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STRUCTURAL OPTIMIZATION OF BOX WING AIRCRAFT

The box wing system is an unconventional way to connect the lifting surfaces that the designers willingly to use in prototypes of new aircrafts. The article present a way to quickly optimize the wing structure of box wing airplane that can be useful during conceptual design. At the beginning, there is presented theory and methods used to code optimization program. Structure analysis is based on FEM beam model, which is sufficient in conceptual design. Optimization is performed using hybrid method, connection of simple iteration and gradient descent methods. Finally, the program is validated by case study.

Notation

C_x	–	drag coefficient
C_m	–	moment coefficient
C_z	–	lift coefficient
ρ	–	air density
n	–	load factor
$I_x,\ I_y,\ I_{xy}$	–	section moments of inertia
q	–	dynamic pressure
V	–	aircraft speed
σ_{allow}	–	allowable stress

1. Introduction

The general concept of box wing configuration was presented by Prandtl in 1924 [1]. Many potential advantages as well as innovation of the design are the reasons why this configuration has become more and more popular.

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Theoretically, it should allow for reducing of both the empty weight and aerodynamic drag [2]. Both effects can reduce the fuel consumption, thus decreasing the operational costs and make aircraft more ecological. But, one should take into account a number of problems that are confronted with when calculating and choosing dimensions of such a structure, before deciding to use that solution. Static indetermination and non-linearity can seriously complicate the task. An additional difficulty consists in the design optimization taking into account the imposed requirements. Given these problems, it becomes practically impossible to carry out the task of choosing dimensions of the real structure with the use of analytical calculations only. Fortunately, modern technology supports us yielding various programming capabilities.

Research of Gallman, Smith and Kroo [2] could serve as an example of automated optimization of similar joined wing configuration. They used several of commercial programs to perform the task. The optimization method adopted by them was based on flow calculations using the LinAir software package. The NPSOL program was responsible for the optimization process. The interface created by Gallman at all. controlled the calculation process. The flow around the wing was calculated using the vortex-lattice method (LinAir). As a result, one could get the aerodynamic forces acting on the wing. These were used as an input to another software. Gallman at all. considered multiple load cases and performed the calculations of basic flight mechanics. That allowed for optimization of 17 design variables, including mechanical features, as constraints.

A different approach to solving optimization problems is presented in [3]. The authors decided to include phenomena such as aeroelastic failures and instability, in calculations relevant to statically indeterminate configuration. Ensuring from the very beginning of project that such a phenomena will not appear, is the key to success.

Linear approach to the problem of highly loaded surface leads to significant errors. There is therefore a need to develop a method of the structure optimization taking structural nonlinearity into account. The NROEL method [4, 5, 6] could be an example. This is a modified gradient method, which transforms the nonlinear problem to the linear one and then solves it. This method consists in calculating the equivalent loads for the linear approach that causes the same displacements as in nonlinear models.

2. General Survey

The mission is the first assumption which is made during the design of new aircraft, which in turn defines the required parameters of the airplane. In most cases, the main requirement consist in the lowest possible empty weight and the largest payload with the specified size. The loads acting on the structure are defined mainly by its size. Larger forces require a stronger structure which may cause an increase in mass. As a consequence, the design process becomes iterative. There is therefore a need to reduce the structure mass to a minimum. Such an action provides convergence to the solution and reduces costs.

The POSUPS is an example of a software code that solves this task. The code was written by the author in Delphi and its interface allows for co-operation with commercial software. A three-dimensional linear FEM model was created, in which the wings are represented by beams of variable cross sections. The wing cross-section was modeled as one-cell wing box with four flanges placed in the corners. The opposite walls have the same thickness, and the cross-sections of the flanges located on the opposite corners are identical (Fig. 3). This configuration of structure is considered in the literature as optimal for the joined wing [7]. For this reason the author decided to use it in POSUPS program.

