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Identification and modeling the pulsatile blood flow in the cardio-vascular system using a zero-dimensional model in an electrical analogy

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Key words: Windkessel Model, boundary conditions, arterial pressure, blood flow, aorta

Abstract

Advanced computer technology tools, like computational fluid dynamics (CFD) and knowledge about the functioning of the human blood circulatory system, structure of blood, behavior of vessels – allow improving understanding the process of blood distribution in human body. Complex simulation has to assume multiphase approach, walls elasticity and pulsating blood flow conditions, resulting from work of the heart [1]. In the presented work the blood flow and pressure were simulated.

The characteristic impedance, peripheral resistance, capacitance in the systemic peripheral vascular beds had been taken into the consideration in electrical analogues as a lumped parameter model (LPM) of the circulatory system which will be implemented as a boundary conditions in the complete CFD model on the inlet of ascending aorta and on the outlets of descending aorta, innominate artery, left common artery and left subclavian artery [2].

The resistance in a vessel was modeled by electronic component – resistor. The blood flow is not stationary, it is stored (e.g. in the vessels and kidneys) just like energy in the capacitors. Coil is an analog to the inert tendency of blood, which mass resists to move due to the pressure difference. Furthermore it can be assumed that flow has only one direction – so vessels act like a diodes in electronic circles. Additionally the conservation of the mass principle has been applied converted into the Kirchhoff's law [3].

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The realistic, time-course, lumped parameter (0D) model represents the cardiovascular system and is written in Matlab code, however it can be implemented in the external numerical modeling application (CFD) in the future.

1 Introduction

The design and development of the medical equipment requires the precise and accurate numerical model reproducing the physical phenomena inside the vessel. Moreover computer analysis are less harmful then in-vivo methods and overall less expensive. Information about vulnerable zones can enhance diagnose. But the first step to generate specific models for an individual is to prepare model as a generalization of the average person.

To prepare any blood flow model a vast database is necessary; it has been understood as a knowledge about properties of blood components, a description of interactions between them and interactions between blood and vessels. Moreover blood flow model requires also transient boundary conditions which are capable to describe the pulsatile blood flow.

1.1 The scope of the work

The work presents process of creating the lumped parameter model of the blood flow in the cardiovascular system based on its electrical analog. LPM, including main characteristics of blood and vascular tree is able to mimic response of the whole system on the analyzing section.

The heart rate, duration of the systolic and diastolic phases can be adopted to patient data. The resistances and compliances of vessels must be assumed so the validation of the model is dependent on further data. Model was prepared in the Matlab application.

1.2 Cardiovascular system

The heart, deformable vessels and blood are essential components of the cardiovascular system. The greatest function of the system is the transport of oxygen, nutrients, metabolites and hormones and removal of the carbon dioxide, waste products and toxins. It also regulates the temperature and protects body from pathogens.

It is divided for pulmonary and systemic circuits. The function of the pulmonary part is to deliver blood to the lungs for exchange of carbon dioxide and oxygen. It is then returned into the left part of the heart. The left chamber pumps the blood through the systemic circuit for circulation around the body so that oxygen can be used and harmful substances can be taken away and then returns it to the right heart's atrium. Blood is carried again to the lungs [4].

The heart is a two synchronized muscular pumps with four chambers. Atria collect the blood from the veins and ventricles pump the blood to the arteries. The heart is equipped with four valves – one per each cavity – which prevent blood from flowing backwards. When blood is being pushed by contracting left ventricular the mitral valve is closed preventing blood back – flow to atrium and the aortic valve is opened allowing blood to get to aorta. Valves in the right part of the heart behave similarly. The ventricles are separated by a septum so oxygenated and de-oxygenated blood do not mix [4].

Each of the circuits begin with relatively high pressure with the pulsatile flow from the ventricles through a tree-like network of arteries. The size of cross-section of the vessels decrease gradually from the aorta to the arteries then arterioles and finally they turn into the capillaries which task is to exchange substances between blood and tissues. The mass transfer in this microcirculation includes, inter alia, transport though the intercellular gaps, active transport across the membranes or via vesicles. Then blood with modified composition is collected and returned to the heart at low pressure by veins. The pressure in vena cava equals 0.5 kPa so it is much lower than pressure of blood in aorta, which is 12.5 kPa [5]. Thus the structure of vessels is different, those vessels have to resist specific hydrostatic pressure, tension and shear stress. Arteries carrying blood away from the heart have thick walls, thin veins carry blood to it and unlike the arteries have valves. Moreover vessels respond to changes in pressure and flow rate changing temporarily the diameter by smooth muscle cells. It can be described as a pulse wave propagation and characterized by specific value of Reynolds number:

$$Re = \frac{Dw}{\vartheta} \tag{1.1}$$

where D is a diameter of vessel, m; w mean velocity of the blood, $m \cdot s^{-1}$ and ϑ is its kinematic viscosity, $m^2 \cdot s^{-1}$.

Considering the blood viscosity Fahraeus – Lindquist effect should be mentioned. It turns out that in the capillary the plasma create the layer covering the vessel wall and red cells stay in the central part which helps erythrocytes to move and decrease the apparent viscosity. So blood shows viscoelastic behavior which is a result of cells deformation and shear rate. In other words it has non-Newtonian model, where the viscosity is not constant and depends on the strain rate. In the large and medium sized vessels the Navier – Stokes equation can be applied.

Blood is suspension of particles in the matrix. Plasma which is the fluid component of the blood represents 55% of its volume. The other parts are: red and white blood cells, platelets and inter alia, electrolytes, small sugars, carbohydrates, lipids, proteins, hormone and others. All of them interact with each other and with vessel’s wall. Blood transports most of the substances in the body, removes heat and is responsible for acidic balance. Blood pressure depends on the oxygen need. During exercises or stress heart pump it faster.

One normal cardiac cycle lasts about 0.8s and is divided for two stages: systolic and diastolic. During systole the blood is ejected from the heart, during diastole the heart muscle is relaxing. Both of this phases can be recorded as an electrical activity of the heart on the commonly performed cardiology tests which is electrocardiography. The results shows specific points: P, R, Q, S, T per each rhythm [4].

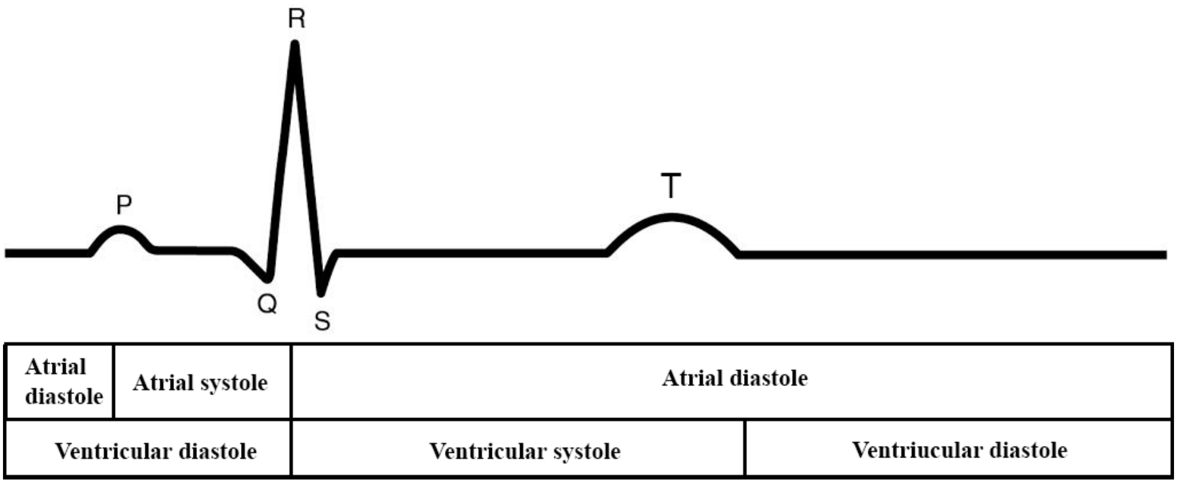


Figure 1: Systolic and diastolic phases presented against ECG [4].

