Solving the Stationary Hydroaeroelasticity Problem for Dynamic Deflection Elements of Separation Devices

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Abstract

The paper presents a method for expanding the working range of separation elements, where the separation is conducted through the use of inertia particles. The presented dynamic separation elements work as the automatic control system (the regulating action is the elastic energy; the regulation object is the hydraulic resistance). It was taken the first step to the engineering method development for their calculation using analytical dependences of the finite element method. The critical velocity of the gas-liquid flow was determined, that causes a divergence phenomenon of dynamic separation device elements and expressions for generalized forces for the system "gas-liquid flow is a dynamic deflection element." Two-knot finite elements with two degrees of freedom (transverse displacement and angle of the cross-section rotation) were used for dynamic deflection elements. The given number of degrees of freedom of the mechanical system "gas-liquid flow is a dynamic deflection element" due to the consideration of the transverse deformations of the plate allows simplifying the mathematical model. It was suggested to use aerohydroplastic phenomena of dynamic non-stability of dynamic deflection elements of separating devices, analogous to the method of applying acoustic oscillations to a heterogeneous stream, for the coagulation of dispersed particles in the flow.

1. Introduction

Technological and natural gases used in production processes are often heterogeneous in its composition and contain unwanted admixtures in the form of liquid and solid particles. These third-party inclusions can lead to equipment wear and emergency situations. Today separation methods based on the use of inertia of particles [1, 2] are the most applicable for the heterogeneous mixtures separation. When changing the direction of the deposition contaminated flow of existing inclusions occurs on the walls of the channel. These separation methods combine a single general disadvantage, namely, the operating modes

are limited to the critical values of the hydraulic resistance, where a sharp decrease in the separation efficiency occurs due to the removal of trapped particles from the deposition surfaces. Dynamic separation elements [3, 4] were proposed for expansion of working modes, which work is based on the principles of the automatic regulation system (the regulating action is the elastic energy; the regulation object is the hydraulic resistance). The flow, that is directed to the channel with elastic plates (deposition surfaces) makes a hydrodynamic pressure on them, in turn, internal stresses arise in the walls of the channel, that leads to the deformation of channel's form, and hence the hydraulic resistance. This paper considers the dynamic separation element, which elastic deposition surfaces have a form of an inclined parabolic semicylinder [4].

In respect that under the influence of the flow there will be significant deformations of the channel, which in turn will cause a change in the flow parameters, the static and dynamic hydro-aerosol effects, which can reduce the limits of the use of these elements, will be appropriate. Such hydroelectric static phenomena include the flow velocity at which divergence will take place, and hence a significant reduction in the heterogeneous system separation efficiency.

Thus, this paper is objected to the determination of the critical velocity of the gas-liquid flow that causes the divergence phenomenon of dynamic deflection elements of separation devices.

2. General problem description and mathematical model

In the stationary formulation, the deformation analysis of the separation device deflection element as a result of the action of the gas-liquid flow is based on the determination and further study of the equations of bending deformations of the plate relative to the predetermined static deformed state in accordance with the calculation scheme shown in Figure 1. The research of the previously deformed state was conducted in [5].
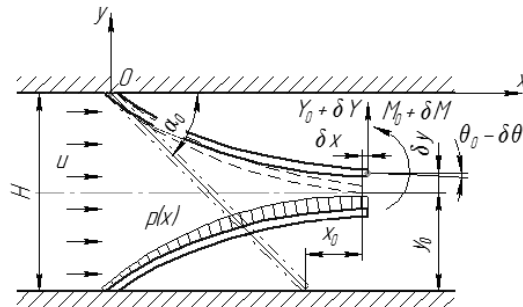

Figure 1. Design scheme

The follow-up research is based on the application of analytic dependencies of the finite element method [6, 7]. In particular, the deformation of dynamic deflection elements in the result of the gas-flow in the global coordinate system *x* - *y* can be described by the matrix equation:

$$[C]\{U\} = \{F\}, \tag{1}$$

where $[C]$ is the stiffness matrix of the dynamic deflection element, which was defined in [5]. The vector of generalized knot forces $\{F\}$ can be represented as the sum of the generalized forces vector $\{F\}_0$, determined as a result of the previous static calculation in [5], and the vector $\{\delta F\}$ of stationary components of generalized external forces [8]:

$$\{F\} = \{F\}_0 + \{\delta F\}. \tag{2}$$

The latter is determined by the distributed load $p(x)$, obtained by integrating the height of the pressure plate obtained as a result of experimental studies or as a result of numerical simulation of the interaction of the gas-liquid flow with the dynamic detachable element of the separation device in the subcritical mode, when the flow rate does not cause the flutter to appear.