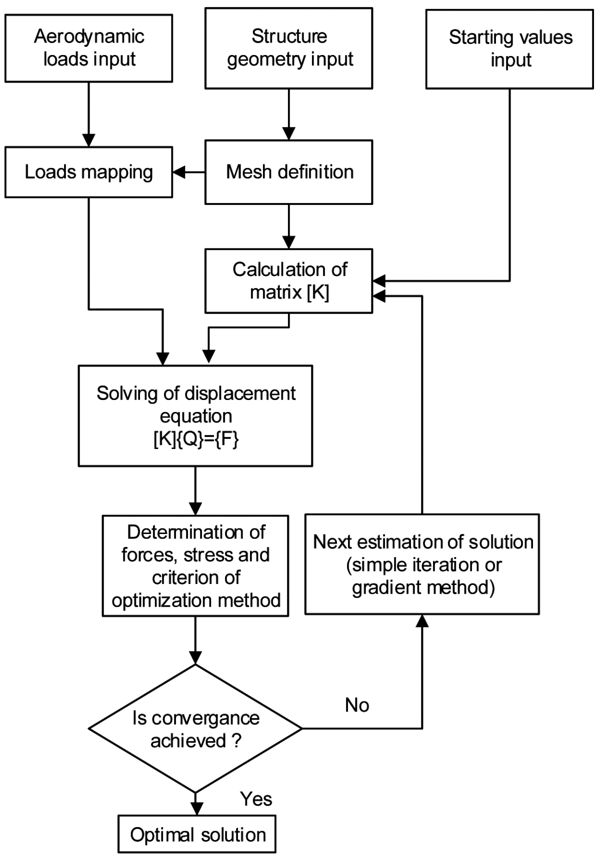

Fig. 1. Block diagram of POSUPS program

The aerodynamic loads can be obtained from any program that calculates the aerodynamic coefficients distributions along on the wing span. When the loads are introduced from the file, they are approximated for the FEM model using the Legendre polynomials. The final static analysis solution is obtained using the Gaussian elimination method. This method provides a reliable solution.

Optimization of the structure is performed by taking the value of actual mass as the objective function. On the other hand, wall thickness and flanges cross-section area serve as optimization variables. Searching for the optimal structure is carried out for the critical load. To limit the area of analysis, the penalty function is applied. It is a quadratic form that uses stress to determine the value of penalty. The algorithm which searches for a minimum weight is based on two methods. The first one is the gradient descent method, being one of gradient methods. The second one is a simple iteration method based on the fixed-point theorem. A combination of both methods in order to take advantage of both algorithms at once is the best way to optimize the structure of the aircraft. Distributions of cross sections, forces, displacements and stresses corresponding to the minimum weight of the structure are obtained as a result of searching for the optimal solution. Due to the simplification and numerous assumptions accepted, the presented software code is suitable for the analysis of small unmanned aircraft, or a conceptual analysis of larger structures.

3. Wing Model

The model used in conceptual design should be the simplest one, in which the number of variables is small and most intuitive. As a result, quick, rough estimation of the structure is possible. In the next calculation stages a simple model is subject to comprehensive local analysis. Therefore, the box wing configuration (Fig. 2) is represented in terms of a spatial arrangement of beams [3, 7]. Each beam represents a single lifting surface: two front surfaces, two rear and tip plates, respectively. The front surfaces are arranged in a high wing configuration, since in the literature some suggestions about additional aerodynamic advantages of that configuration [8] can be found. Each beam is fixed at the quarter of chord from its leading edge. This three-dimensional beam model adequately represents the design for the conceptual design purposes.

To maintain the problem simplicity, it is assumed that the internal structure of the wing can be represented by a one-cell torsion box only. The rationale for this approach consists in the fact that the stiffness of other elements of the wing (including wing skins) is usually much lower than the

