

Atypical application of the parametric method for track infrastructure inventory

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Abstract

For many years, satellite systems have seen widespread use in a variety of technical applications, as well as in operations related to setting-out and the exploitation of track infrastructure. Their main applications include an inventory of the trackage course and detecting changes in its position. In both of these tasks, the most important element that determines the quality of an analyses is the high accuracy of the determinations being carried out. Satellite surveying techniques are not always sufficiently accurate, and in such cases, it is necessary to employ other land surveying methods to process surveying data.

This article presents the authors' considerations with regards to the possibility of applying one of the most common land surveying adjustment methods, the parametric method, to operations related to an inventory of tram infrastructure in Gdańsk. The results are based on surveys carried out during a surveying campaign in the autumn of 2018. The considerations presented in the article concern a small part of the research conducted under project No. POIR.04.01.01-00-0017/17 entitled "Development of an innovative method for determining the precise trajectory of a railborne vehicle" which is being implemented by a consortium of Gdansk University of Technology and Gdynia Maritime University.

Introduction

For many years, satellite systems have facilitated the daily lives of people and are one of the most important research tools used in the technical sciences. Nowadays, many typical technical operations cannot be carried out without using satellite surveying techniques, which offer user accuracy levels that are higher than those in the 1970s. Research centers all over the world are also looking for opportunities to use satellite systems in non-standard ways in order to solve research problems, such as

in maritime navigation. In earlier publications, the authors indicated the possibility of applying satellite techniques to model the vessel traffic in navigation and maneuvering simulators (Czaplewski & Zwolan, 2016). They also demonstrated the application of satellite systems along with onshore radio stations for increasing the accuracy of determining a vessel's position in areas under the operation of the VTS system (Czaplewski, Guze & Świerczyński, 2018). Many examples of atypical applications of satellite systems can also be found in publications by other authors (Felski, Naus & Wąż, 2016).

In aviation navigation, ensuring the highest possible level of aircraft safety has been a common research problem; therefore, most studies have focused on increasing the accuracy of determining an aircraft's position during its flight (Grzegorzewski et al., 2008). On the other hand, the issue of track position stability for inland railway navigation is of extreme importance. This is why the authors have attempted to address the issue of inventorying both railway and tram trackage for several years. A method was developed to diagnose the stability of railway track geometry and analyze their quality using both dynamic and continuous recording and satellite techniques (Specht et al., 2014, 2019; Koc et al., 2019). However, increasingly often, the operation of satellite systems is limited, not only by natural obstacles but also by deliberate human actions aimed at eliminating satellite systems from daily life. Therefore, the countries that own these systems have for years been taking measures to make them immune to deliberate interference (Czaplewski, 2015). Moreover, research centers are working to increase the accuracy of satellite surveys and their wider use in mobile surveys, other than classical geodetic surveys. Therefore, as part of project No. POIR.04.01.01-00-0017/17, entitled "Development of an innovative method for determining a precise trajectory of a railborne vehicle" which is being implemented by a consortium of Gdańsk University of Technology and Gdynia Maritime University, the authors have adapted the methods of observation result adjustment that are used in land surveying. This article presents the possibility of adopting one of the most frequently applied adjustment methods, the parametric method, whose properties indicate that it can be easily applied in mobile surveys.

An adjustment task and its solution using the parametric method

The parametric method is the most commonly used observation adjustment method in land surveying. It is very well described in numerous scientific publications in the field of land surveying and cartography (Findeisen, Szymanowski & Wierzbicki, 1980; Wiśniewski, 2013; 2016). Due to its properties, it is often adapted to other scientific areas as well. Since the essence of the method is to search for increments to the expected values of the parameters being surveyed, the method is often used in navigational tasks that involve searching for a shift in the coordinates of the observed position of a moving vessel in relation to its calculated position on a map

