ENERGY ANALYSIS OF THE PROPULSION SHAFT FATIGUE PROCESS IN A ROTATING MECHANICAL SYSTEM
PART II
IDENTIFICATION STUDIES – DEVELOPING THE FATIGUE DURABILITY MODEL OF A DRIVE SHAFT

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ABSTRACT

The article presents a continuation of research carried out concerning identification of energy consequences of mechanical fatigue within a propeller shaft in a rotating mechanical system, while working under conditions of the loss of the required alignment of shaft lines. Experimental research was carried out on a physical model reflecting a full-sized real object: i.e., the propulsion system of the ship. It is proven, by means of an active experiment, that changes in propeller shaft deflection are reflected in the amount of dissipated kinetic energy of masses in rotational motion and the accumulated internal energy in its construction material. Adoption of a high-cycle fatigue syndrome, consisting of diagnostic symptoms determined from the action of the propeller shaft associated with the transformation of mechanical energy into work and heat, as well as with the generation of mechanical vibrations and elastic waves of acoustic emission, is proposed. To assess the diagnostic information quantity brought about by the defined features of propeller shaft fatigue, an experimental research program was developed and implemented, in which two statistical hypotheses are verified: the significance of the impact of the values enforcing the fatigue process, presented in the first part of the article, and the adequacy of the regression equation describing the fatigue durability of the propeller shaft in the energy aspect, constituting the second part of the article. This finally gives us the opportunity, after the appropriate translation of the model test results into full-sized real objects, to develop a methodology to diagnose marine propeller shaft fatigue in operating conditions. The third part of the article is devoted to this issue

Keywords: rotating mechanical system, modelling energy processes, propulsion shaft fatigue, testing statistical hypotheses

INTRODUCTION

Identification tests of the rotating mechanical system experimentally confirmed the lack of significance of the impact of one variable (of two) enforcing the drive shaft fatigue process associated with its rotational speed (statistically proven in the first part of this article); while searching for a mathematical function describing the process of shaft fatigue with respect to its durability, only one significant quantity forcing its deflection (fatigue) was taken into account: specifically, the load mass m_{load} [2,5,6].

Thus, an identification experiment based on the statically determined complete study plan of the physical system was carried out. In this plan, the range of changes in the input value m_{loadc} was determined in the same way as for the elimination experiment – i.e., from 30 to 40 kg, at six levels of variability for the assumed values (every 2 kg), as shown in Table 1. The experiment also anticipated keeping the propeller shaft rotational speed constant at 1500 min^{-1}. In each experiment, five repetitions of the recording of the observed control parameters of the rotary mechanical system were performed.

No.	Loading mass m_{load} [kg]	Rotational speed of drive shaft n [min^{-1}]
1	30	1500
2	32	1500
3	34	1500
4	36	1500
5	38	1500
6	40	1500

IDENTIFICATION STUDIES

The main aim of the identification tests was to find a mathematical function describing the fatigue durability of the drive shaft τ_W in terms of input (forcing) quantity values associated with an action of the rotary mechanical system in the form of work D_W, heat D_U, mechanical vibration generation D_V, and generation of acoustic emission elastic waves D_{EA} [1,4]. Assuming that the values of the individual forms of action are determined from measurements as extortions affecting the propeller shaft, the physical model of its fatigue process might be presented as the physical model shown in Fig. 1.
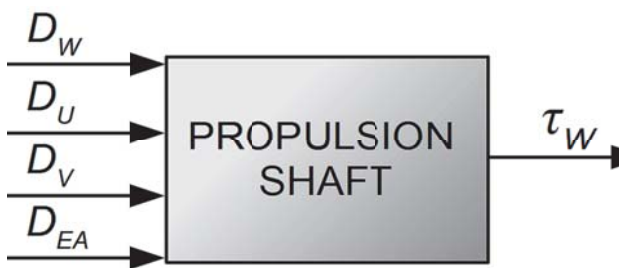

Fig 1. Physical model of propeller shaft fatigue process in a rotating mechanical system developed to determine its durability: D_W – propulsion shaft action associated with mechanical energy conversion in the form of working rotational motion; D_U – propulsion shaft action associated with mechanical energy conversion in the form of heat; D_V – propulsion shaft action associated with mechanical vibration generation; D_{EA} – propulsion shaft action associated with the generation of acoustic emission elastic waves; τ_W – propeller shaft durability.

According to the adopted procedure, the determined plan of the experiment, and the set of input variables selected for the elaborated physical model, an analysis of their impact on the achieved fatigue durability of the propeller shaft was carried out. The results of testing the mechanical fatigue process of the shaft material, after conducting all the measurement sequences assumed in the experiment plan, are presented in Table 2. The result of each measurement series, each consisting of five repetitions, represents the arithmetic mean of the action value of the considered mechanical system expressed in J·s and its fatigue durability τ_{WB}, expressed in s.