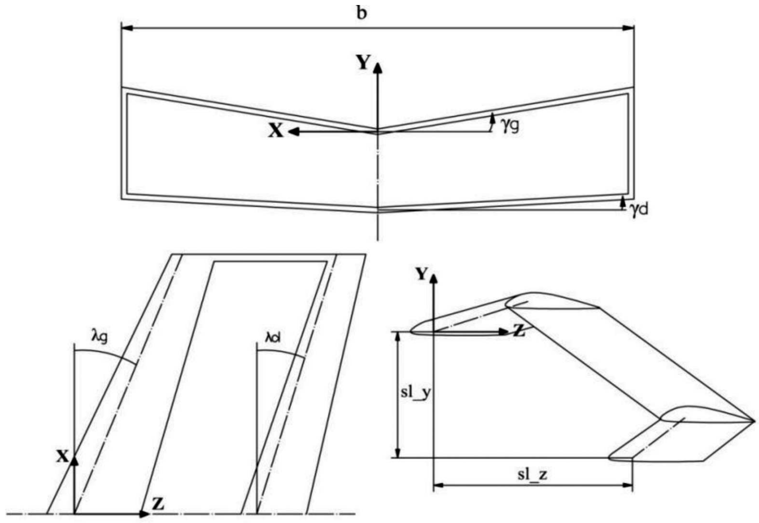

Fig. 2. Geometry of the analyzed box wing

stiffness of structural elements. It means that the loads carried by them are small as compared to the forces acting upon the structural elements. The torsion box (Fig. 3) consists of two horizontal and two vertical webs, and four flanges that are placed in the corners. It was assumed that the opposite flanges have the same cross-sectional area, which is due to the fact that the resultant moment acting on the cross-section lies in the plane of the section but does not coincide with the axes of local reference system. The objective is therefore to ensure that the resultant moment acts along one of the principal axes of inertia.

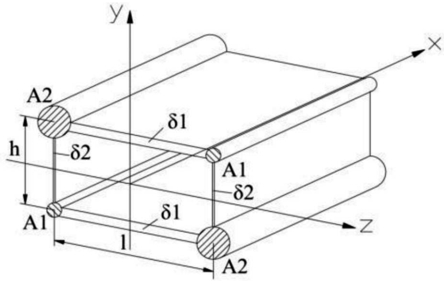

Fig. 3. Geometry of the wing box section

Assuming structure model illustrated above, one can determine the approximate values of the moments of inertia for each of the current wing section. These values are determined from the formulas [3]:

$$I_x(\xi) = \frac{2h^2 l^2 \delta_1(\xi)\,\delta_2(\xi)}{l\delta_{2(\xi)} + h\delta_1(\xi)} \tag{1}$$

$$I_y(\xi) = 2\left(\frac{l}{2}\right)^2 (A_1(\xi) + A_2(\xi)) \tag{2}$$

$$I_z(\xi) = 2\left(\frac{h}{2}\right)^2 (A_1(\xi) + A_2(\xi)) \tag{3}$$

$$I_{yz}(\xi) = \frac{hl}{2} (A_1(\xi) - A_2(\xi)) \tag{4}$$

Note that the cross section is asymmetric. As a result, directions of the principal axes of inertia should be determined. Moments of inertia should be obtained with respect to these axes. The best way to do this is to use Mohr's circle for the moments of inertia.

4. Loads Mapping Onto FEM Model

The analysis is performed for the predefined load distributions, which result from the distributions of aerodynamic coefficients along each lifting surface. They can be obtained by using the aerodynamic software for calculation of aerodynamic loads for specified flight conditions (e.g. AVL). The distributions obtained from an available aerodynamic software are usually defined for a mesh different from that used in the FEM software. For that reason, it is necessary to transform the distributions of coefficients resulting from the aerodynamic software to the mesh applied by the POSUPS.

The transformation of loads is performed using the 8^{th} and 9^{th} degree Legendre polynomial approximation [9]. The following recursive formula is used to obtain the polynomials:

$$\begin{gathered} P_0(x) = 1 \\ P_1(x) = x \\ P_{n+1}(x) = \frac{2n+1}{n+1} x P_n(x) - \frac{n}{n+1} x P_{n-1}(x) \end{gathered} \tag{5}$$

The main advantage of this method consists in its correct performance even in the case when the matrix used to determine multipliers of polynomial is ill-conditioned.

