigma_{\xi}(\tau, f) = \xi \delta(f)$  [14] vector  $S^{(0)\prime} = [1 -1 -1 1 1 1]$  and calculated filter

$$W^{(0)\prime} = [1 -0,5 -0,5 1 -0,5 0,5].$$

$S_2^{(0)\prime} = [1 1 1]$  have set of signals,  $S_1^{(0)\prime} = [1 -1 -1]$  and set of filters  $W_1^{(0)\prime} = [1 -0,5 -0,5]$ ,  $W_2^{(0)\prime} = [1 -0,5 0,5]$ .

Cross correlation functions

$$R^{S_1 W_1} = [-0,5;0;2;-0,5;-1]$$

$$R^{S_2 W_2} = [0,5; 0; 1; 0,5; 1].$$

As we can see this pairs of signals and filters are complementary (the sum of cross correlation functions has zero side-lobes). This example is interesting because the classical Golay complementary pair for  $N=3$  doesn't exist, but for mismatched case it does [13, 14].

In this example  $N=M=3$ ,  $P=2$  signal-to-noise ratio loses [5]  $\rho=0,5$ . But if we increase memory of filter  $N=3$ ,  $M=5$  we get  $\rho=0,7$  [14]. So signal-to-noise ratio loses are decreased. For this case we have

$$S_1^{(0)j} = [1-1-1]; S_2^{(0)j} = [1 1 1];$$

$$W_1^{(0)j} = [-0,5;1,0;-0,9;-0,4;0,2];$$

$$W_2^{(0)j} = [-0,5;1,0;0,1;0,8;-0,2]$$

Cross correlation functions

$$R_k^{S_1 W_1} = [0; 0,2;-0,6;-0,7; 2,3;-0,6;-0,5; 0,5; 0];$$

$$R_k^{S_2 W_2} = [0;-0,2; 0,6; 0,7; 1,9; 0,6; 0,5;-0,5;0].$$

Another example  $N=5$ ,  $M=5$ ,  $P=2$  [11]

$$S_1 = [1 1 1 1-1]; S_2 = [1 1 1-1 1]$$

$$W_1 = [1 4 1 6-4]; W_2 = [1 4-1 6 4].$$

Cross correlation functions

$$RSW_1 = [-4;2;3;7;16;0;4;-3;-1];$$

$$RSW_2 = [4;-2;-3;-7;14;0;-4;3;1]; \rho=0,64.$$

For increased filter memory  $N=5$ ,  $M=7$  we have [11]

$$W_1 = [-1,5;3,5;2;2;6;-6,5;-1,5];$$

$$W_2 = [-1,5;3,5;5;1;-6;6,5;1,5]; \rho=0,79.$$

So, signal-to-noise ratio loses are also decrease.

Consider a few examples else:  $N=6$ ,  $M=6$ ,  $P=2$

$$S_1 = [1 1 1 1 1 1]; S_2 = [1 1 1 1-1];$$

$$W_1 = [-1;7;-1;5;11;-7]; W_2 = [-1;7;1;-5;-11;7]; \rho=0,39.$$

$$N=7, M=7, P=2$$

$$S_1 = [1 1-1-1 1-1 1]; S_2 = [1 1 1-1-1 1-1];$$

$$W_1 = [1;0,75;1;1;2,5;0,75;0,75];$$

$$W_2 = [1;0,75;1;-1;-1;-0,75;-0,75]; \rho=0,21,$$

but for increased memory  $N=7$ ,  $M=9$ , we get  $\rho=0,24$ .

