ISSN: 1231-4005
e-ISSN: 2354-0133
DOI: 10.5604/01.3001.0010.3126

VERIFICATION OF THE PRECISE POSITION OF THE AIRCRAFT IN AIR NAVIGATION BASED ON THE SOLUTION OF THE RTK-OTF TECHNIQUE

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Abstract

The article presents the possibility of applying the differential technique RTK-OTF to recover the position of the aircraft in the post-processing mode. Within the framework of the conducted research, the authors designated the geocentric coordinates XYZ of the aircraft and compared them. In the research experiment, they used archive materials from the test flight of the aircraft Cessna 172 around the airfield in Deblin on 1 June 2010. The actual position of the aircraft Cessna 172 was recovered on the basis of GPS kinematic observations registered by the receiver Topcon HiperPro mounted on board the aircraft. In the calculations, the authors also used static GPS observations from the reference station REF1 as well as virtual reference stations VirA and VirB. The final coordinates of the aircraft Cessna 172 with three independent determinations RTK-OTF were defined in the AOSS v.2.0 programme. On this basis, they made verification of accuracy in determining XYZ coordinates of the aircraft Cessna 172. The dispersion of results for the difference in the designation of the X coordinate of the aircraft ranges from –0.19 m to +0.05 m. On the other hand, the size of the difference in the designation of Y-coordinate of the aircraft ranges from –0.07 m to +0.11 m. In addition, the dispersion of the results for the difference in the designation of the Z coordinate of the aircraft is from 0.19 m to +0.12 m.

Keywords: *GPS, RTK-OTF, accuracy, air navigation, RMS*

1. Introduction

In the GPS satellite measurements, the highest precision in determining the actual position of the aircraft is made possible due to the application of the precise positioning method PPP or the differential method RTK-OTF. In the PPP method, the position of the aircraft is determined on the basis of indifference code-phase observations in the GPS system [1]. In the RTK-OTF differential method, the aircraft position is determined on the basis of the double difference technique for GPS code-phase measurements [2]. Moreover, the differential method also facilitates the elimination of systematic errors from GPS code-phase observations such as satellite clock error, instrumental biases of the on-board oscillator of the satellite, relativistic effects, receiver clock error, instrumental biases of the receiver [3].

The RTK-OTF differential technique assumes the stability of ionospheric and tropospheric corrections in the area of up to 20-25 km. Such an assumption greatly simplifies the observation model and reduces the number of unknown parameters to be determined [4]. The effect of the ionospheric delay is reduced by the use of the „Geometry-Free" linear combination for double differences of phase observations. On the other hand, the tropospheric delay parameter is described by means of the deterministic tropospheric model (e.g. Hopfield or Saastamoinen models) separately for the hydrostatic and wet parts. In addition, the effect of the multipath GPS signal is minimized by applying the differentiation technique of the GPS observations. The position of GPS satellites in the RTK-OTF method are calculated based on Kepler orbit data from the GPS navigation message. It should be noted that the values of GPS satellite coordinates based on Kepler orbit parameters are related to the average position of the satellite antenna phase centre. The phase ambiguity parameter for the phase observations is determined from the mathematical model of the RTK-OTF method. The determination of the ambiguity values in phase measurements is usually sought in the field of ambiguity, observations or coordinates [5].

In the RTK-OTF method, the determined coordinates of the rover receiver antenna are referenced and expressed in the coordinate reference frame of the reference station. Typically, in such a case, the coordinates of the rover receiver antenna are determined globally (e.g. ITRF or IGS) or regionally (e.g. ETRF). The stochastic model for determining the rover antenna coordinates in the RTK-OTF technique is usually conducted using the Kalman filtering method. This solution ensures high accuracy of the determined antenna coordinates of the receiver, even at the level of several cm [6]

The purpose of the present work is to show the possibility of implementation of the RTK-OTF differential technique to recover a reliable position of the aircraft in air navigation as well as presenting the research findings on the subject in question. The computations of the aircraft coordinates were made in the RTKLIB programme, in the module RTKPOST. In the calculations, GPS code and phase observations were obtained from the Topcon HiperPro rover receiver and the REF1 reference station, and also VirA and VirB virtual stations. The article is divided into 5 parts: introduction, mathematical model, the research experiment, research results and discussion, conclusions. The article is completed with a list of research literature references.