5. FEM Formulation

The algorithm that solves the problem of static analysis employs the finite element method based on linear displacement equation (6). Currently, in most available codes this approach is used in analysis of the structure. The main advantages of this algorithm are simplicity of implementation into any programming language and the possibility of calculation of complex shapes.

$$[K]\{Q\} = \{F\} \tag{6}$$

The FEM model consists of a frame formed from beam elements. Each element has a characteristic cross section defined in Fig. 3. For these cross sections the stiffness matrix $[K]$ is determined.

The spatial arrangement of beams is loaded by fluxes of aerodynamic forces:

$$\begin{aligned} f_z &= qC_z(x)\,c(x) \\ f_x &= qC_x(x)\,c(x) \\ f_m &= qC_m(x)\,c^2(x) \end{aligned} \tag{7}$$

where $q = 0.5\rho V^2$ represents the dynamic pressure.

Since a continuous load distributed along each element is considered, there is a need to define the corresponding nodal forces according to the relation [10]:

$$P_i = \int_0^{le} f_j(\xi)\,N_k(\xi)\,d\xi \tag{8}$$

Applying the loads to the nodes, one should pay attention to vector directions.

The box wing is a symmetrical system. It allows for a half-oft-the-wing analysis only, assuming the necessary boundary conditions. The above simplification reduces significantly the computation time because there is no need to analyze the entire model.

The Gauss elimination method is used to solve displacement equation (6). This method is used because of its simplicity and versatility. In contrast to the iterative methods, we always obtain the solution of the system of equations (provided that it exists). There is no problem with the selection of initial values or convergence of the solution.

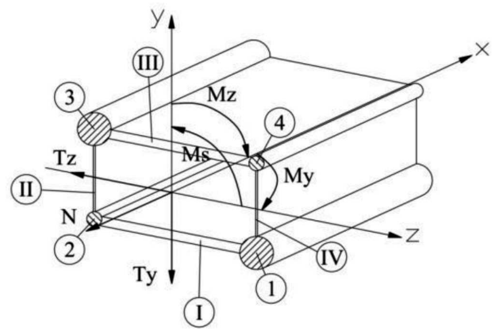

Fig. 4. Description of the current section

One can calculate the internal forces acting upon sections based on the displacements obtained using the Gaussian elimination. Using these forces, one can determine the stresses in flanges and webs in every cross section of the torsion box.

Normal stresses in flanges:

$$\begin{aligned}
(1):\ \sigma &= \sigma_M\left(-\frac{h}{2}, \frac{l}{2}\right) + \frac{N}{2\left(A_1 + A_2\right)}\\
(2):\ \sigma &= \sigma_M\left(-\frac{h}{2}, -\frac{l}{2}\right) + \frac{N}{2\left(A_1 + A_2\right)}\\
(3):\ \sigma &= \sigma_M\left(\frac{h}{2}, -\frac{l}{2}\right) + \frac{N}{2\left(A_1 + A_2\right)}\\
(4):\ \sigma &= \sigma_M\left(\frac{h}{2}, \frac{l}{2}\right) + \frac{N}{2\left(A_1 + A_2\right)}
\end{aligned} \tag{9}$$

where [11]:

$$\sigma_M\left(y, z\right) = \frac{M_z I_{yz} + M_y I_z}{I_z I_y - I_{yz}^2} z - \frac{M_z I_y + M_y I_{yz}}{I_z I_y - I_{yz}^2} y \tag{10}$$

Shear stresses in webs:

$$\begin{aligned}
(\mathrm{I}):\ \tau &= \frac{1}{2\delta_1 l}\left(T_z - \frac{M_s}{h}\right)\\
(\mathrm{III}):\ \tau &= \frac{1}{2\delta_1 l}\left(T_z + \frac{M_s}{h}\right)\\
(\mathrm{II}):\ \tau &= \frac{1}{2\delta_2 h}\left(T_y - \frac{M_s}{l}\right)\\
(\mathrm{IV}):\ \tau &= \frac{1}{2\delta_2 h}\left(T_y - \frac{M_s}{l}\right)
\end{aligned} \tag{11}$$

6. Optimization Algorithms

One should use various types of tricks to cope effectively with the task consisting in finding a minimum of a function of several variables. Most commonly those tricks are used in the problem definition (e.g. solving an equivalent problem) and then the approach is applicable only to the given specific case.