$$S_1 = [1111111]; S_2 = [111-1-1-1];$$

$$W_1 = [0,11;-0,11;0,11;-0,14;0,14;-0,11;0,14];$$

$$W_2 = [0,11;0,11;0,11;-0,14;-0,14;0,11;-0,14];$$

$$\rho=0,33.$$

$$N=5, M=5, P=6$$

$$S_1 = [111-11]; S_2 = [11-111]; S_3 = [1-1111];$$

$$S_4 = [11111]; S_5 = [1111-1]; S_6 = [11111]; W_1 = S_1;$$

$$W_2 = S_2; W_3 = S_3; W_4 = S_4; W_5 = S_5; W_6 = -S_6; \rho=0,44$$

$$N=5, M=5, P=4$$

$$S_1 = [111-11]; S_2 = [1-111-1]; S_3 = [1-1-1-11];$$

$$S_4 = [1-1-1-1-1]; W_1 = S_1; W_2 = S_2; W_3 = S_3; W_4 = S_4.$$

$$N=6, M=6, P=4$$

$$S_1 = [1-111-11]; S_2 = [1-1111-1]; S_3 = [111-1-1-1];$$

$$S_4 = [111-111];$$

$$W_1 = S_1; W_2 = S_2; W_3 = S_3; W_4 = S_4.$$

$$N=7, M=7, P=8 [13]$$

$$S_1 = [111-1-11-1]; S_2, S_3, S_4, S_5, S_6, S_7, S_8 \text{ are cyclic shifts of signal } S_1;$$

$$S_8 = [1111111];$$

$$W_i = S_i (i=1,2,\dots,P).$$

$$N=11, M=11, P=4$$

$$S_1 = [111-111-11-1-11]; S_2 = [111-11-1-1-1-111];$$

$$S_3 = [111-1111-111-1]; S_4 = [111-11-1111-1-1];$$

$$W_i = S_i (i=1,2,\dots,P); \rho=1.$$

Cross correlation functions are represented on fig.1-5.

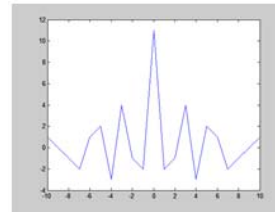


Figure 1. for  $S_1$

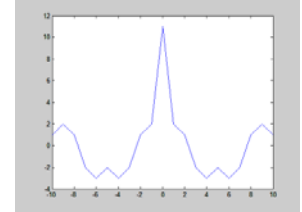


Figure 2. for  $S_2$

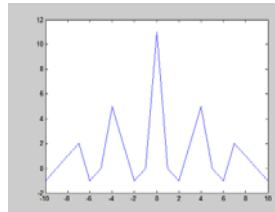


Figure 3. for  $S_3$

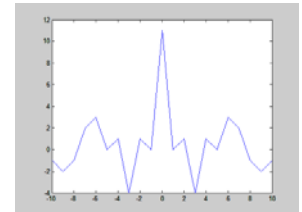


Figure 4 for  $S_4$

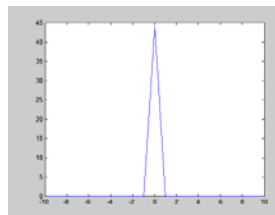


Figure 5. for the sum of the correlation functions

In last four examples we get the maximum value of  $p=1$ , which corresponds to matched filters (without signal-to-noise ratio loses).

For considered cases  $\sigma_{\xi}(\tau, f) = \xi \delta(f)$  matrix  $D_{\theta_{\xi}}, W^* S$  in (7) is formed by the next way:

$$D_{\theta_{\xi}} = \xi \sum_{k=1-N}^{N-1} \begin{bmatrix} S_{1k} \\ \dots \\ S_{pk} \end{bmatrix} \begin{bmatrix} S_{1k}^* \\ \dots \\ S_{pk}^* \end{bmatrix} \quad (10)$$

were  $S_{pk}$  - vector of signal with number  $p$  in set, which is shifted on  $k$  position:

$$S_{pk} = \begin{cases} \begin{bmatrix} 0 \\ \vdots \\ S_{0p} \\ \vdots \\ S_{2p} \end{bmatrix}, & \text{if } k \geq 0 \\ \begin{bmatrix} S_{N-3p} \\ S_{N-2p} \\ S_{N-1p} \\ \vdots \\ 0 \end{bmatrix}, & \text{if } k \leq 0 \end{cases}$$

$$W * S = \begin{bmatrix} W_1^* & \dots & W_{P-2}^* & W_{P-1}^* & W_P^* \end{bmatrix} \begin{bmatrix} S_1 \\ \vdots \\ S_{P-2} \\ S_{P-1} \\ S_P \end{bmatrix} \quad (11)$$