2. The mathematical model of designation of the aircraft position based on the RTK-OTF technique

The basic equation of the observation model in the differential technique RTK-OTF in the post-processing mode for GPS code-phase measurements may be written as follows [7]:

$$\begin{cases} \nabla\Delta\lambda_1 \cdot L^{ij}_{AB,1} = \rho^{ij}_{AB} - \nabla\Delta I^{ij}_{AB,1} + \nabla\Delta T^{ij}_{AB} + \nabla\Delta B^{ij}_{AB,1} + \nabla\Delta M_{L1}, \\ \nabla\Delta\lambda_2 \cdot L^{ij}_{AB,2} = \rho^{ij}_{AB} - \nabla\Delta I^{ij}_{AB,2} + \nabla\Delta T^{ij}_{AB} + \nabla\Delta B^{ij}_{AB,2} + \nabla\Delta M_{L2}, \\ \nabla\Delta P^{ij}_{AB,1} = \rho^{ij}_{AB} + \nabla\Delta I^{ij}_{AB,1} + \nabla\Delta T^{ij}_{AB} + \nabla\Delta M_{P1}, \\ \nabla\Delta P^{ij}_{AB,2} = \rho^{ij}_{AB} + \nabla\Delta I^{ij}_{AB,2} + \nabla\Delta T^{ij}_{AB} + \nabla\Delta M_{P2}, \end{cases} \tag{1}$$

where:
∇ – operator of a double difference for code and phase measurements allows comparing the difference in code and phase measurements for two satellites tracked by two receivers; Δ – operator of a single difference for code and phase measurements allows determining the difference in code and phase measurements for two satellites tracked by one receiver; λ_1 – wavelength frequency L1 in the GPS system; λ_2 – wavelength frequency L2 in the GPS system; $A\vec{B}$ – vector in the space between the base station ($\vec{A}$) and the rover receiver ($\vec{B}$) mounted on-board the aircraft; $L^{ij}_{AB,1}$ – value of double phase difference (expressed in cycles) on the vector $A\vec{B}$ between the satellites *i* and *j* on the L1 frequency in the GPS system; $L^{ij}_{AB,2}$ – value of double phase difference (expressed in cycles) on the vector $A\vec{B}$ between the satellites *i* and *j* on the L2 frequency in the GPS system; $P^{ij}_{AB,1}$ – value of double code difference (expressed in metres) on the vector $A\vec{B}$ between the satellites *i* and *j* on the L1 frequency in the GPS system;

$P_{AB,2}^{ij}$ – value of double code difference (expressed in metres) on the vector $\vec{AB}$ between the satellites i and j on the L2 frequency in the GPS system; ρ_{AB}^{ij} – geometric distance of the vector $\vec{AB}$ for the double code and phase difference (expressed in geocentric coordinates XYZ); $I_{AB,1}^{ij}$ – value of ionospheric delay on L1 frequency for double code and phase difference; $I_{AB,2}^{ij}$ – value of ionospheric delay on L2 frequency for double code and phase difference; $I_{AB,2}^{ij} = \gamma \cdot I_{AB,1}^{ij}$ relationship of ionospheric delay on L1 and L2 frequency; $\gamma = (f_1/f_2)^2$ scaling coefficient; f_1 – frequency L1 in the GPS system; f_2 – frequency L2 in the GPS system; T_{AB}^{ij} – value of tropospheric delay for double code and phase difference; $B_{AB,1}^{ij}$ – real value of phase ambiguity on L1 frequency for double code difference; $B_{AB,1}^{ij} = \lambda_1 \cdot N_{AB,1}^{ij}$; $N_{AB,1}^{ij}$ – total value of phase ambiguity on L1 frequency for double code difference; $B_{AB,2}^{ij}$ – real value of phase ambiguity on L2 frequency for double code difference; $B_{AB,2}^{ij} = \lambda_2 \cdot N_{AB,2}^{ij}$; $N_{AB,2}^{ij}$ – total value of phase ambiguity on L2 frequency for double code difference; M_{L1} – multipath effect and measurement noise at frequency L1 for phase measurements; M_{L2} – multipath effect and measurement noise at frequency L2 for phase measurements; M_{P1} – multipath effect and measurement noise at frequency L1 for code measurements; M_{P2} – multipath effect and measurement noise at frequency L2 for code measurements.