To simplify the current task, one can replace the weight minimization with an equivalent problem. Searching for optimum cross sections of a pre-defined structure is equivalent to the problem of searching for the maximum material effort. At first glance, this is a very similar task. However, in this case cross sections are selected to achieve the predetermined allowable stress in each section.

In the present optimization problem two methods are used: simple iteration method [12] and gradient descent method [13]. The simple iteration method is used to estimate the optimum quickly but not accurately, while the gradient descent method allows for reaching the final solution. The simple iteration algorithm is based on the Banach theorem of fixed point. This is a very fast convergent algorithm and its computational cost is proportional to the number of variables. Another but not less important advantage of this method consists in its flexibility in choosing the starting point. The convergence is always achieved, no matter from which point the calculation will start. This is the convergence to the same solution.

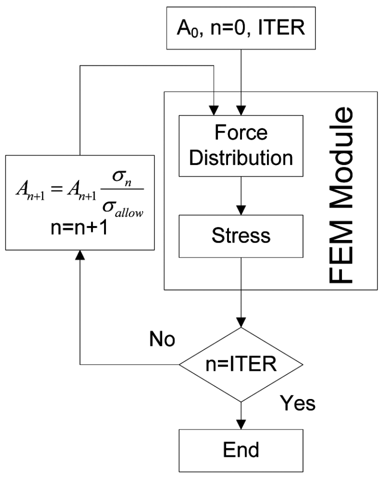

Fig. 5. Simple iteration algorithm

Searching for the solution is performed following the formula:

$$A_{n+1} = A_n \frac{\sigma_n}{\sigma_{dop}} \tag{12}$$

From relation (12) it can be clearly seen that the optimal solution can be reached just after the first step of approximation for statically determinate systems,. Unfortunately, the considered box wing configuration is statically indeterminate. In reality, changes in cross-sections are not linear as those obtained from equation (12). Stresses in the current cross section depend on the stiffness of the wing in each section. But when one considers this problem step by step according to (12) one can expect convergence to minimum values of the variables in each section. However, there is one more obstacle. While for the statically determined case the assumptions of Banach theorem are always true, checking if these assumptions are fulfilled for complex and statically indeterminate cases is practically impossible. Nevertheless, based on experience gained in solving similar problems, one can assume that this algorithm does not tend to diverge. The worst case happens when the solution oscillates around the final solution. Then, a different method has to be used to carry out several successive iterations.

The best method to complete the iteration is the simplest one. In this case the gradient descent method was chosen. This algorithm has the lowest computational cost among all gradient methods, but its convergence can be very slow. The solution is searched in the opposite direction to that indicated by the objective function gradient.

The gradient descent algorithm is much less effective than a simple iteration method because one has to determine not only the value of the objective function but also its derivatives at each step. However, this method suits perfectly for finishing the iterative solution. The objective function $f(x)$ given above provides constraints due to the maximum allowable stress in the cross section for the gradient descent method.

$$f(x) = m(x) + c \cdot S(\mathbf{x}) \tag{13}$$

Objective function (13) is the sum of structure mass and penalty function. The latter is a quadratic form of stresses appearing in the cross section. Its value is determined from the formula:

$$S(\mathbf{x}) = \begin{cases} \sum_{i=1}^{N} \left(\sigma^i(\mathbf{x}) - \sigma_{dop}\right)^2 & \text{if } \sigma^i(\mathbf{x}) > \sigma_{allow} \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

Solution to the problem consists in finding an approximate solution (using simple iteration algorithm) and then obtaining more accurate solution with the use of gradient descent method.