(Świerczyński & Czaplewski, 2015) or enhancing the quality of the measurements being carried out (Czaplewski, Guze & Świerczyński, 2018; Czaplewski, Wąż & Zienkiewicz, 2019). This article proposes the application of the parametric method for land navigation with regards to the positioning of railborne vehicles. In order to illustrate the possibility of applying this method to determine the *r*-coordinates of antennas on a measurement platform (X_{Ri}, Y_{Ri}), for $i = 1, \dots, k$, the distances between receivers on the platform (d_z) for $z = 1, \dots, q$, and the distances to the reference stations (d_j) for $j = 1, \dots, m$ were used. This allowed us to obtain n -observations (where $n > 2$). The coordinates (X_{Sj}, Y_{Sj}) of the reference stations are known. Such a geometric layout enables the creation of linear and then matrix equations of corrections, in an analogue way as in (Wiśniewski, 2016; Czaplewski, Wąż & Zienkiewicz, 2019):

$$d_n + v_n = F_n(\hat{X}_{R_i}, \hat{Y}_{R_i}) = \sqrt{(\mathbf{X}_{S_j} - \hat{\mathbf{X}}_{R_i}) - (\mathbf{Y}_{S_j} - \hat{\mathbf{Y}}_{R_i})} \Big\}_{\substack{i=1, \dots, n \\ j=1, \dots, m}} \Leftrightarrow \mathbf{D} + \mathbf{V} = \mathbf{F}(\hat{\mathbf{X}}_R) \tag{1}$$

where:

$\mathbf{V} = [v_1, v_2, \dots, v_n]^T$ – a vector of corrections to the distances measured,
 $\hat{\mathbf{X}}_R = [\hat{X}_{R_i}, \hat{Y}_{R_i}]^T$ – a vector of adjusted coordinates of antennas on the measurement platform.

We assume that the vector of approximate coordinates of antennas on the platform is known:

$$\mathbf{X}_R^o = [X_{R_i}^o, Y_{R_i}^o]^T.$$

In the task being presented, the coordinates determined based on satellite surveys will be assumed as approximate coordinates. Then, bringing the function $\mathbf{F}(\hat{\mathbf{X}}_R)$ to the linear form by developing it into a Taylor series limited to the first terms (Wiśniewski, 2016) results in the following:

$$\mathbf{F}(\hat{\mathbf{X}}_R) = \mathbf{F}(\mathbf{X}_R^o) + \mathbf{A} \hat{\mathbf{d}}\mathbf{x}_R \tag{2}$$

where: $\mathbf{A} = \partial_X \mathbf{F}(\mathbf{X}_R^o)$ whereas $\hat{\mathbf{X}}_R = \mathbf{X}_R^o + \hat{\mathbf{d}}\mathbf{x}_R$.

Having considered the above assumptions, the system of correction equations can be presented in the following form:

$$\mathbf{D} + \mathbf{V} = \mathbf{F}(\hat{\mathbf{X}}_R) \Leftrightarrow \mathbf{D} + \mathbf{V} = \mathbf{F}(\mathbf{X}_R^o) + \mathbf{A} \hat{\mathbf{d}}\mathbf{x}_R \Leftrightarrow \mathbf{V} = \mathbf{A} \hat{\mathbf{d}}\mathbf{x}_R + \mathbf{L} \tag{3}$$

where: $\mathbf{L} = \mathbf{F}(\mathbf{X}_R^o) - \mathbf{D}$.

Let us assume that the mean errors m_1, m_2, \dots, m_n of the mutually independent results of distance measurement is d_1, d_2, \dots, d_n . The diagonal measurement results in a cofactor matrix $\mathbf{Q}_D = \text{Diag}(m_1^2, m_2^2, \dots, m_n^2)$ that will then serve as an approximation of the covariance matrix $\mathbf{C}_D = \text{Diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_n^2)$ (Wiśniewski, 2016):

$$\mathbf{C}_D = m_0^2 \mathbf{Q}_D = m_0^2 \mathbf{P}^{-1} \quad (4)$$

where: m_0^2 – is an unknown coefficient of variance,

$$\mathbf{P} = \mathbf{Q}_D^{-1} = \begin{bmatrix} p_1 & & & \\ & p_2 & & \\ & & \ddots & \\ & & & p_n \end{bmatrix}$$

– the known weight matrix
(p_n – the weight of n -th observation) (5)