Stroke volume of the ejected blood is the difference between the end diastolic volume (which means the volume remaining in the ventricle after diastole) and end systolic volume. The difference between pressure during systole and diastole, so called: pulse pressure, is usually given in millimeters of mercury unit (mmHg).

During systole the pressure generated by heart creates the pressure wave that distend the arterial walls due to maintenance a steady mean arterial pressure. Because of their elasticity, vessels buffer the blood flow and hence the wave can be observed as a pulse. For children over 10 years and adults normal pulse rate is 60–100 beats per min, for younger children: 70–130 [6].

1.3 Hydraulic analogy

The real cardiac cycle can be described as the hydraulic Windkessel Model developed by Otto Frank in 1899 [8]. It consists a water pump and an air–filled chamber, which affect the velocity and pressure of fluid in pipes. The arterial compliance is represented by compressible air in the pocket of air-chamber (in German language – Windkessel).

The water is pumped to the chamber, compresses the air which pushes water out from the chamber, working against the resistance water encounters in the canals – just like arterial compliance. This resistance refers to the peripheral resistance.

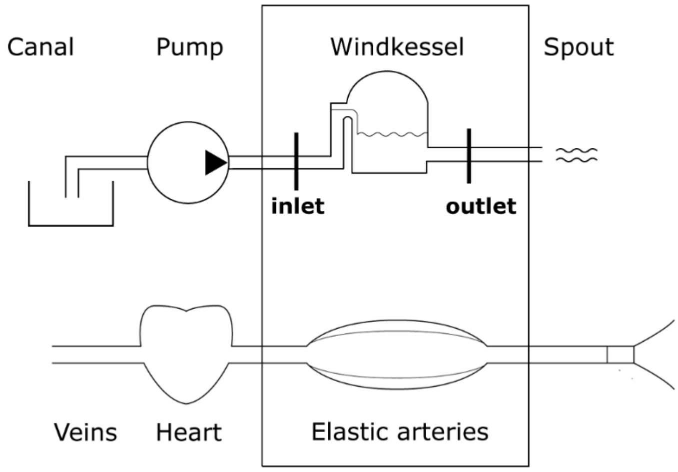


Figure 2: Hydraulic analogy of cardiovascular system. Figure made on the basis of [7].

The Windkessel model takes into account an unsteady flow, conservation of mass and assumes that the water (as a blood) is incompressible.

Considering the section marked on the Figure 2. between point 'inlet' and 'outlet' during systole ($0 \leq t \leq t_s$),

where:

t is the current time, s; and t_s is the systole duration, s; the relation between the water flow and volume of the water that is stored in the chamber (windkessel) can be described as the principle of the mass conservation:

$$Q(t)_{in} = Q(t)_{out} + Q(t)_w = Q(t)_{out} + \frac{dV}{dt} \tag{1.2}$$

where:

$Q(t)_{in}$ is a water volumetric flow as a function of time at the point 'inlet', $m^3 \cdot s^{-1}$; $Q(t)_{out}$ is a water volumetric flow at the point 'outlet', $m^3 \cdot s^{-1}$; $Q(t)_w$ is a water volume inside the chamber, $m^3 \cdot s^{-1}$; and $\frac{dV}{dt}$ is the time derivative of the volume, $m^3 \cdot s^{-1}$. Moreover the Hagen – Poiseuille law says that:

$$Q(t) = \frac{\Delta P \pi r^4}{8 l \eta} \tag{1.3}$$

And the resistance, $Pa \cdot s \cdot m^{-3}$ can be calculated:

$$R = \frac{8 l \eta}{\pi r^4} \tag{1.4}$$

where: ΔP is a pressure loss, Pa; r is the geometric factor depending on cross-section of the canal, m; l is the length of the canal, m; η is dynamic viscosity, Pa·s.

Therefore the flow in general equals:

$$Q(t) = \frac{P(t)}{R} + \frac{dV(t)}{dP(t)}\frac{dP(t)}{dt} = \frac{P(t)}{R} + C\frac{dP(t)}{dt} \tag{1.5}$$

where:

$C = \frac{dV}{dP}$ is a constant ratio, $\frac{m^4 \cdot s}{g}$. In case of cardiovascular system it is an arterial compliance connected to the vasodilation which is the widening of elastic vessel [8].

1.4 Electrical analogy

In the project the relationship between pressure and volume in certain points is considering therefore the model can be simplified into a lumped parameter model to get reliable results and to decrease computational cost at the same time.

In zero-dimensional models the space is discretized and solution is based on differential equations.

In this approach the blood flow rate is presented by a current, and the pressure difference analogy by a voltage. The elements corresponding to cardiovascular parameters are presented in the Table 1 where relations are presented to prove the validity of using the analogy.

They are simplified because of assumption of the linearity of the equations and of the laminar blood flow [3].

Comparison of both analogies can be made with the same assumptions:

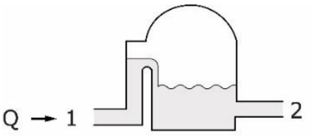


Figure 3: Hydraulic analog of the circulatory system.

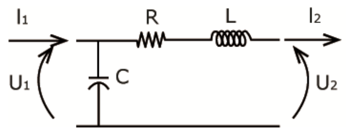


Figure 4: Electrical analog of the circulatory system. Figure made on the basis of [5].

For the flow in the section presented above the governing equation of the system are:

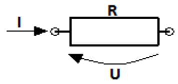
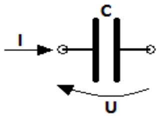
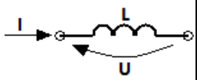
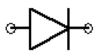
$$C_c \frac{d\hat{p}}{dt} + Q_2 - Q_1 = 0 \quad \rightarrow \quad C_e \frac{d\hat{u}}{dt} + U_2 - U_1 = 0 \tag{1.6}$$

$$L_c \frac{d\hat{Q}}{dt} + R\hat{Q} + P_2 - P_1 = 0 \quad \rightarrow \quad L_e \frac{d\hat{I}}{dt} + R\hat{I} + U_2 - U_1 = 0 \tag{1.7}$$

with the initial values:

Qi and Pi with i as 1 and 2 [5].

Table 1: The summary of the corresponding equations in cardiovascular system and its electrical analog [3].