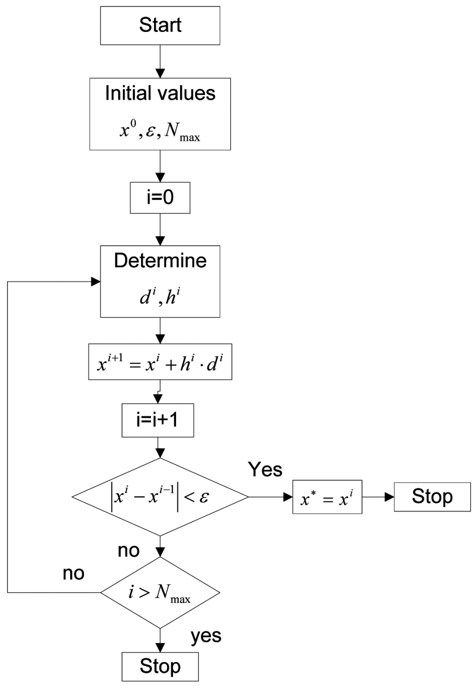

Fig. 6. Gradient descent algorithm

7. Case Study

The Author decided to optimize the structure of an unmanned aircraft wing shown in Fig. 7 to verify working correctness of the POSUPS software.

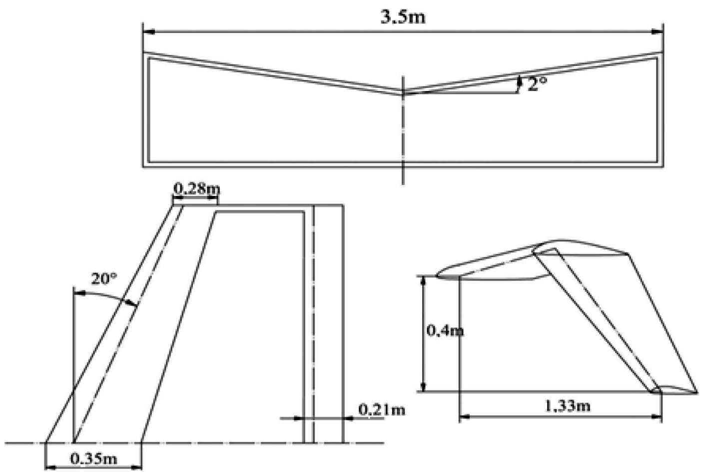

Fig. 7. Geometry of analyzed UAV

NACA 23012 airfoil is used in this aircraft. The analysis was carried out for critical flight conditions (maximum lift and maximum load factor):

- $n = 6.58$
- $V = 44.5$ m/s

In the calculations it was assumed that the length of torsion box section is 35% of chord, while the height is 11% of chord. To simplify the problem, aluminum alloy 2024 was selected for the wing structure. The only constraint for optimization is an allowable stress of the material.

The AVL software was used to determine the loads acting upon the wing. The following distributions of aerodynamic coefficients [14] were obtained.