Moreover, let

$\Phi(\hat{\mathbf{d}}\mathbf{x}_R) = \mathbf{V}^T \mathbf{C}_D \mathbf{V} = \min \Leftrightarrow \Phi(\hat{\mathbf{d}}\mathbf{x}_R) = \mathbf{V}^T \mathbf{P} \mathbf{V} = \min$
be a criterion for estimation by the least-squares method. The process of determining an unknown vector of coordinates of the position of antennas on the platform can then be equated with the solution to a classical optimisation task using the least-squares method (Wiśniewski, 2013, 2016; Czaplewski, Wąż & Zienkiewicz, 2019):

$$\begin{cases} \mathbf{V} = \mathbf{A} \hat{\mathbf{d}}\mathbf{x}_R + \mathbf{L} \\ \mathbf{C}_D = m_0^2 \mathbf{Q}_D = m_0^2 \mathbf{P}^{-1} \\ \Phi(\hat{\mathbf{d}}\mathbf{x}_R) = \mathbf{V}^T \mathbf{P} \mathbf{V} = \min \end{cases} \quad (6)$$

Its solution is (provided that $|\mathbf{A} \mathbf{P} \mathbf{A}| \neq 0$):

$$\hat{\mathbf{d}}\mathbf{x}_R = -(\mathbf{A}^T \mathbf{P} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{P} \mathbf{L} \quad (7)$$

Since $\hat{\mathbf{X}}_R = \mathbf{X}_R^o + \hat{\mathbf{d}}\mathbf{x}_R$, the vector of approximate parameters \mathbf{X}_R^o is not random, therefore:

$$\mathbf{C}_{\hat{\mathbf{X}}_R} = \mathbf{C}_{\hat{\mathbf{d}}\mathbf{x}_R} = \sigma_0^2 (\mathbf{A}^T \mathbf{P} \mathbf{A})^{-1} = \sigma_0^2 \mathbf{Q}_{\hat{\mathbf{X}}_R} \quad (8)$$

The matrix $\mathbf{Q}_{\hat{\mathbf{X}}_R} = (\mathbf{A}^T \mathbf{P} \mathbf{A})^{-1}$ is an estimator's cofactor matrix $\hat{\mathbf{X}}_R$, while $\mathbf{P}_{\hat{\mathbf{X}}_R} = \mathbf{Q}_{\hat{\mathbf{X}}_R}^{-1} = \mathbf{A}^T \mathbf{P} \mathbf{A}$ is its weight matrix.

By replacing the variance coefficient σ_0^2 with its estimator $\hat{\sigma}_0^2 = (\mathbf{V}^T \mathbf{P} \mathbf{V}) / (n - r)$, we obtain the estimator of the covariance matrix of the vector of adjusted coordinates $\hat{\mathbf{X}}_R$:

$$\hat{\mathbf{C}}_{\hat{\mathbf{X}}_R} = \hat{\sigma}_0^2 (\mathbf{A}^T \mathbf{P} \mathbf{A})^{-1} = \hat{\sigma}_0^2 \mathbf{Q}_{\hat{\mathbf{X}}_R} \quad (9)$$

On the diagonal of the main matrix $\hat{\mathbf{C}}_{\hat{\mathbf{X}}_R}$, there are squares of mean errors of determined coordinates of antennas on the platform. Therefore, the mean error is:

$$m_{\hat{x}_i} = \sqrt{[\hat{\mathbf{C}}_{\hat{\mathbf{X}}_R}]_{ii}} \quad (10)$$

Surveying campaign

In order to verify the adopted theoretical assumptions, the authors of the article conducted a surveying campaign to record the coordinates of the position of satellite receiver antennas and other measurement platform location parameters not described in this article. The recordings were carried out on 29 November 2018 between 23:00 and 04:00 hours in the Gdańsk agglomeration area. The study used a set of railborne vehicles comprised of a Bombardier NGT6 tram (Figure 1) and two mobile measurement platforms (Figure 2).



Figure 1. Bombardier NGT6 Tram (Wikimedia, 2009)

The GNSS surveying receivers of two leading land surveying instrument manufacturers were mounted on the measurement platforms, and five receivers of the same manufacturer were mounted on each platform. Devices with very similar technical parameters were selected for the study (Table 1).