Parameters of the real model	Electrical analog	The symbol of the analog	Characteristic of the cardiovascular system	Description of the analog
The flow resistance	Resistor		$R_c = \frac{\Delta P}{Q}$	$R_e = \frac{U}{I}$ (the Ohm's law)
Vessel compliance	Capacitor		$Q = C_c \frac{dP}{dt}$	$I = C_e \frac{dU}{dt}$
Blood inertia	Coil		$\Delta P = L_c \frac{dQ}{dt}$ Relationship referring to the Newton's second law	$U = L_e \frac{dI}{dt}$
The valve position	Diode		$Q = 0 \; if \; \Delta P < \Delta P_{cr}$ $Q = \frac{\Delta P}{R_c} \; if \; \Delta P \geq \Delta P_{cr}$ The valve position depends on the pressure difference. If it is lower than critical value the valve is closed.	$I = 0 \; if \; U < U_{cr}$ $I = \frac{U}{R_e} \; if \; U \geq U_{cr}$ The diode behaves similarly as it blocks the current flow in the wrong direction due to the potential difference.
Reference pressure	Ground		$P = P_{min}$ Minimal pressure occurring in the cardiovascular system	$U = 0$

where: t is the time, s; R is the resistance, Ω; P – pressure, Q – flow rate, U – potential difference, V; I – current flow, A; C_c – compliance coefficient, C_e – capacitance, F; L_c – inertia coefficient, L_e – inductance,Ω; c' subscript refers to cardiovascular system, 'e' subscript refers to electrical analogy. The units are written only for the electrical analogue, because hydraulic analogy was already mentioned.

The most often used electrical models are the variants of Windkessel Models [8]:

a) 2–element Windkessel Model

$$I(t) = \frac{P(t)}{R} + C\frac{dP(t)}{dt} \tag{1.8}$$

when $I(t) = 0$ (during diastole):

$$P(t) = P(t_d) \cdot e^{\frac{-(t)}{RC}} \tag{1.9}$$

Resistor mimes the peripheral resistance of the systemic circuit. Capacitor refers to the vessel compliance.

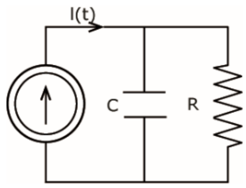


Figure 5: Two-element Windkessel Model. Figure made on the basis of [8].

b) 3–element Windkessel Model

$$\left(1 + \frac{R_1}{R_2}\right) I(t) + CR_1\frac{dI(t)}{dt} = \frac{P(t)}{R_2} + C\frac{dP(t)}{dt} \tag{1.10}$$

when $I(t) = 0$ (during diastole):

$$P(t) = P(t_d) \cdot e^{\frac{-(t)}{R_2 C}} \tag{1.11}$$

Additional resistance is an analogue to the resistance occurring due to the blood flow through the valve.

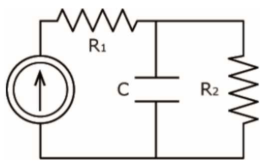


Figure 6: Three element Windkessel Model. Figure made on the basis of [8].

a) 4–element Windkessel Model

$$\left(1 + \frac{R_1}{R_2}\right) I(t) + \left(R_1 C + \frac{L}{R_2}\right)\frac{dI(t)}{dt} + LC\frac{d^2 I(t)}{dt^2} = \frac{P(tP}{R_2} + C\frac{dP(t)}{dt} \tag{1.12}$$

when $I(t) = 0$ (during diastole):

$$P(t) = P(t_d) \cdot e^{\frac{-(t)}{R_2 C}} \tag{1.13}$$

Inductor mimes the blood inertia.

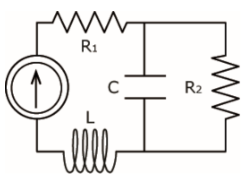


Figure 7: Four element Windkessel Model. Figure made on the basis of [8].

2 Comparison between two- and three-element Windkessel Models and analysis of the analytical and numerical approaches.

In the present work the comparison between two- and three-element Windkessel Models has been done including the analytical and numerical approaches in case of two–element Windkessel Model.

The calculations using the electrical analogy base on the relations between the pressure and flow. The outflows are read from the CFD model (what will be discussed later) however on the inlets the function describing the ejection of the blood from the heart must be assumed.

2.1 Analytical solution of the two-element Windkessel Model [9]

Before calculating the value of the blood flow rate I(t), $ml \cdot s^{-1}$, is important to define in which phase the heart is at the current time t, s. According to the literature systole takes about two–fifths of the cardiac cycle. Thus if the remainder of division t by cardiac period – T_c, s; is smaller than $T_s = \frac{2}{5} T_c, s$ it means that systole is in progress [7] and I(t) can be modeled as an sinusoidal flow:

$$I(t) = I_0 \, sin\left(\pi \cdot \frac{t}{T_s}\right) \tag{2.1}$$

Where I_0 [ml] is a half of the amplitude of the sinusoidal function within the limits: 0 – T_c. Blood flow during diastole had been assumed to be zero. Integral of I(t) represents cardiac output (CO).

$$CO = \int_0^{Ts} I_0 \, sin\left(\pi \cdot \frac{t}{T_s}\right) dt \tag{2.2}$$

$$CO = I_0 \int_0^{Ts} sin\left(\pi \cdot \frac{t}{T_s}\right) dt \tag{2.3}$$

If the heart rate and normal cardiac output are given and with assumptions that the cycle period is $T_c = \frac{60}{heart\ rate}$ s and systole lasts $T_s = \frac{2}{5} \cdot T_c\ s$, maximum value of sinusoidal function can be counted:

$$I_0 = \frac{CO}{\int_0^{Ts} \sin\left(\pi \cdot \frac{t}{T_s}\right) dt} \tag{2.4}$$

$$I_0 = \frac{CO}{-\frac{T_s}{\pi} cos(\pi) + \frac{T_s}{\pi} cos(0)} \tag{2.5}$$

Windkessel first order ordinary differential equation:

$$\frac{P(t)}{R} + C\frac{dP(t)}{dt} = I(t) \tag{2.6}$$

To find function of pressure Integrating Factor Method was used. The method was described below (2.2.7 –2.2.13). *W* and *Y* are functions of the same variable

$$a\frac{Y}{dt} + b \cdot Y = W \tag{2.7}$$

With both sides divided by *a*:

$$\frac{Y}{dt} + \frac{b}{a} \cdot Y = \frac{W}{a} \tag{2.8}$$

Method involves defining the integrating factor:

$$IF = e^{\int \frac{b}{a} dt} \tag{2.9}$$

and multiplying every element of the equation by integrating factor:

$$IF \cdot \frac{W}{a} = IF\frac{yY}{dt} + IF \cdot \frac{b}{a} \cdot Y \tag{2.10}$$

Then equation should be integrated:

$$\int IF\frac{W}{a}dt = \int \left(IF \cdot \frac{dY}{dt} + IF \cdot \frac{b}{a}Y\right)dt \tag{2.11}$$

taking into account the product rule :

$$\frac{d}{dt}(IF \cdot Y) = IF\frac{Y}{dt} + Y\frac{IF}{dt}$$

$$\frac{d}{dt}(IF \cdot Y) = IF\frac{dY}{dt} + IF \cdot Y \quad since\text{: } IF' = \left(e^{\int \frac{b}{a}dx}\right)' = e^{\int \frac{b}{a}dx} \tag{2.12}$$