In equation (1), the unknowns parameters are the coordinates of the aircraft involved in the geometrical distance factor. In addition, the total phase ambiguity values for phase measurements at L1 and L2 frequencies are also determined. The parameters of the ionosphere and tropospheric delays are determined at the initial stage of working out the GPS observations. The effect of multipath are determined on the basis of the relationship between the code and phase measurements. The observation model from equation (1) is usually solved using Kalman filtering, see below [8]:

a) process of „prediction":

$$\begin{cases} x_p = A_0 \cdot x_0, \\ P_p = A_0 \cdot P_0 \cdot A_0^T + Q_o, \end{cases} \tag{2}$$

b) process of „correction":

$$\begin{cases} K_k = P_p \cdot H^T \cdot (H \cdot P_p \cdot H^T + R)^{-1}, \\ x_k = x_p + K_k \cdot (z - H \cdot x_p), \\ P_k = (I - K_k \cdot H) \cdot P_p, \end{cases} \tag{3}$$

where:

A_0 – matrix of coefficients; x_0 – estimating the values of the designated parameters a priori from the previous step; P_0 – estimating the values of covariance a priori from the previous step; x_p – prediction of state value; P_p – predicted covariance values; Q_0 – variance matrix of the disturbances of the measurement process; R – covariance matrix of measurements; H – matrix of partial derivatives; K_k – Kalman gain matrix; z – vector of measured values; I – unit matrix; x_k – parameters determined a posteriori; P_k – covariance matrix of parameters determined a posteriori.

3. The research experiment

The aircraft position parameters determined in equation (3) are expressed in ortho-Cartesian coordinates XYZ in the geocentric frame ECEF (e.g. ITRF or ETRF). For the purpose of the test, attempts were made to reconstruct the precise geocentric coordinates XYZ of the Cessna 172 on the basis of the RTK-OTF differential method in post-processing. The aircraft Cessna 172 executed a test flight around the military airfield in Deblin on 1 June 2010. On board, the aircraft there was mounted a rover Topcon HiperPro dual frequency geodetic receiver, which recorded GPS observations at a frequency of 1 second. The GPS observations collected in the geodetic receiver Topcon HiperPro were used to recover the actual position of the aircraft with the use of the RTK-OTF differential technique. In addition, on the flight route of Cessna 172, three base stations (REF1, VirA and VirB) were located, which also collected raw GPS observations for the needs of preparation of post-processing satellite data. The REF1 reference station was a physical station at Deblin airfield, and the VirA and VirB stations constituted virtual reference stations for which observations were generated in the POZGEO-D service in the ASG-EUPOS system. The interval of recording the observations for 3 reference stations was 1 second, similarly to the Topcon HiperPro rover receiver installed on-board the Cessna 172. At the reference station in Deblin, the Topcon HiperPro receiver was responsible for the operation and collection of GPS/GLONASS observations. On the VirA and VirB, stations there was installed the TRIMBLE NETRS receiver (with the TRM41249.00 TZGD antenna) which recorded only GPS observations. The reference stations REF1, VirA and VirB had precisely designated coordinates in the geodetic frame ETRF'89. In Fig. 1, there is a visual sketch of the location of REF1, VirA and VirB base stations against the flight route of Cessna 172.

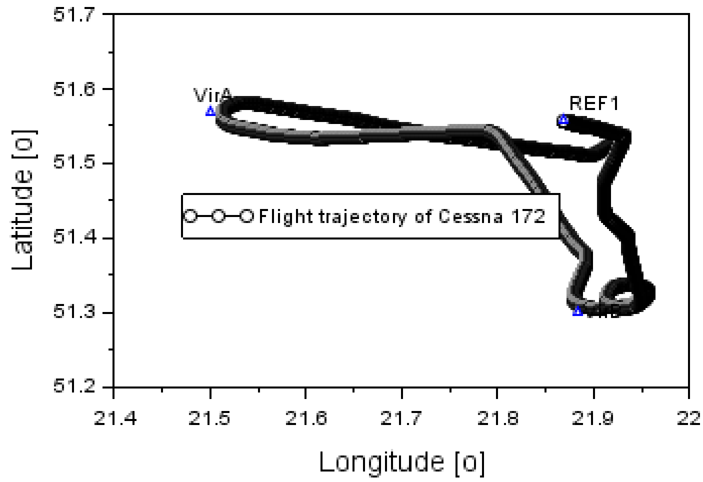

Fig. 1. Flight trajectory of Cessna 172 plane

Simultaneous synchronization of the GPS observations from the rover receiver Topcon HiperPro and from the base stations REF1, VirA and VirB allowed accurate designation of aircraft position Cessna 172 in the geocentric coordinates XYZ. The coordinates of the aircraft were recovered on the basis of a single baseline (space vector $A\vec{B}$) for three independent determinations in RTK-OTF mode, i.e.:

- vector (Baseline 1) REF1-Cessna,
- vector (Baseline 2) VirA-Cessna,
- vector (Baseline 3) VirB-Cessna.