Moreover, in order to conduct other research not described in this article, an inclinometer, accelerator, and compass were mounted (Figure 2).

The study was conducted on a 3-km long tram loop whose course is shown in Figure 3. The surveying set passed the section of trackage in Gdańsk used for the study several times, which was characterized by the presence of various degrees of development, which affects the accessibility of satellite systems. The average speed of the set used to carry out surveys was 10 km/h.

Table 1. Selected technical data of the receivers used in the study

Parameter	Receivers on the first MMP	Receivers on the second MMP
Signal	GPS: L1, L2, L2C, L5	GPS: L1, L2, L2C, L5
Tracking	Glonass: L1, L2, L3 BeiDou: B1, B2, B3 Galileo: E1, E5a, E5b, Alt-BOC, E6 QZSS: L1, L2, L5 SBAS: WAAS, EGNOS, MSAS, GAGAN, L-band	Glonass: L1, L2, L3 BeiDou: B1, B2 Galileo: E1, E5a, E5b SBAS: QZSS, WAAS, EGNOS, GAGAN
Accuracy	Single baseline: Hz 8 mm + 1 ppm / V 15 mm + 1 ppm	Single baseline: Hz 8 mm + 1 ppm / V 15 mm + 1 ppm
Real Time	Network RTK: Hz 8 mm + 0.5 ppm / V 15 mm + 0.5 ppm	Network RTK: Hz 8 mm + 0.5 ppm / V 15 mm + 0.5 ppm
Accuracy	Static (phase) with long observations:	Static (phase) with long observations:
Post-	Hz 3 mm + 0.1 ppm / V 3.5 mm + 0.4 ppm	Hz 3 mm + 0.1 ppm / V 3.5 mm + 0.4 ppm
-processing	Static and rapid static (phase):	Static and rapid static (phase):
	Hz 3 mm + 0.5 ppm / V 5 mm + 0.5 ppm	Hz 3 mm + 0.5 ppm / V 5 mm + 0.5 ppm

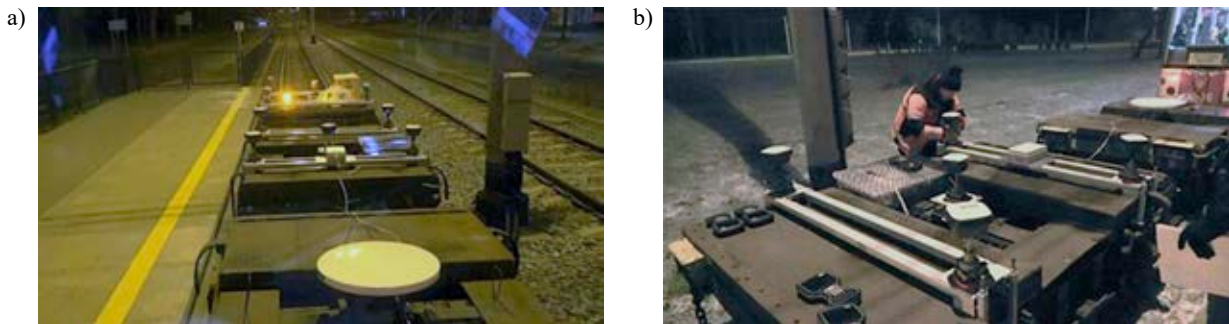


Figure 2. a) Mobile measurement platforms, b) GNSS receiver guides



Figure 3. Planned measurement set passage (Google Maps, 2019)

A schematic diagram of the constructed measurement platforms is shown in Figure 4 in which receivers were distributed so that four of them were situated in the vertices of the square, and the fifth was placed at the diagonal intersection. The design of the mobile measurement platform enabled the

construction of a square-shaped geometric surveying structure with side lengths ranging from 155 cm to 170 cm. The precise placement of four GNSS receivers above the tracks and one receiver on the track axis was performed using a local reference system that used an electronic tacheometer and a prismatic mirror placed on a specially-dedicated pin in levelling heads with an accuracy of approx. 1 mm.