General solution can be calculated by integrating:

$$y = IF^{-1}\int \frac{W}{a}IFdt \tag{2.13}$$

Rearranged Windkessel equation analogically to (2.2.8) is:

$$\frac{dP(t)}{dt} + \frac{1}{CR}P(t) = \frac{I(t)}{C} \tag{2.14}$$

For sinusoidal function of the blood flow:

$$\frac{dP(t)}{dt} + \frac{P(t)}{CR} = \frac{I_0}{C} \cdot sin\left(\pi \ \cdot \frac{t}{T_s}\right) \tag{2.15}$$

Analogically parameters a, b and integrating factor are:

$$a = C\ , \quad b = \frac{1}{R}\ , \ IF = e^{\int \frac{1}{RC}dt} = e^{\frac{t}{RC}} \tag{2.16}$$

General solution for two-element Windkessel Model:

$$P(t) = e^{-\frac{t}{RC}} \int \frac{I_0}{C} sin\left(\frac{\pi \cdot t}{T_s}\right) e^{\frac{t}{RC}} dt \tag{2.17}$$

The integral was solved by parts and the solution can be presented as follows:

a) for systole:

$$P(t) = -\frac{I_0 R T_s e^{\frac{t}{RC}} \left(\pi CR\ cos\left(\frac{\pi t}{T_s}\right) - T_s\ sin\left(\frac{\pi t}{T_s}\right)\right)}{\pi^2 C^2 R^2 + {T_s}^2} + c_1 e^{\frac{-t}{RC}} \tag{2.18}$$

Integration constant was calculated for pressure at the beginning of the systole which equals to the pressure at the end of the diastole from previous cycle. This value is assumed for initialization of calculations.

$$c_1 = P_{ss} e^{\frac{t}{RC}} + \frac{e^{\frac{t}{RC}} I_0 R T_s \left(\pi CR\ cos\left(\frac{\pi t}{T_s}\right) - T_s\ sin\left(\frac{\pi t}{T_s}\right)\right)}{\pi^2 C^2 R^2 + {T_s}^2} \tag{2.19}$$

where P_{ss} is pressure at the start of systole and t_{ss} is a time when systole starts so it equals 0 for the first cycle. It gives us:

$$c_1 = P_{ss} + \frac{I_0 R T_s C \pi R}{\pi^2 C^2 R^2 + {T_s}^2} \tag{2.20}$$

b) for diastole:

$$P(t) = e^{-\frac{t}{RC}} \int I(t) dt \tag{2.21}$$

Since the blood flow for this part of the cycle is assumed to be zero:

$$P(t) = c_2 e^{\frac{-t}{RC}} \tag{2.22}$$

Pressure at the start of diastole equals the pressure at the end of systole so the constants c_1 and c_2 are different.

The calculations for the three-element Windkessel model can be done using Integrating Factor:

$$IF = e^{\frac{t}{R_2 C}} \tag{2.23}$$

where the peripheral resistance is described with the R_2 symbol.

2.2 Numerical solution of the two-element Windkessel Model [10]

Numerical model of the inlet pressure was performed using the Runge –Kutta method for solving differential equations. This method involves the initial value and approximates solution for each step using FSAL strategy (First Same as Last) and has a wide range of versions.

ODE23 is a version of Runge –Kutta method implemented in Matlab software. In the ODE23 algorithm, pressure is calculated iteratively, each step gives pressure for the time enlarged by the time step (h). Step contains three stages.

Initial condition is the blood pressure at the start of systolic phase. Blood flow is calculated using equation (2.2.1).

Derivative of pressure is counted:

$$s_1 = -\frac{P(t)}{RC} + \frac{I(t)}{C} \tag{2.24}$$

For the next stage with the time enlarged by half of the time step (h) pressure equals:

$$P\left(t + \frac{h}{2}\right) = P(t) + \frac{h}{2} s_1 \tag{2.25}$$

blood flow for the same time: $$I\left(t + \frac{h}{2}\right) = I_0 \, sin \frac{\pi\left(t + \frac{h}{2}\right)}{T_s} \tag{2.26}$$

And it is used to count next derivative:

$$s_2 = -\frac{P\left(t + \frac{h}{2}\right)}{RC} + \frac{I\left(t + \frac{h}{2}\right)}{C} \tag{2.27}$$

Next stage is calculated for the time enlarged by three fourth of the time step:

$$P\left(t+\frac{3}{4}h\right) = P\left(t+\frac{h}{2}\right) + \frac{3}{4}h \cdot s_2 \tag{2.28}$$

$$I\left(t+\frac{3}{2}h\right) = I_0 \, sin \frac{\pi\left(t+\frac{3}{2}h\right)}{T_s} \tag{2.29}$$

$$s_3 = -\frac{P\left(t+\frac{3}{4}h\right)}{RC} + \frac{I\left(t+\frac{3}{2}h\right)}{C} \tag{2.30}$$

The pressure for next step equals:

$$P(t+h) = P(t) + \frac{h}{9}(2s_1 + 3s_2 + 4s_3) \tag{2.31}$$

Next derivative must be calculated: $s_4 = -\frac{P(t+h)}{RC} + \frac{I(t+h)}{C}$ (2.32)

it is used in next step as s_1 due to FSAL strategy [10].

The estimation error equals:

$$e_{n+1} = \frac{h}{72}(-5s_1 + 6s_2 + 8s_3 - 9s_4) \tag{2.33}$$

where n is the number of the step. The numerical calculations for three-element Windkessel Model can be solved using the same method.

2.3 Model of the healthy heart

The model was prepared for the data used in the [9] and presented in the Table 2.

Table 2: Data used for model validation.

	heart rate	72 beats per minute
	cardiac output	90 $\frac{ml}{cycle}$
	duration of cardiac cycle	$\frac{60}{heart\ rate}$ s
	duration of systolic phase	$\frac{2}{5}$ of cardiac cycle s
2WM	resistance	0.95 $\frac{g}{cm^4 \cdot s}$
	compliance	1.0666 $\frac{cm^4 \cdot s}{g}$
3WM	resistance 1	0.05 $\frac{g}{cm^4 \cdot s}$
	resistance 2	0.9 $\frac{g}{cm^4 \cdot s}$
	compliance	1.0666 $\frac{cm^4 \cdot s}{g}$

where: 2WM and 3WM are respectively: two- and three-element Windkessel Model. With step size: 0.01.

Results for every used method discussed in the present chapter are close to those obtained by the authors of the article [9] and are presented on the graph (Figure 8.). Figure presents one of calculated cycle.