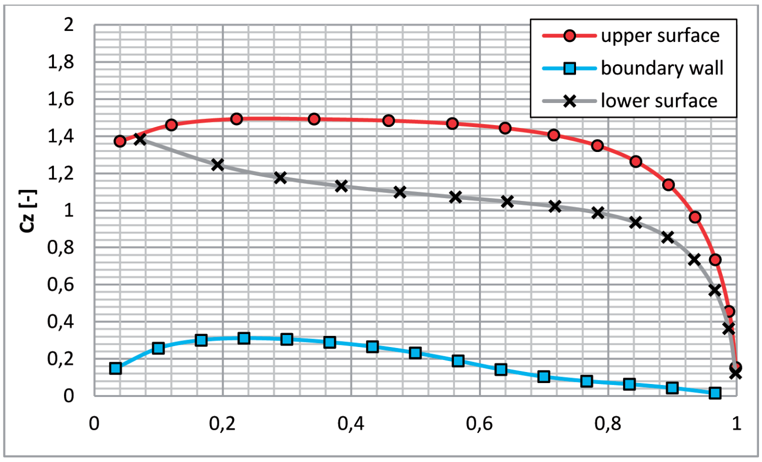

Fig. 8. Distribution of Cz coefficient along the wing span

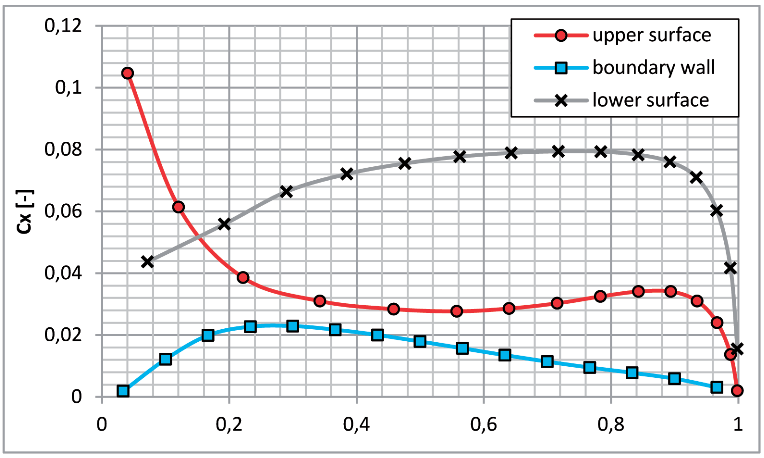

Fig. 9. Distribution of Cx coefficient along the wing span

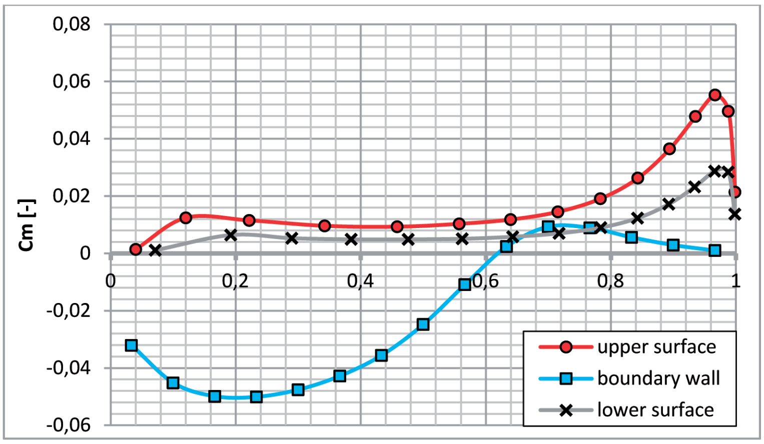

Fig. 10. Distribution of Cm coefficient along the wing span

The FEM model developed for optimization purposes consisted of 30 finite elements, 10 on each lifting surface. This effectively gives 120 design variables (spar caps cross section area and webs thickness). Iterative calculations of the structure dimensions started from the same fixed initial values in each section. The wall thickness of 0.5 mm was applied, the initial cross section area of the flanges was 40 mm^2 each. The starting values were chosen based on stress values in each element. It was assumed that half of them should give stress values greater than the allowable stress. There were 450 iterations of POSUPS program performed. First 50 iterations were performed using the simple iteration, while the descent gradient method was employed in the rest part of them. The results of the program are shown in Fig. 11 and Fig. 12.

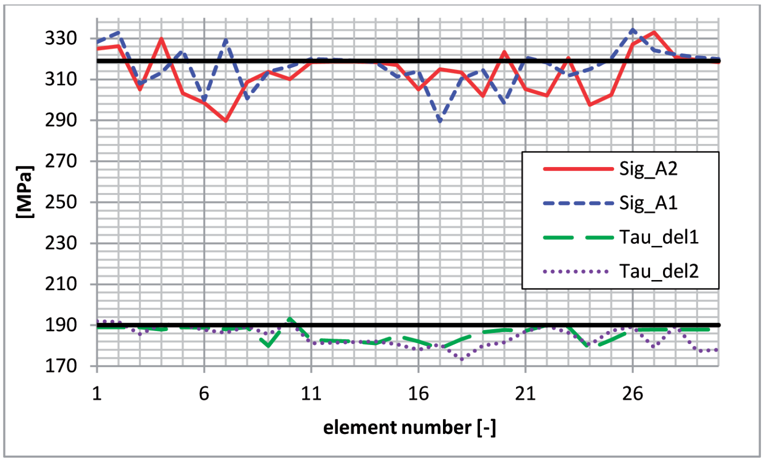

Fig. 11. Distribution of stress in all elements

As it can be seen from Fig. 12 simple iteration method quickly leads to the solution area and then oscillates around it. Further calculations using gradient method lead to convergence solutions. Fig. 11 shows that the optimal solution has not been finally achieved. Moreover, there are areas where the stress exceeds the allowable values. This error does not exceed 5%. To get rid of these areas the optimization should be continued.