The calculations of Cessna 172 coordinates for a single baseline were made in the AOSS v.2.0 programme (Ashtech Office Suite for Survey) for each second of the flight in the OTF mode [9].

4. Research results and discussion

The first step in verifying the determination of XYZ coordinates of the Cessna 172 was to specify the PDOP coefficients for each differential RTK-OTF solution (see Fig. 2). To this end, there were specified PDOP coefficients for the base solution REF1-Cessna (Baseline 1), the base solution VirA-Cessna (Baseline 2), the base solution VirB-Cessna (Baseline 3).

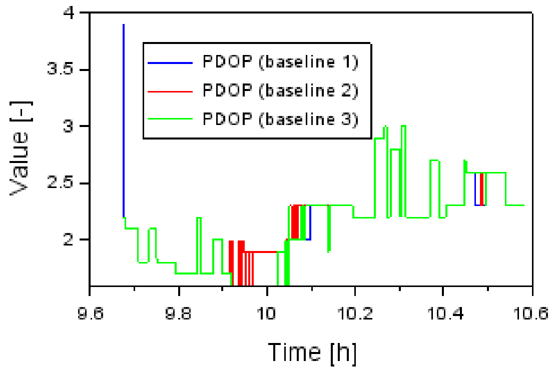

Fig. 2. Values of PDOP parameters

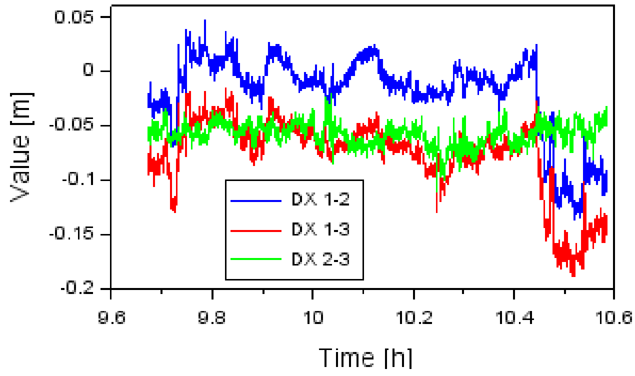

Fig. 3. Dispersion of coordinates difference along to X axis

The mean value of the PDOP coefficient for the RTK-OTF solution from baseline 1 equalled 2.1, whereas for the median it was equal to 2.2. Besides, the dispersion of the obtained PDOP results ranged between 1.6 and 3.9. The mean value of the PDOP coefficient for the RTK-OTF solution from baseline 2 equalled 2.2, similarly to the statistic parameter of the median. In contrast, the dispersion of the obtained PDOP results ranged from 1.6 to 3.0. The mean value of the PDOP coefficient for the RTK-OTF solution from baseline 3 equalled 2.1, whereas for the median it was equal to 2.2. The range of values for all PDOP results was from 1.6 to 3.0.

The second stage of verifying the actual position of the Cessna 172 was to determine the accuracy of the XYZ coordinates on the basis of the RTK-OTF differential solution. To this end, the difference in the coordinates of the aircraft along each XYZ axis was determined, as follows:

a) along the X axis:

$$\begin{cases} DX_{1-2} = X_{Baseline1} - X_{Baseline2}, \\ DX_{1-3} = X_{Baseline1} - X_{Baseline3}, \\ DX_{2-3} = X_{Baseline2} - X_{Baseline3}, \end{cases} \tag{4}$$

b) along the Y axis:

$$\begin{cases} DY_{1-2} = Y_{Baseline1} - Y_{Baseline2}, \\ DY_{1-3} = Y_{Baseline1} - Y_{Baseline3}, \\ DY_{2-3} = Y_{Baseline2} - Y_{Baseline3}, \end{cases} \tag{5}$$

c) along the Z axis:

$$\begin{cases} DZ_{1-2} = Z_{Baseline1} - Z_{Baseline2}, \\ DZ_{1-3} = Z_{Baseline1} - Z_{Baseline3}, \\ DZ_{2-3} = Z_{Baseline2} - Z_{Baseline3}, \end{cases} \tag{6}$$

where:

DX_{1-2} – the difference between the X-coordinate value of the aircraft between the RTK-OTF solution from baseline 1 and 2; DX_{1-3} – the difference between the X-coordinate value of the aircraft between the RTK-OTF solution from baseline 1 and 3; DX_{2-3} – the difference between the X-coordinate value of the aircraft between the RTK-OTF solution from baseline 2 and 3; DY_{1-2} – the difference between the Y-coordinate value of the aircraft between the RTK-OTF solution from baseline 1 and 2; DY_{1-3} – the difference between the Y-coordinate value of the aircraft between the RTK-OTF solution from baseline 1 and 3; DY_{2-3} – the difference between the Y-coordinate value of the aircraft between the RTK-OTF solution from baseline 2 and 3; DZ_{1-2} – the difference between the Z-coordinate value of the aircraft between the RTK-OTF solution from baseline 1 and 2; DZ_{1-3} – the difference between the Z-coordinate value of the aircraft between the RTK-OTF solution from baseline 1 and 3; DZ_{2-3} – the difference between the Z-coordinate value of the aircraft between the RTK-OTF solution from baseline 2 and 3.

Fig. 3 shows the results of the accuracy in the determination of the difference in the X coordinate of the aircraft between individual solutions in the OTF mode. The average value of the parameter DX_{1-2} is equal to –0.02 m, and of the median to –0.01 m, respectively. In addition, the dispersion of the obtained results of the parameter DX_{1-2} ranges from –0.14 m to +0.05 m. It is worth adding that approximately 84% of the parameter results DX_{1-2} is in the range of ±0.05 m. The mean value of the parameter DX_{1-3} is –0.08 m, and of the median –0.07 m, respectively. Moreover, the dispersion of the obtained results of the parameter DX_{1-3} is from –0.19 m to –0.02 m. It should be noted that about 82% of the parameter results DX_{1-3} is in the number interval of ±0.1 m. The mean value of the parameter DX_{2-3} is equal to –0.06 m, similarly to the median parameter. On the other hand, the range of the obtained results of the parameter DX_{2-3} is between –0.11 m to –0.02 m. It should be stressed that approximately 98% results of the parameter DX_{2-3} is in the number interval of ±0.1 m.

Fig. 4 shows the results of the accuracy in the determination of the difference in the Y coordinate of the aircraft between individual solutions in the OTF mode. The average value of the parameter DY_{1-2} is equal to +0.06 m, and of the median to +0.05 m, respectively. In addition, the dispersion of the obtained results of the parameter DY_{1-2} ranges from –0.01 m to +0.11 m. It is worth adding that approximately 99% of the parameter results DY_{1-2} is in the range of ±0.1 m. The mean value of the parameter DY_{1-3} equals +0.04 m, and of the median +0.03 m, respectively. Moreover, the dispersion of the obtained results of the parameter DY_{1-3} is from –0.01 m do +0.09 m. It should be noted that 100% of the parameter results DY_{1-3} is in the number interval of ±0.1 m. The mean value of the parameter DY_{2-3} is equal to –0.02 m, similarly to the median parameter. On the other hand, the range of the obtained results of the parameter DY_{2-3} is between –0.07 m to +0.03 m. It should be stressed that 100 % of the parameter results DY_{2-3} is in the number interval of ±0.1 m.