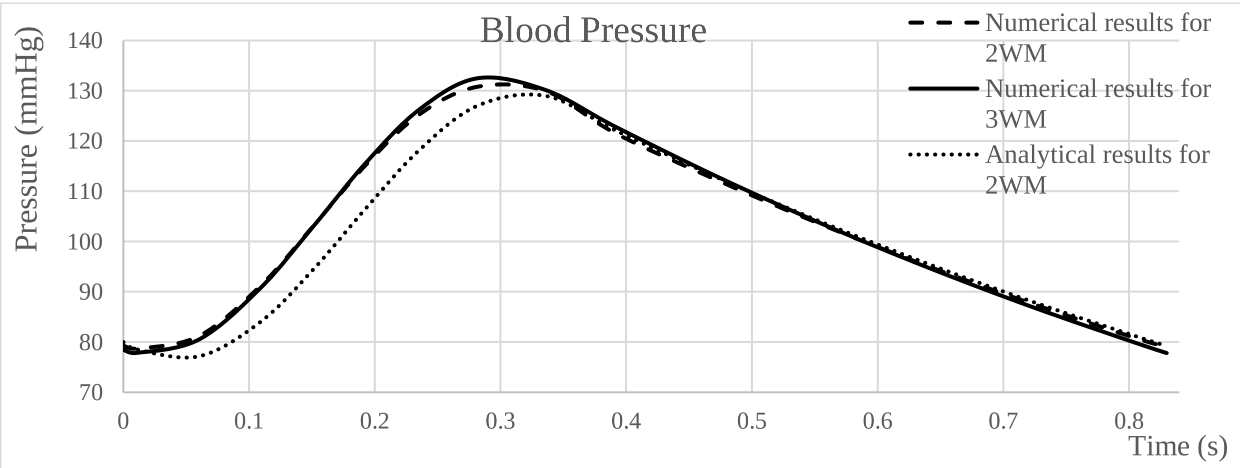


Figure 8: Results for analytical and numerical approaches to two- and three-element Windkessel Models of the healthy cardiovascular system during first calculated cycle.

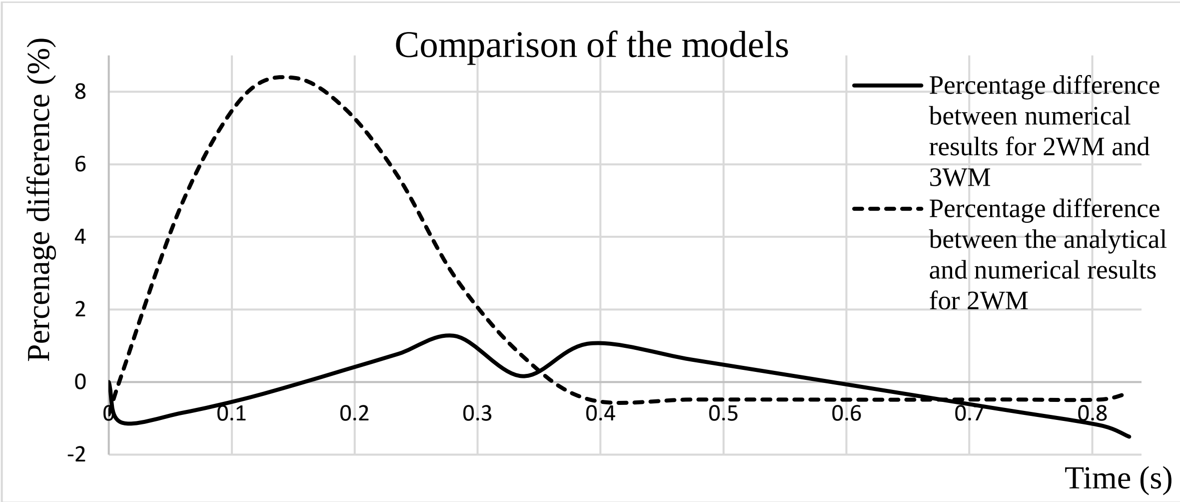


Figure 9: Comparison of the results for analytical and numerical approaches to two- and three-element Windkessel Models.

As it is presented on the Figure 9. the difference between numerical solutions for 2–element and 3-element Windkessel Models are negligible. The absolute value of maximum percentage difference is 1.51% at the time 0.83s (which is the end of the cycle). At the time 0.28s the difference is 1.27%. On the beginning of diastole (0.34s of the cycle) the difference is only 0.16%.

Results for the diastolic phase (0.33s – 0.83s) calculated numerically and analytically for the 2–elements Windkessel Model are close to each other. The absolute values of percentage differences are within the limits: 0.49% – 0.73%. For the systole those limits are higher: 0.89% – 8.38%. This difference reaches its peaks at 0.15s, which the half of the systole.

3 Case study

In the project simulations of the blood flow for eight years old girl is being made.

Girls weight was assumed to be 25 kg thus cardiac output (CO) can be calculated according to [11]:

$$co = 933 \cdot weight^{0,38}, \qquad \frac{ml}{min} \tag{3.1}$$

$$CO = co \cdot \frac{T_c}{60} = 36.9, \qquad \frac{ml}{cycle} \tag{3.2}$$

The heart rate has a value measured on the patient and equals 86 beats per minute [12].

With the assumptions [12] that:

$$T_c = \frac{60}{HR} = 0.7, \qquad s \tag{3.3}$$

$$T_s = \frac{2}{5} \cdot T_c = 0.28, \qquad s \tag{3.4}$$

the maximum blood flow according to the equation (2.2.5) equals:

$$I_0 = \frac{CO}{-\frac{T_s}{\pi} cos(\pi) + \frac{T_s}{\pi} cos(0)} = 207.49, \qquad \frac{ml}{s} \tag{3.5}$$

Blood pressure at rest in the upper-body had been tested via a sphygmometer and for systole was equal to 115 mmHg, for diastole to 65 mmHg [12].

High blood pressure is one of the consequences of the pathology. Using this specific data the resistance and compliance of the circulatory system were chosen in order to obtain values of pressure mentioned above.

3.1 Estimation of resistance and compliance coefficients

The choice of the R and C parameters was made using the Matlab script prepared for this purpose. Assumptions were done:

-the minimum values of R and C: 0.1,

-the maximum values of R and C: 1.9,

-the maximum tolerance: 5,

-the expected results for diastolic and systolic pressure: 65±5 mmHg and 115±5 mmHg.

Program chose the set of random values of R and C from the range: 0.1 to 1.9, and run the algorithm based on Windkessel Models to compute the pressures. If the values of pressure matched the chosen ranges, results were written into the table. 100000 iterations were calculated. Through this time-consuming process the set of R and C were obtained. For the calculations the following values were chosen:

- resistance = 1.812621 $\frac{g}{cm^4 \cdot s}$,
- compliance = 0.550099 $\frac{cm^4 \cdot s}{g}$,

which provide the result for diastolic and systolic pressure respectively: 62.12 and 114.60 mmHg.

3.2 Results for the case study

Summarized data and assumptions that were used in computing the model of blood flow out of the heart are presented in the table 3.