For taking into account the load distributed along the length of the deformed deflection element under the arbitrary (or prescribed) law $p_s(x)$, the transition to the equivalent system of forces in node j (Figure 1) is carried out. For this reason, the work of the actual loading system on the corresponding displacements should be equal the work of equivalent system of forces and moments on the knot j motions.

The vector of generalized nodal forces of a finite element in a local coordinate system is determined by the following formula:

$$\{\delta F\}_{sl} = \int_0^L \Phi^T p_s(x_l) dx_l. \tag{3}$$

In the stationary problem formulation with the representation of function distribution of a load in the form of a quadric polynomial

$$p_s(x_l) = p_1 - \Delta p \cdot \frac{x_l}{l} - \frac{a}{2} \cdot \frac{x_l}{L}\left(1 - \frac{x_l}{L}\right), \tag{4}$$

where $\Delta p = p_1 - p_2$ – pressure drop across the length of the element; $a = d^2 p(x_l)/dx_l^2$ – the curvature of the distributed load diagram (for the linear distribution law, the coefficient $a = 0$), the vector of generalized nodal forces will be determined by the formula:

$$\{\delta F\}_l = L \cdot \int_0^1 \begin{Bmatrix} 1 - 3\xi^2 + 2\xi^3 \\ L\left(\xi - 2\xi^2 + \xi^3\right) \\ 3\xi^2 - 2\xi^3 \\ L\left(-\xi^2 + \xi^3\right) \end{Bmatrix} \left[p_1 - \Delta p \xi + \frac{1}{2} a \xi(1 - \xi)\right] d\xi, \tag{5}$$

where $\xi = x / L$ – relative coordinate.

After integration, the components of the vector of generalized nodal forces $\{\delta F\}_{sl} = \{\delta Y_{1l}, \delta M_{1l}, \delta Y_{2l}, \delta M_{2l}\}^T$ are determined by such dependencies:

$$\delta Y_{1l} = \frac{1}{2}L\left(p_1 - 0.3\Delta p + \frac{a}{12}\right); \qquad \delta Y_{2l} = \frac{1}{2}L\left(p_1 - 0.7\Delta p + \frac{a}{12}\right);$$

$$\delta M_{1l} = \frac{1}{12}L^2\left(p_1 - 0.4\Delta p + \frac{a}{10}\right); \quad \delta M_{2l} = -\frac{1}{12}L^2\left(p_1 - 0.6\Delta p + \frac{a}{10}\right) \tag{6}$$

In the global coordinate system, the components of the vector of generalized nodal forces $\{\delta F\}_s = \{\delta X_1, \delta Y_1, \delta M_1, \delta X_2, \delta Y_2, \delta M_2\}^T$, determined by the formula

$$\{F\}_s = [T]\{F\}_{sl}, \tag{7}$$

take the following form:

$$\delta X_1 = -\frac{1}{2}L\left(p_1 - 0.3\Delta p + \frac{a}{12}\right)\sin\alpha_0; \quad \delta M_1 = \frac{1}{12}L^2\left(p_1 - 0.4\Delta p + \frac{a}{10}\right);$$

$$\delta Y_1 = \frac{1}{2}L\left(p_1 - 0.3\Delta p + \frac{a}{12}\right)\cos\alpha_0; \quad \delta X_2 = -\frac{1}{2}L\left(p_1 - 0.7\Delta p + \frac{a}{12}\right)\sin\alpha_0; \tag{8}$$

$$\delta Y_2 = \frac{1}{2}L\left(p_1 - 0.7\Delta p + \frac{a}{12}\right)\cos\alpha_0; \quad \delta M_2 = -\frac{1}{12}L^2\left(p_1 - 0.6\Delta p + \frac{a}{10}\right).$$

With account of kinematic boundary conditions for the node i ($x_{10} = y_{20} = 0$; $\theta_{10} = 0$) the column vector of stationary values of the generalized forces

$$\{\delta F\} = \left\{-\frac{1}{2}L\left(p_1 - 0.7\Delta p + \frac{a}{12}\right)\sin\alpha_0; \frac{1}{2}L\left(p_1 - 0.7\Delta p + \frac{a}{12}\right)\cos\alpha_0; -\frac{1}{12}L^2\left(p_1 - 0.6\Delta p + \frac{a}{10}\right)\right\}^T. \tag{9}$$

The nodal displacement vector $\{U\}_s$ can be represented as the sum of the displacement vector $\{U\}_0$ as a result of the previous static loading of the plate and the vector $\{\delta U\}_s$ of the forced displacement components [8]:

$$\{U\}_s = \{U\}_0 + \{\delta U\}_s. \tag{10}$$

The expressions (2), (10) allow rewriting the equation (1) in the next form:

$$[C]\{U\}_0 + [C]\{\delta U\}_s = \{F\}_0 + \{\delta F\}_s, \tag{11}$$

which in view of the matrix equation of elastic equilibrium given in [5], reduces to the matrix differential equation of the forced oscillations of the separation device's dynamic deflection element:

$$[C]\{\delta U\}_s = \{\delta F\}_s, \tag{12}$$

containing exclusively forced components of external forces $\{\delta F\}_s$ and corresponding displacements $\{\delta U\}_s$.