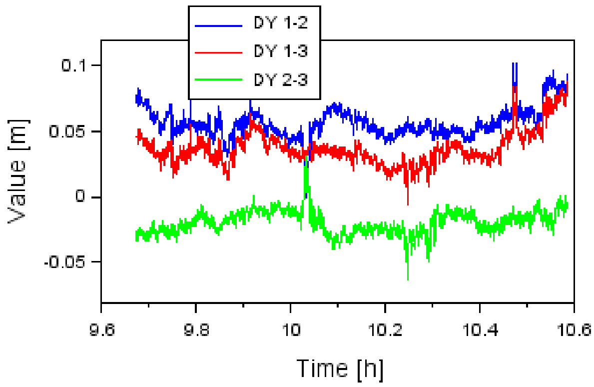

Fig. 4. Dispersion of coordinates difference along to Y axis

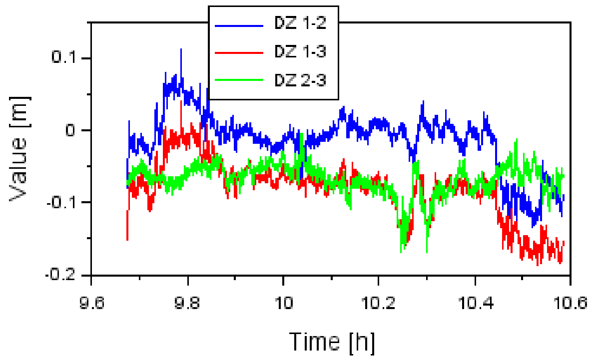

Fig. 5. Dispersion of coordinates difference along to Z axis

Fig. 5 shows the results of the accuracy in the determination of the difference in the Z coordinate of the aircraft between individual solutions in the OTF mode. The mean value of the parameter DZ_{1-2} is equal to –0.01 m, similarly to the parameter of the median. Furthermore, the dispersion of the obtained results of parameter DZ_{1-2} is between –0.14 m to +0.12 m. It is worth adding that approximately 94% of the parameter results DZ_{1-2} is in the range ±0.1 m. The average value of the parameter DZ_{1-3} is equal to –0.08 m, and of the median to –0.07 m, respectively. Besides, the dispersion of the obtained results of the parameter DZ_{1-3} is between –0.19 m to +0.04 m. It should be underlined that 77% of the parameter results DZ_{1-3} is in the number interval of ±0.1 m. The average value of parameter DZ_{2-3} is equal to –0.07 m, and of the median to –0.06 m, respectively. The range of the obtained results of parameter DZ_{2-3} is between –0.17 m to –0.01 m. It should be stressed that approximately 93% of the parameter results DZ_{2-3} is in the number interval ±0.1 m.

In the third and final stage of the analysis of the verification of the obtained results with regard to the XYZ coordinates of the aircraft, we determined the RMS parameter. The RMS parameter along each axis of the XYZ coordinate was determined by the dependence [10]:
a) along the X axis:

$$RMSx_{1-2} = \sqrt{\frac{\left(DX_{1-2}\right)^2}{n}}, \quad RMSx_{1-3} = \sqrt{\frac{\left(DX_{1-3}\right)^2}{n}}, \quad RMSx_{2-3} = \sqrt{\frac{\left(DX_{2-3}\right)^2}{n}}, \tag{7}$$

b) along the Y axis:

$$RMSy_{1-2} = \sqrt{\frac{\left(DY_{1-2}\right)^2}{n}}, \quad RMSy_{1-3} = \sqrt{\frac{\left(DY_{1-3}\right)^2}{n}}, \quad RMSy_{2-3} = \sqrt{\frac{\left(DY_{2-3}\right)^2}{n}}, \tag{8}$$

c) along the Z axis:

$$RMSz_{1-2} = \sqrt{\frac{\left(DZ_{1-2}\right)^2}{n}}, \quad RMSz_{1-3} = \sqrt{\frac{\left(DZ_{1-3}\right)^2}{n}}, \quad RMSz_{2-3} = \sqrt{\frac{\left(DZ_{2-3}\right)^2}{n}}, \tag{9}$$

where:
n – number of measurements.

The RMS bias for X axis equals to 0.04 m for baselines 1-2 and 1-3, whereas for baseline 2-3 it is about 0.01 m. The value of the parameter RMS along the X-axis is the largest for baselines 1-2 and 1-3. The value of the parameter RMS along the X-axis is the smallest for baseline 2-3. The value of the parameter RMS along the Y-axis is the same and equals 0.01 m for all three obtained results. The RMS bias for Z-axis amounts to 0.04 m for baselines 1-2 and 1-3, whereas for baseline 2-3 it is about 0.02 m. The value of the parameter RMS along the Z-axis is the largest for baselines 1-2 and 1-3. On the other hand, the value of the parameter RMS along the Z-axis is the smallest for baseline 2-3.

5. Conclusions

The article discusses the results of designating geocentric coordinates XYZ of the aircraft Cessna 172 based on the differential solutions RTK-OTF. Therefore, we recovered the accurate coordinates of the aircraft Cessna 172 with GPS observations in post-processing. The accurate coordinates of the Cessna 172 were determined on the basis of three independent position determinations in OTF mode. The geocentric coordinates XYZ of the aircraft were designated in relation to the position of the reference station REF1 and virtual reference stations VirA and VirB. In the article, we compared the accuracy of the determined coordinates XYZ of the aircraft Cessna 172. We specified the difference of XYZ coordinates of the aircraft among three independent OTF designations, determining the RMS parameter along each axis XYZ. In addition, in the article we specified the coefficients of the dilution of precision PDOP.

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