Table 3: Summarized blood flow and pressure data.

	systolic pressure	114.60 mmHg
	diastolic pressure	62.12 mmHg
	heart rate	86 beats per minute
	weight	25 kg
	cardiac output	36.9 $\frac{ml}{cycle}$
	duration of cardiac cycle	0.7 s
	duration of systolic phase	0.28 s
	maximum blood flow	207.49 $\frac{ml}{s}$
2WM	resistance	1.812621 $\frac{g}{cm^4 \cdot s}$
	compliance	0.550099 $\frac{cm^4 \cdot s}{g}$
3WM	resistance 1	0.05 $\frac{g}{cm^4 \cdot s}$
	resistance 2	1.812621 $\frac{g}{cm^4 \cdot s}$
	compliance	0.550099 $\frac{cm^4 \cdot s}{g}$

where: 2WM and 3WM are respectively: two- and three-element Windkessel Model. With the step size: 0.01.

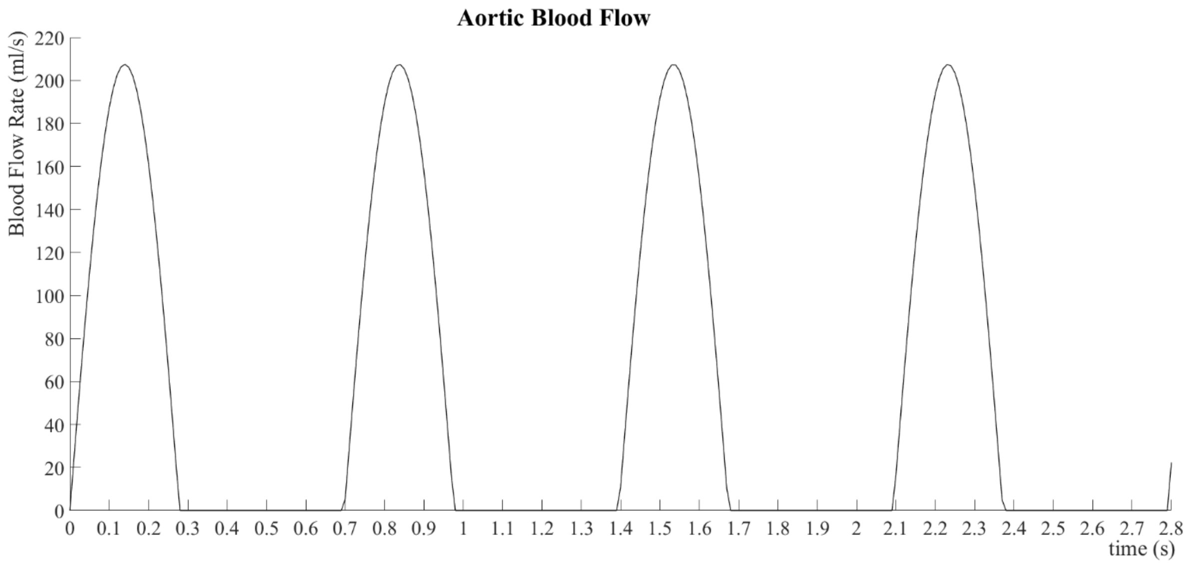


Figure 10: Blood flow for the parameters presented in Table 1. Chart includes four cycles.

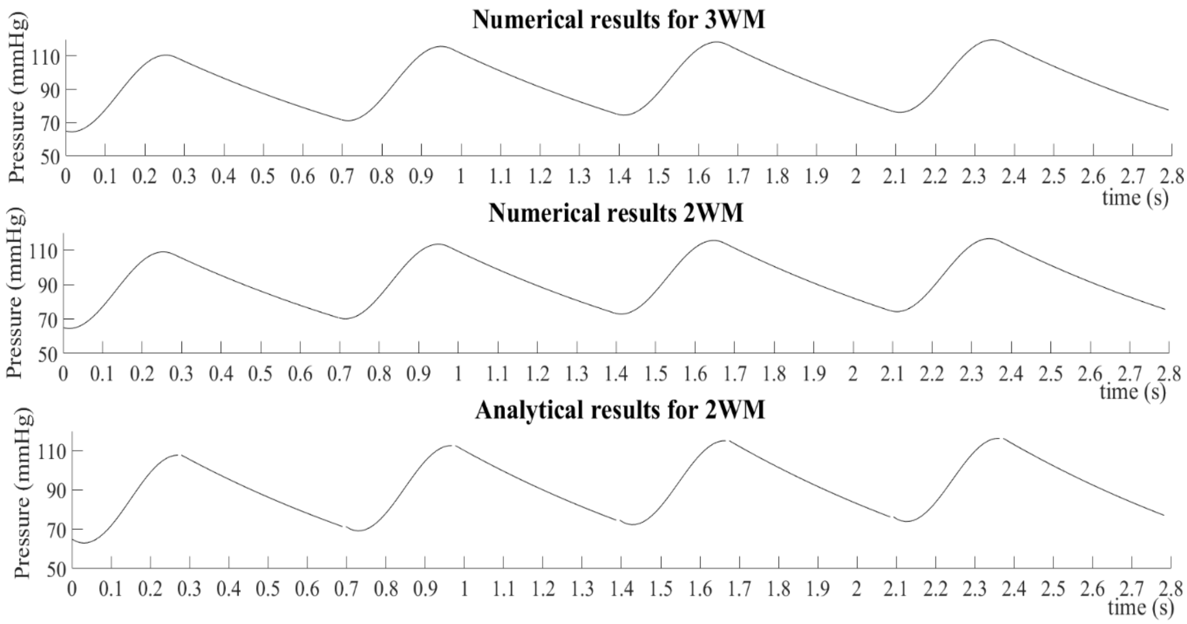


Figure 11: Results of the analizys of the case study.

Multivariate analyze included observation of pressure changes due to changes in heart rate.

According to [11] among the girls under the age of seventeen heart rate that occurs is between 52 and 136 beats per minute. In Table 4. the results for this range have been summarized.

Table 4: Maximum and minimum values of cardiac pressure in respect of changing pulse.

heart rate, bpm	maximum value of pressure, mmHg	minimum value of pressure, mmHg
50	131.34	58,77
60	122.55	60.8
70	115.77	61.95
80	110.42	62,71
90	106.05	63.16
100	102.47	63.55
110	99.45	63.75
120	96.87	63.93
130	94.62	64.1
140	92.68	64.26

The systolic pressure decreases exponentially with increase of heart rate. The opposite behavior is observed in case of diastolic pressure. The changes are illustrated on the Figure 12 and Figure 13.