Considered method of optimization set of filters for given set of waveforms, as a first step of joint waveform-filter sets optimization, suggested in [13], gives the possibility to get the solutions for arbitrary discrete signals and arbitrary number of signals in set. The minimum losses in signal-to-noise ratio are provided for given sets of signals. For existing non trivial solutions only no singularity of matrix (10) should be provided. For example in the case of identical signals in set the matrix (10) is singular and no trivial solutions for set of filters, which provided complementary property, doesn't exist. Increasing memory of filters leads to decreasing of signal-to-noise ratio losses. It is very important to note that getting solutions allow getting new sets of signals and filters without any additional calculations. Really, if we have set of two signals with equal  $N$  and set of two corresponding mismatched filters, which are complementary, we may create new complementary sets of two signals and two filters, but with twice lengths by the next way[11]:

$$S_1(2N)=[S_1(N); S_2(N)]; S_2(2N)=[S_1(N); -S_2(N)];$$

$$W_1(2N)=[W_1(N); W_2(N)]; W_2(2N)=[W_1(N); -W_2(N)].$$

The value of  $p$  is reserved as for the length  $N$ .

We can demonstrate it for example which are considered above for  $N=3, M=3, P=2$

$$S_1(6)=[1111-1-1]; S_2(6)=[111-111];$$

$$W_1(6)=[2-112-1-1]; W_2(6)=[2-11-211]; \rho=0,5.$$

If we have set of two signals with different  $N_1; N_2$  and set of two corresponding mismatched filters, which are complementary, we may create new complementary sets of four signals and four filters with lengths  $N_1+N_2$  by the next way[11]:

$$S_1(N_1+N_2)=[S_1(N_1); S_1(N_2)];$$

$$W_1(N_1+N_2)=[W_1(N_1); W_1(N_2)];$$

$$S_2(N_1+N_2)=[S_1(N_1); -S_1(N_2)];$$

$$W_2(N_1+N_2)=[W_1(N_1); -W_1(N_2)];$$

$$S_3(N_1+N_2)=[S_2(N_1); S_2(N_2)];$$

$$W_3(N_1+N_2)=[W_2(N_1); W_2(N_2)];$$

$$S_4(N_1+N_2)=[S_2(N_1); -S_2(N_2)];$$

$$W_4(N_1+N_2)=[W_2(N_1); -W_2(N_2)].$$

We can demonstrate it for examples which were calculated above for  $N=M=6=N_1=6, P=2$  and  $N=M=3=N_2, P=2$ .

$$S_1(9)=[11111-11-1-1];$$

$$W_1(9)=[-1;7;-1;5;11;-7;1;-0,5;-0,5]$$

$$S_2(9)=[11111-1-111];$$

$$W_2(9)=[-1;7;-1;5;11;-7;-1;0,5;0,5];$$

$$S_3(9)=[1111-11111];$$

$$W_3(9)=[-1;7;1;-5;-11;7;1;-0,5;0,5];$$

$$S_4(9)=[1111-11-1-1-1];$$

$$W_4(9)=[-1;7;1;-5;-11;7;-1;0,5;-0,5]; \rho = 0,37$$

This suggested approach to construction new complementary sets of filters and signals is an extension of known approach for the mismatched case.

Resembling task is considered in [17] where the filter design technique was worked out for given set of signals and considered only signal-to-noise losses and complementary properties and doesn't considered another type of interference. The task of maximizing signal-to-noise ratio under constraints on fulfilling complementary properties was solved. Number of equations which must be solved for that is equal to  $PN+2N-1$ . This is much more than we have in our case. Although our approaches are also maximizing signal-to-noise ratio and besides suppressing another types of interference.

In [18] also is considering the task of maximization signal-to-noise ratio under constraints on complimentary property, but only for two signals in set. Approach is similar to [17].

Beside in [17] a few questions of using some properties of shifting m-sequences for designing the array of Golay's sequences are considered, but without any background of it. In [13] the back grounding of this property on the base of analyzes the fundamental properties of Cross ambiguity function and extension this property on another wide class of signals have been done.

All signals and filters considered may be used for as group-complementary sets of waveforms and filters for the case of antenna with electronically scanning.