Table 2. Results of identification tests of the drive shaft mechanical fatigue process.

m_{load} [kg]	D_W [J·s]	D_U [J·s]	D_V [J·s]	D_{EA} [J·s]	τ_{WB} (tests) [s]
30	$2.91 \cdot 10^5$	$3.53 \cdot 10^7$	$3.84 \cdot 10^{-2}$	$1.73 \cdot 10^{-2}$	8718.2
32	$1.79 \cdot 10^5$	$2.18 \cdot 10^7$	$2.31 \cdot 10^{-2}$	$1.04 \cdot 10^{-2}$	5348.8
34	$1.76 \cdot 10^5$	$1.77 \cdot 10^7$	$1.12 \cdot 10^{-2}$	$9.91 \cdot 10^{-3}$	4258.2
36	$8.71 \cdot 10^4$	$8.76 \cdot 10^6$	$1.76 \cdot 10^{-3}$	$2.12 \cdot 10^{-3}$	2086.2
38	$8.23 \cdot 10^4$	$7.74 \cdot 10^6$	$1.50 \cdot 10^{-3}$	$1.21 \cdot 10^{-3}$	1828.6
40	$6.19 \cdot 10^4$	$5.94 \cdot 10^6$	$7.77 \cdot 10^{-4}$	$1.48 \cdot 10^{-3}$	1372.4

The final result of the experimental tests performed on the physical model of a real object (made on a scale) is a function describing the fatigue durability of the propeller shaft, which in a general form is expressed as follows:

$$\tau_W = f\left(D_W, D_U, D_V, D_{EA}\right) \quad (1)$$

In order to determine the fatigue durability function of the propeller shaft, the multiple regression analysis method was applied [9]. The purpose of a multiple regression is to quantify the relationships between many independent variables: i.e., the so-called explanatory ones. In the considered issue, there were variables characterising the action of the propeller shaft: D_W, D_U, D_V and D_{EA}. The dependent variable (the so-called explained one) is its fatigue durability τ_W. In order to assess the impact of the input factors on the fatigue durability of the propeller shaft (with six levels of variation), the function of the test object takes a linear form, which for the considered case can be written as follows:

$$\tau_{Wi} = \alpha_0 + \alpha_1 \cdot D_{Wi} + \alpha_2 \cdot D_{Ui} + \alpha_3 \cdot D_{Vi} + \alpha_4 \cdot D_{EAi} + \varepsilon \quad (2)$$

where:
$\alpha_0 \div \alpha_4$ – estimated parameters of regression model;
i – level number of input factor variability;
ε – random component.
In the matrix form, this function is given by the following formula:

$$T_w = D \cdot \alpha + \varepsilon \quad (3)$$

where:

$$T_w = \begin{bmatrix} \tau_{W1} \\ \tau_{W2} \\ \tau_{W3} \\ \tau_{W4} \\ \tau_{W5} \\ \tau_{W6} \end{bmatrix} \quad D = \begin{bmatrix} 1 & D_{W1} & D_{W2} & \dots & D_{W6} \\ 1 & D_{U1} & D_{U2} & \dots & D_{U6} \\ 1 & D_{V1} & D_{V2} & \dots & D_{V6} \\ 1 & D_{EA1} & D_{EA2} & \dots & D_{EA6} \end{bmatrix}$$

$$\alpha = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_5 \end{bmatrix}$$

In order to determine the parameters of the function describing the fatigue durability of the propeller shaft, the GRETL computer program, which is widely applied in econometrics [5], was used. The computational algorithms built into the program allow the user to quickly develop their own econometric models by means of using many methods of measurement data approximation. Due to the fact that the identification experiment program had only six levels of variability and the results obtained were characterised by a linear course, the least sum of squares method was applied to estimate the parameters of the linear model. As a result of the conducted analysis of many mathematical models using the GRETL program, only one was chosen because the best one describes statistically (with the highest coefficient of determination R^2) the fatigue durability of a propeller shaft subjected to mechanical fatigue [10]. After taking into account the measurement data summarised in Table 2, the assumed function describing the fatigue durability of the propeller shaft is as follows:

$$\tau_W = -136.172 - 0.00155504 \cdot D_W + 0.000273696 \cdot D_U - 1116.11 \cdot D_V - 17602.4 \cdot D_{EA} \tag{4}$$

STATISTICAL ANALYSIS OF OBTAINED RESULTS

It was assumed a priori that the measurement results of all the control parameters (D_W, D_U, D_V, and D_{EA}) characterising the propulsion shaft's fatigue are subject to random errors and the studied process is affected by various types of disturbances, also of an accidental nature [4]. Therefore, they were modelled as random variables of a normal distribution, with a specific, expected value and variation, as a measure of dispersion of the carried out measurements around the average value. It was also assumed that the variances of statistical data were equal or similar in value. Hence, a parametric statistical test

for variance was used to assess the adequacy of the regression equation describing the considered fatigue process. Because these are tests with a one-sided critical area, verification of the hypotheses was based on Fisher–Snedecor F distribution statistics, comparing the calculated value of the F_{cal} test statistics with the critical (table) value F_{cr} of the distribution, calculated at the assumed significance level, $\alpha = 0.05$, and the number of degrees of freedom, f_1 and f_2 [7].