The research of the multicomponent mixtures separation process is inextricably linked with the solution of the problem of hydroaeroelastic gas-liquid flow interaction with the separator's dynamic deflection elements. For this purpose, the mathematical model (12) should be supplemented with an expression describing the dependence of the vector $\{F\}_s$

on the generalized dynamic components of the motion from the motion vector $\{U\}_s$ and the flow velocity u.

Further, the equation (12) will be considered as the system with two degrees of freedom - the transverse displacement δy and the cross section rotation angle $\delta \theta$. In this approach, the rigidity matrix acquires the next form:

$$[C] = \frac{EI}{L^3} \begin{bmatrix} 12\cos^2\alpha_0 & -6L\cos\alpha_0 \\ -6L\cos\alpha_0 & 4L^2 \end{bmatrix}. \tag{13}$$

Reducing the number of freedom degree of the mechanical system "gas-liquid flow - dynamic deflection element" is determined through the consideration of the prevailing transverse deformations of the plate that simplifies the mathematical model.

From the perspective of the hydroaeroelasticity theory [8] the vector of generalized stationary forces can be represented by the following dependence:

$$\{\delta F\}_s = [\Delta C]\{\delta U\}_s, \tag{14}$$

where $[\Delta C]$ – matrix of added rigidity, which elements are the flow velocity function u.

The Theory of Flow Past Immersed Bodies is taken as a basis for determining the components of this matrix. This problem is solved by methods of conformal transformations [9], hydrodynamic peculiarities [10], and operational calculus [11] with the use of Laplace [12] and Fourier [13] transforms.

The specific values of the generalized forces for the system "gas-liquid flow - dynamic deflection element" are determined by this dependence [14]:

$$\delta Y = 2\pi\rho u^2 c(k)\delta\theta; \delta M = -\pi\rho u^2 c(k)\delta\theta, \tag{15}$$

where ρ – liquid density; $c(k)$ – Theodorsen's function [15], which can be determined by the functions of Bessel [16] or Hankel [17].

In the general case, the vector components of generalized forces can be written as:

$$\delta Y = c_1 u^2 \delta\theta; \delta M = c_2 u^2 \delta\theta, \tag{16}$$

where c_1, c_2 – attached stiffness coefficients, which can be calculated by the methods of evaluation theory based on the results of physical experiment, numerical simulation or a combination of them.

The stationary hydroaeroelasticity equation of dynamic separation elements (12) in view of the formulas (14) and (16) takes the next form:

$$[C]_\Sigma \{\delta U\}_s = \{0\}, \tag{17}$$

where $\{0\}$ – zero column vector; $[C]_\Sigma$ – matrix of system's total rigidity, according to formulas (13) and (16):

$$[C]_\Sigma = [C] - [\Delta C] = \begin{bmatrix} \dfrac{12EI}{L^3}\cos^2\alpha_0 & -\left(\dfrac{6EI}{L^2}\cos\alpha_0 + c_1 u^2\right) \\ -\dfrac{6EI}{L^2}\cos\alpha_0 & \dfrac{4EI}{L} - c_2 u^2 \end{bmatrix}; \tag{18}$$

The critical velocity of the gas-liquid flow, in which the phenomenon of divergence of the dynamic deflection elements occurs

$$u_{div} = \sqrt{EI / \left[L \left(c_2 + 0,5 c_1 L \sec \alpha_0 \right) \right]}. \tag{19}$$

3. Conclusions

This work demonstrates the definition of gas-liquid flow critical velocity that causes the divergence phenomenon of dynamic deflection elements of the separation devices, and expressions for generalized forces for the system "gas-liquid flow – dynamic deflection element." Further research will be focused on determining the distributed load $p(x)$ by numerical simulation of the interaction of the gas-liquid flow with the dynamic deflection element in a subcritical mode when the flow rate does not cause the flatter. Also, we will consider the possibility of using dynamic instability (buffeting and flutter) for vibrocoagulation analogously to the method of applying acoustic oscillations to a heterogeneous flow in order to reduce the dispersion of the weighed phase.

Acknowledgments

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