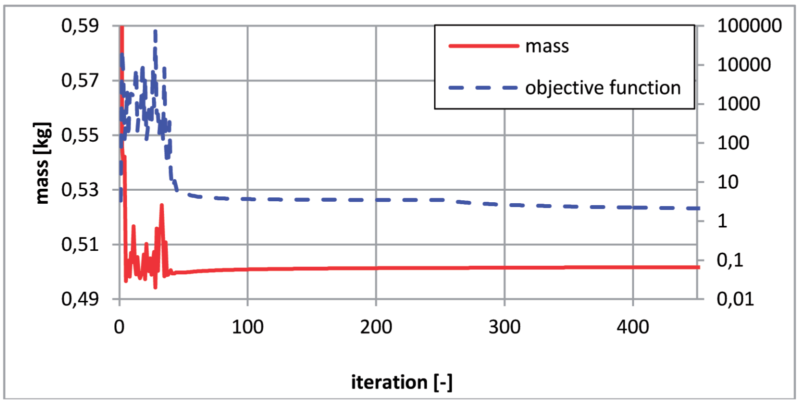

Fig. 12. Changes of mass and objective function

During the optimization it turned out that the stiffness of regions connecting the lifting surfaces is practically the most important in terms of a deformation of the entire box wing system. The less stiff joining region the more it resembles articulated connection. It causes high sensitivity in the whole structure to any changes in this region. Even a small change in this region causes rapid changes of forces and stresses in the whole structure and poses problems with convergence.

8. Conclusions

The software code based on linear formulas has been developed to optimize the structure of box wing aircraft. The code employs the aerodynamic loads obtained using the aerodynamic software available on the internet. The approximation method based on the Legendre polynomials gives good results for approximation of aerodynamic coefficient distributions, therefore it is well suited for mapping loads from external software onto the FEM model created in the code. The strength calculations were performed using the FEM based on the 3D beam model. The Gauss elimination method was used to obtain the solution. Although it is rather a time-consuming algorithm, there are no such problems with convergence as in the case of iterative algorithms.

The optimization process was performed with the application of two methods: simple iteration and gradient descent. While these methods solve statically determinate problems very well, statically indeterminate analysis can pose serious obstacles for both of them. The hybrid method which is a connection of both methods combines good qualities of both algorithms and can cope with the problem of optimizing the structure of box wing aircraft.

The structure optimization of UAV wing gave reasonable results, paving a good way for the concept of an aircraft wing structure in the box wing system. The analysis also showed a disturbing tendency of the solution in the areas of lifting surfaces connection.

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Optymalizacja strukturalna samolotu w układzie skrzydeł zamkniętych

S t r e s z c z e n i e

Układ zamkniętych skrzydeł to niekonwencjonalne rozwiązanie połączenia powierzchni nośnych, które coraz częściej konstruktorzy starają się stosować w prototypach nowych konstrukcji.

Ten artykuł prezentuje przykładowy sposób realizacji optymalizacji strukturalnej struktury nośnej skrzydeł w rozpatrywanym układzie, który może być użyteczny w trakcie projektowania wstępnego samolotu. Na wstępie zaprezentowano metody oraz teorię wykorzystane do stworzenia algorytmu optymalizacji. Struktura analizowana jest przy użyciu belkowego modelu MES. Optymalizacja została przeprowadzona z wykorzystaniem połączenia metod iteracji prostych i gradientowych. Wyniki działania algorytmu przedstawione są na prostym przypadku obliczeniowym.