In the case of rotating antenna the group-complementary properties of sets of waveform and filters (figure a) are destroyed due to amplitude modulation. So the construction waveforms and filters may be realise in other way (figure b), which guarantees zero side-lobes level in nearby peak of correlation function zone independently of rotating antenna effect.

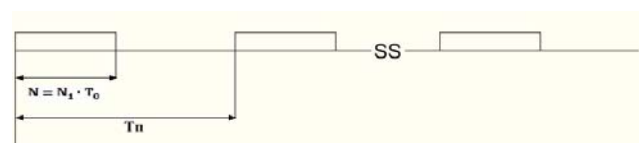


Figure a Diagram of signals for the case of an electronic scanning antenna

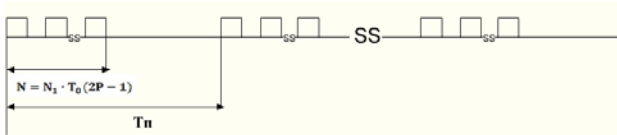


Figure b. Diagram of signals for the case of the rotating antenna

We demonstrate it on the example of  $N=15$  on the base of waveforms and filters set for  $N_1=5, M_1=5, P=2$ , which we were considered above:  $S=[1111-100000111-11]; W=[1;4;1;6;-4;00000;1;4;-1;-6;4]$ . Cross correlation functions for these signal and filter (filter tuned on different Doppler frequencies  $L=0; L=1; L=2$ ) are shown on Fig. 6. On this Fig. we can see zero side-lobes level in the nearby zone of the cross correlation function central peak.  $\rho = 0,64$ .  $L = Fw4NT_0$  ( $Fw$  – frequency of filter tuned).

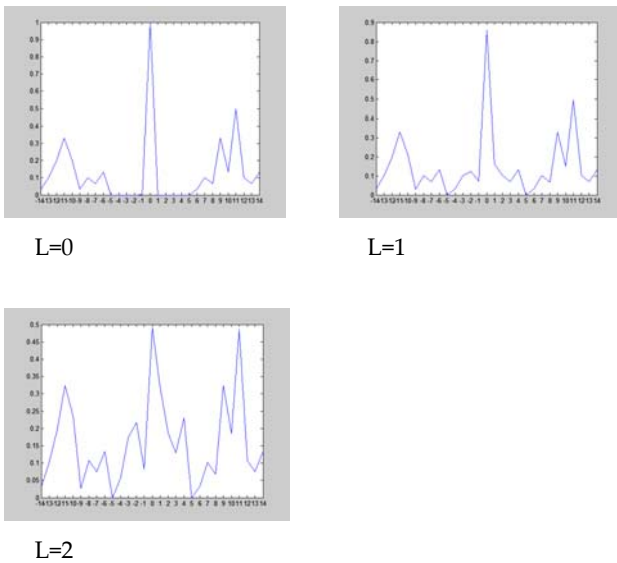


Figure 6. When the Doppler shift of signal  $l_1=0$  for  $N=15$

We also may consider the signal and filter  $N=77$ , which are constructed on the base of waveforms and filters set for  $N_1=1; M_1=1; P=4$ .

$S=[111-111-11-1-1100000000000111-11-1-1-1-1100000000000111-111-111-100000000000111-11-1111-1-1]; W=S$ . Cross correlation functions are shown on Fig.7 ( $L=0; L=1; L=2$ ).  $\rho = 1$ .