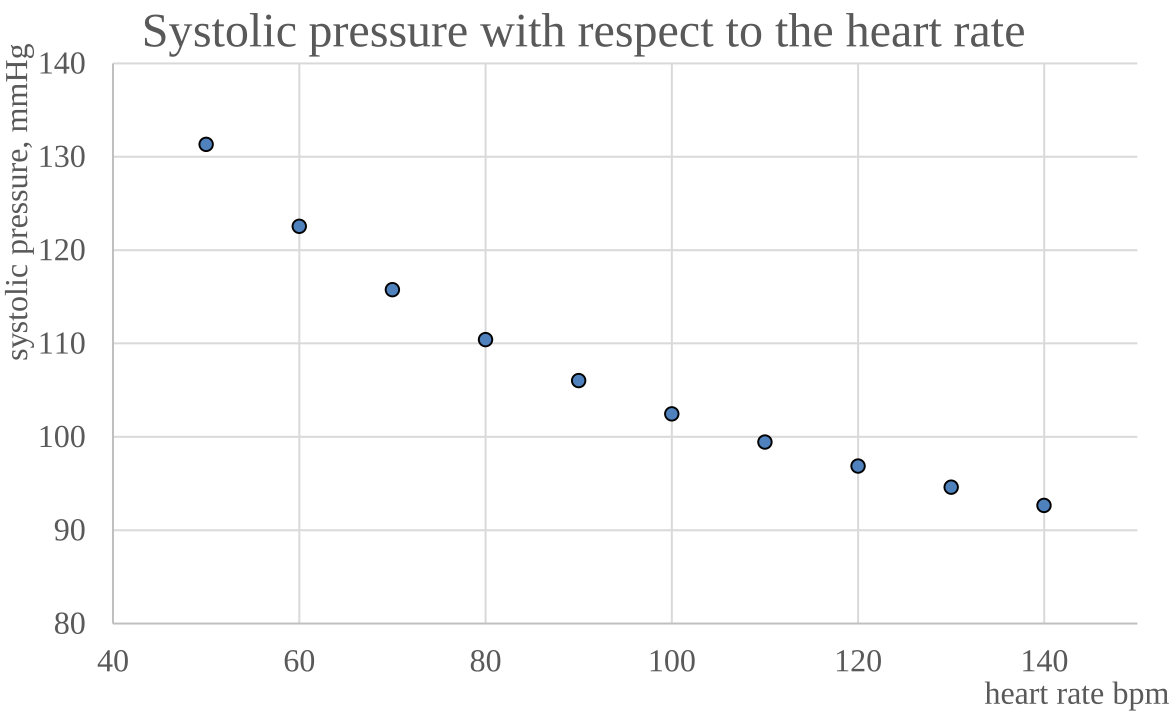


Figure 12: Changes of systolic pressure with heart rate increase.

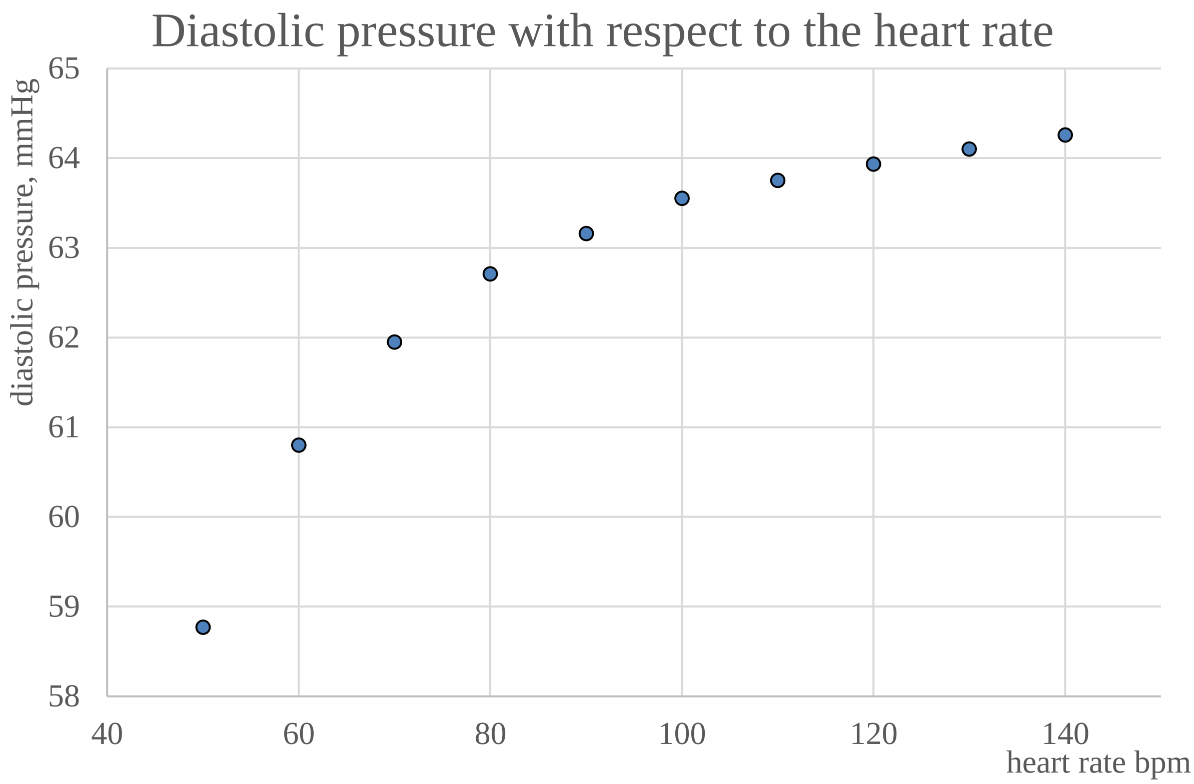


Figure 13: Changes of diastolic pressure with heart rate increase.

Coupling outlet boundary conditions for the model which will be discussed later is carried out iteratively. The blood flow in domain is computed based on predetermined velocity inlet profile. On the respective outflows the volumetric flows are read every time step. Using information about the outflows and coupled Lumped Parameters Model the pressure is periodically calculated. When the results are reliable time step is finished and pressure becomes new boundary condition.

Two – or three – element Wondkessel Model will be used. Parameters of the flow resistance due to the blood flow through the valve (R_1), the peripheral resistance (R_2) and compliance of the patient vessels are presented into the Table 5.

Parameters of the Windkessel Model on the outlets differ from those on the inlets, because they represent distribution of the blood to the certain branches of the vascular system.

Table 5: Parameters of the Windkessel Models coupled on the outlets of the model [12].

Artery name	R_1, $kg \cdot m^{-4} \cdot s^{-1}$	C, $m^4 \cdot s^2 \cdot kg^{-1}$	R_2, $kg \cdot m^{-4} \cdot s^{-1}$
Right Subclavian Artery	$0.885 \cdot 10^8$	$7.13 \cdot 10^{-10}$	$1.3858 \cdot 10^9$
Right Common Carotid Artery	$1.122 \cdot 10^8$	$5.62 \cdot 10^{-10}$	$1.7571 \cdot 10^9$
Left Common Carotid Artery	$1.124 \cdot 10^8$	$5.62 \cdot 10^{-10}$	$1.7611 \cdot 10^9$
Left Subclavian Artery	$2.895 \cdot 10^8$	$2.12 \cdot 10^{-10}$	$4.5361 \cdot 10^9$
Descending Thoracic Aorta	$0.271 \cdot 10^8$	$2.922 \cdot 10^{-9}$	$0.3122 \cdot 10^9$

Using LPM based on those data and blood flow read from the CFD model the boundary conditions are described.

4 Conclusions

The time – depended heart work was identified and modeled with the Lumped Parameter Model which will serve as a boundary conditions of the complete CFD model in the future work. Obtained results are:

- the time dependent blood flow which can be used to prepare the velocity profile as an inlet boundary condition,
- the functions describing the pressure of the blood ejected from the heart,

with the assumptions of heart rate and cardiac output. Moreover the model can be used with another assumptions as it was presented

The analytical and numerical approaches were taken into the consideration.