The following null hypothesis (denoted by H0) was verified in statistical identification tests of the propulsion shaft fatigue process:

H0: The adopted mathematical model (regression equation) describing the fatigue durability of the propeller shaft is adequate.

If the calculated value of the F_{cal} test (empirical) statistics is greater than or equal to the critical value F_{cr} specified from the statistical table for a given level of significance and the number of degrees of freedom ($F_{cal} > F_{cr}$), the null hypothesis should be rejected, considering that the adopted mathematical model is inadequate for the given significance level in the studied range of enforcing quantities.

The matching assessment of the model to the empirical data can be expressed by the determination coefficient which for this model is equal to $R^2 = 0.999998$. The Fisher–Snedecor F function for the adopted model is equal to $F_{cal} = 140576.7$. The critical value of the statistical coefficient for the degrees of freedom of the model – respectively, $f_1 = 4$ and $f_2 = 1$ – is equal to $F_{cr} = 224.583$ [12]. Because $F_{cal} > F_{cr}$, the statistics are located within the critical area of the distribution. Therefore, the zero hypothesis (H0) should be rejected in favour of the alternative hypothesis (H1). Thus, it can be concluded that at least one of the structural parameters of the shaft's fatigue durability model significantly differs from zero and, thus, at least one explanatory variable does not significantly affect the explained variable τ_W.

Adopting the model that is considered to be inadequate creates the possibility of making a second type of error. However, such an error was not made in assuming the model to be adequate, which is confirmed by the numerical values of the fatigue durability of the propeller shaft τ_{WB} obtained from identification tests and the fatigue durability τ_W determined using the mathematical model (4), as shown for comparison in Table 3.

In order to perform a comparative analysis of the determined values of the propulsion shaft's fatigue durability function and the results obtained from the experimental tests, the reference metric δT was applied [3]. Its dimensionless value constitutes a comparative indicator for all the developed mathematical models [10]. The smallest value of the reference metric indicates the mathematical model that best describes (in a quantitative sense) the fatigue durability of the propulsion shaft, as given in Table 3.

$$\delta T = \sqrt{\sum_{i=1}^{6} \left(\frac{\tau_{WB} - \tau_W}{\tau_{WB}} \right)^2} \cdot 100\% \tag{4}$$

Table 3. Results of identification tests of the propulsion shaft's mechanical fatigue process.

D_W [J·s]	D_U [J·s]	D_V [J·s]	D_{EA} [J·s]	τ_{WB} (tests) [s]	τ_W (model) [s]	Reference metric δT [%]
$2.91 \cdot 10^5$	$3.53 \cdot 10^7$	$3.84 \cdot 10^{-2}$	$1.73 \cdot 10^{-2}$	8718.2	8715.0	
$1.79 \cdot 10^5$	$2.18 \cdot 10^7$	$2.31 \cdot 10^{-2}$	$1.04 \cdot 10^{-2}$	5348.8	5353.7	
$1.76 \cdot 10^5$	$1.77 \cdot 10^7$	$1.12 \cdot 10^{-2}$	$9.91 \cdot 10^{-3}$	4258.2	4259.1	
$8.71 \cdot 10^4$	$8.76 \cdot 10^6$	$1.76 \cdot 10^{-3}$	$2.12 \cdot 10^{-3}$	2086.2	2086.7	0.43
$8.23 \cdot 10^4$	$7.74 \cdot 10^6$	$1.50 \cdot 10^{-3}$	$1.21 \cdot 10^{-3}$	1828.6	1830.9	
$6.19 \cdot 10^4$	$5.94 \cdot 10^6$	$7.77 \cdot 10^{-4}$	$1.48 \cdot 10^{-3}$	1372.4	1367.0	

FINAL REMARKS AND CONCLUSIONS

The statistical regression model proposed in this article is based on the linear nature of the fatigue process. The linear character of the propulsion shaft's fatigue durability model, subjected to a loss of required alignment (deflection), results from the application of an action function for its development. This function, in a physical sense, stands for the product of time and work of the fatigue destruction of the shaft's material. Due to the fact that the duration of the experiment is the same and common component for all (four) types of action (input factors), the obtained results of the experiments are characterised with a strong linear relationship and high correlation. For this reason, the application of the least sum of squares method to determine the regression coefficients of the regression equation is justified and allows for determination of the regression equation with a very large (close to unity) determination coefficient R2.

The low value of the reference metric confirms the adequacy of the developed propulsion shaft fatigue model, which may be a prerequisite for further research on its application in diagnosing fatigue of propeller shafts of real objects [11,13].

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