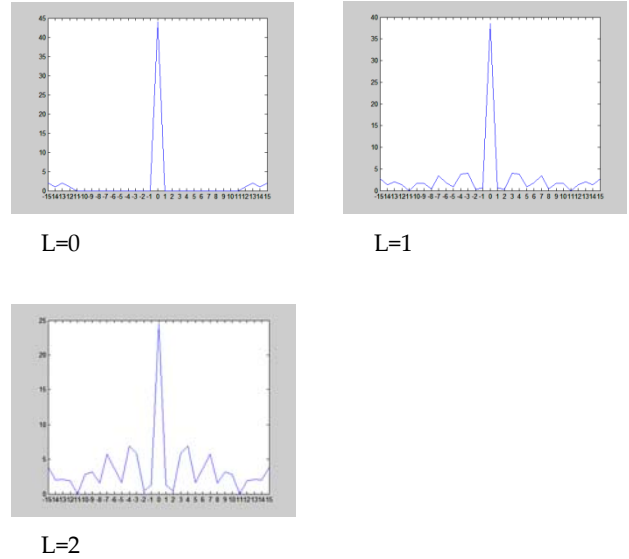


Figure 7 When the Doppler shift of signal  $l_1=0$  for  $N=77$

The tolerance for Doppler shift of signal should be provided for both cases of antenna scanning (electronically and mechanically rotating) by means of special filter counting [16]. Results of such counting for the Doppler shift of signal  $l_1=1$  ( $l_1 = Fw4NT_0$ ) are shown for last two examples on Fig.8 and Fig.9 correspondently. Tolerance to Doppler shift of signal can be seen from comparison of the cross sections  $l_1=0, L=0$  (pictures on Fig.6, Fig.7) and  $l_1=1, L=1$  (pictures on Fig.8, Fig.9).

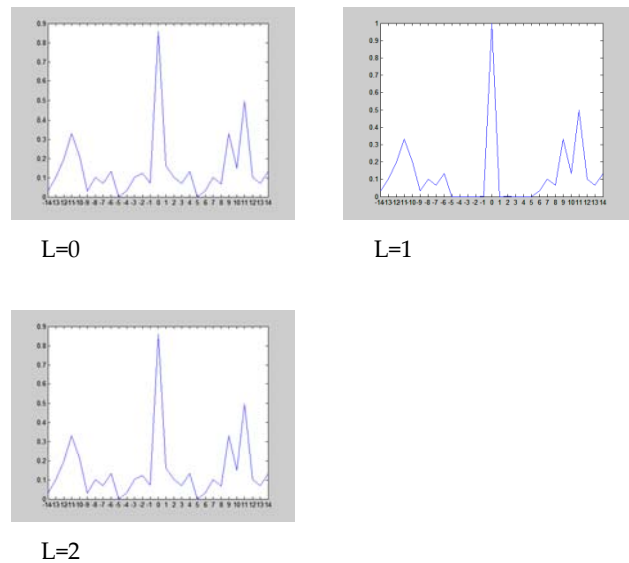


Figure 8 When the Doppler shift of signal  $l_1=1$  for  $N=15$

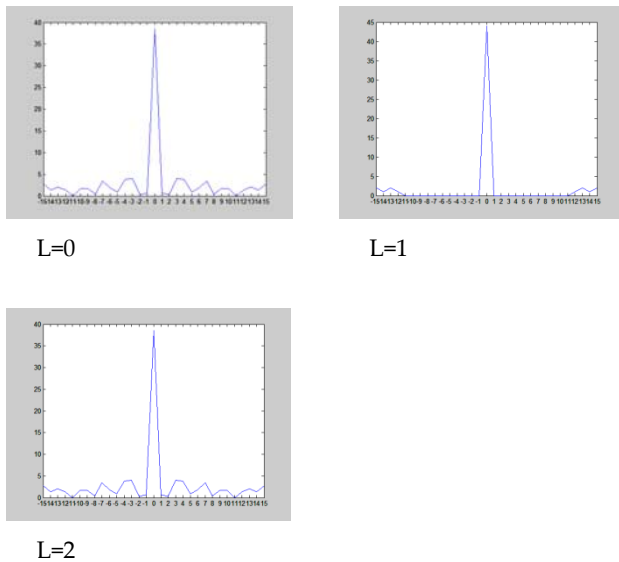


Figure 9. When the Doppler shift of signal  $l_1=1$  for  $N=77$

#### 4 CONCLUSION

This paper demonstrated the efficiency of filter synthesis under additional constraints with group-complementary properties. It was shown that signal-to-noise ratio losses decrease with increasing memory of optimizing filters in set.

Approaches for the construction of new sets of signals and filters on the base of known sets of signals and filters with complementary properties were suggested for different kind of antenna scanning.

The counting of filters, which provides the tolerance for Doppler shifts of signal are also suggested.

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