According to the literature the blood pressure of a healthy adult should not exceed 135/85 mmHg (respectively systolic and diastolic pressure). For the numerical three – element Windkessel Model this rate is equal to: 132/78 mmHg. Hence the claim that the results are reliable. Moreover three methods used to calculate the pressure give the similar results.

The case of eight years old girl with coarctation of the descending aorta was analyzed. The assumed blood pressure was: 115/65 mmHg, what is a consequence of the pathology. Using estimation of the model parameters close results were reached: 114.6/62.1 mmHg.

The work shows that Lumped Parameter Model based on the Windkessel analogue is sufficient tool, give the reliable results and make investigations of the cardiovascular system more precise.

Future work

Figure below presents the geometry of three-dimensional model of aorta with indicated places of the computational domain where boundary conditions have been attached. The data had been taken from the Gadolinium-enhanced MR angiography. Model includes: ascending aorta which starts with 'inlet' (without coronary artery), aorta arch attached with upper branches:

- brachiocephalic artery – 1th outlet (it is a common branch for right subclavian artery and right common carotid artery),
- left common carotid artery - 2nd outlet,
- left subclavian artery – 3th outlet,

and part of descending thoracic aorta (forth outlet) where narrowing called 'coarctation' (approximately 65% aortic area reduction). The geometry is presented on the Figure 14.

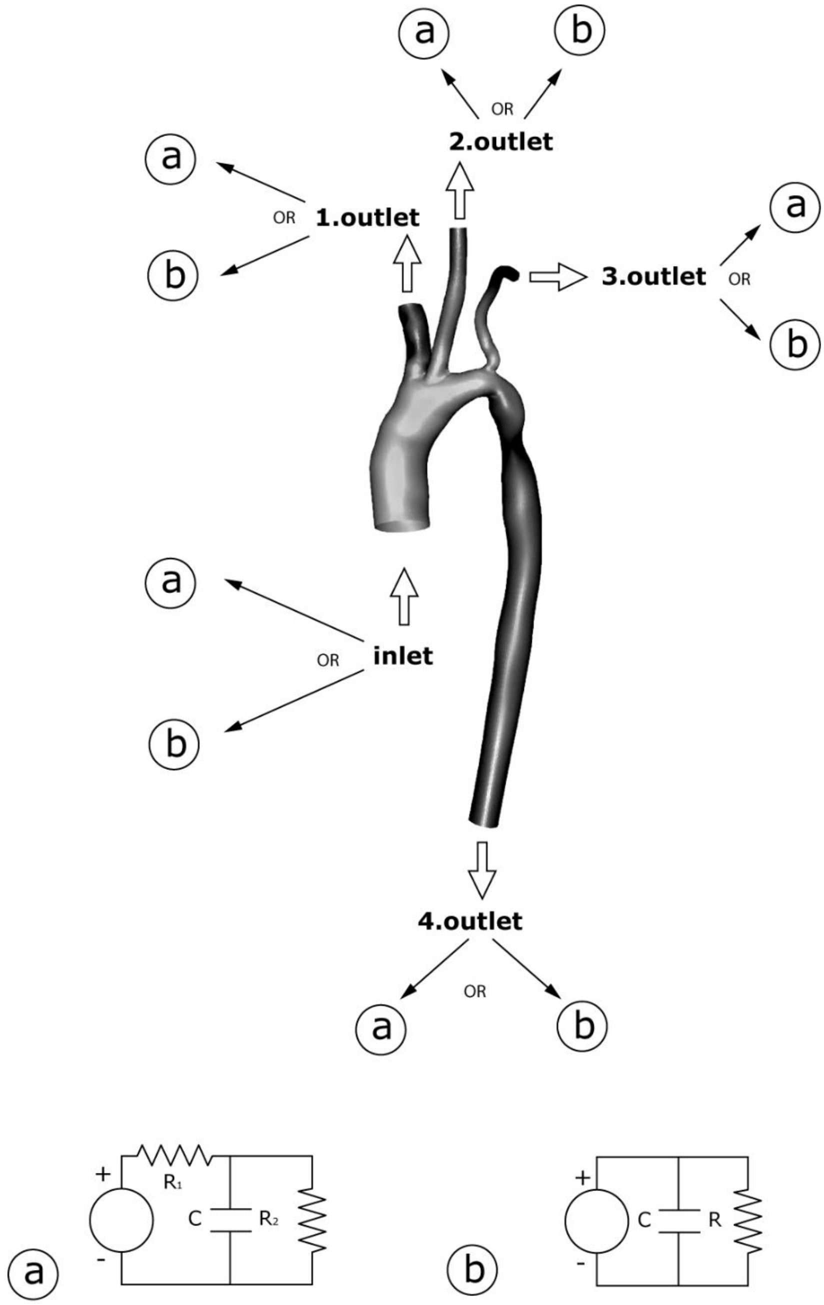


Figure 14: The geometry of the 3D model with indicated boundary conditions as a:
a) three-element Windkessel Model, b) two-element Windkessel model. Figure made on the basis of [13].

Acknowledgments

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Identyfikacja i modelowanie pulsacyjnego przepływu krwi w systemie krwionośnym przy użyciu zero–wymiarowego modelu w analogii elektrycznej.

Dominika Bandoła

Słowa kluczowe: Model Windkessel, warunki brzegowe, ciśnienie tętnicze, przepływ krwi, aorta

Streszczenie

Zaawansowane komputerowe narzędzia wspomagania inżynierskiego, jak numeryczna mechanika płynów (CFD) oraz wiedza na temat funkcjonowania układu krążenia człowieka, struktury krwi, zachowania naczyń krwionośnych pozwalają na lepsze zrozumienie procesu dystrybucji krwi po organizmie. Kompleksowe symulacje muszą zakładać wielofazowość przepływu krwi, elastyczne ściany naczyń krwionośnych oraz pulsacyjny przepływ wynikający z charakterystyki pracy serca [1].

W prezentowanej pracy zamodelowano przepływ oraz ciśnienie krwi.

Charakterystyczna impedancja, rezystancja oraz pojemność zostały wzięte pod uwagę w elektrycznej analogii jako model skupiony dużego układu krążenia, który zostanie zaimplementowany jako warunek brzegowy w kompletnym modelu CFD na wlocie do aorty wstępującej oraz na wylotach z pnia ramienno-głowowego, tętnicy szyjnej wspólnej lewej i tętnicy podobojczykowej lewej.

Opór przepływu krwi w naczyniach jest analogiczny do oporu elektrycznego rezystora. Niestacjonarny przepływ regulowany przez elastyczne naczynia krwionośne można zasymulować używając kondensatora. Za pomocą cewki można opisać inercję krwi.

Co więcej, zakładając, ze krew płynie tylko w jednym kierunku mimo zmiennego ciśnienia dzięki działaniu zastawek, w modelu można użyć w ich miejsce diod, które podobnie działają na przepływ prądu. Dodatkowo analogia elektryczna umożliwia zastosowanie zasady zachowania masy dzięki prawu Kirchhoffa [3].

Powstały model skupiony opisujący układ krążenia został napisany w programie Matlab, jednakże może być przepisany na język programowania C i zaimplementowany w zewnętrznym oprogramowaniu CFD.