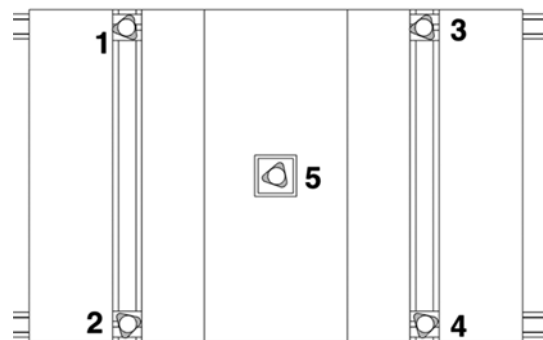


Figure 4. A schematic diagram of the mobile measurement platform (1, ..., 5 – GNSS receiver antennas)

During the surveying campaign, the real-time positioning data was recorded at a frequency of 1 Hz for various configurations of the GNSS receivers which used, jointly or separately, the accessible satellite systems.

Having considered the input data for calculations and after determining the weight matrix (\mathbf{P}), the following estimators of the coordinates of antenna positions on the measurement platform were determined, and the results were rounded to the third decimal place:

$$\hat{\mathbf{X}}_R = \mathbf{X}_R^o + \hat{\mathbf{d}}\mathbf{x}_R = \begin{bmatrix} 6031099.876 \\ 6540722.587 \\ 6031100.474 \\ 6540721.095 \\ 6031099.422 \\ 6540721.550 \\ 6031098.391 \\ 6540721.993 \\ 6031098.965 \\ 6540720.512 \end{bmatrix} + \begin{bmatrix} -0.00041 \\ 0.00104 \\ 0.00385 \\ -0.00009 \\ 0.00016 \\ 0.00043 \\ 0.00082 \\ -0.00213 \\ -0.00377 \\ 0.00083 \end{bmatrix} = \begin{bmatrix} 6031099.876 \\ 6540722.588 \\ 6031100.477 \\ 6540721.095 \\ 6031099.422 \\ 6540721.551 \\ 6031098.392 \\ 6540721.991 \\ 6031098.961 \\ 6540720.513 \end{bmatrix}$$

Figure 5 shows the determined coordinates of the GNSS receiver antennas on the mobile measurement

platform for a 60-second section of the surveying loop.

In order to assess the accuracy of the measurements being carried out, a covariance matrix $\hat{\mathbf{C}}_{\hat{\mathbf{x}}_R}$ determined in accordance with equation (9) was used. At 00:14:11, the covariance matrix took the form (11).

Using equation (10), the mean error of the determined antenna point coordinates can be obtained using the following equation:

$$m_{R_i} = \sqrt{m_{X_{R_i}}^2 + m_{Y_{R_i}}^2} \quad (12)$$

Using equation (10) and the determined matrix $\hat{\mathbf{C}}_{\hat{\mathbf{x}}_R}$, the accuracy of measurements being carried out at 00:14:11 for particular GNSS receiver antennas $m_{R1} = 0.003$ m, $m_{R2} = 0.004$ m, $m_{R3} = 0.004$ m, $m_{R4} = 0.003$ m, and $m_{R5} = 0.004$ m can be determined. For the entire measurement section considered in this article, the mean errors fell within the range of $m_{R_i} \in [0.0004, 0.0053]$ (m).



Figure 5. Results of the adjustment of the coordinates of GNSS receiver antennas on a selected section of the tram loop in Gdańsk

$$\hat{\mathbf{C}}_{\hat{\mathbf{x}}_R} = \begin{bmatrix} 0.0000046 & -0.0000003 & 0.0000002 & -0.0000006 & 0.0000001 & 0.0000004 & 0.000001 & 0.0000005 & 0 & 0.0000001 \\ -0.0000003 & 0.0000005 & -0.0000004 & 0.0000014 & 0.0000004 & 0.0000011 & 0.0000004 & 0.0000002 & 0.0000001 & 0.0000002 \\ 0.0000002 & -0.0000004 & 0.0000077 & 0.0000003 & 0 & 0.0000001 & 0 & 0 & 0.0000014 & 0.0000005 \\ -0.0000006 & 0.0000014 & 0.0000003 & 0.0000088 & 0.0000001 & 0.0000003 & 0 & 0 & 0.0000007 & 0.0000003 \\ 0.0000001 & 0.0000004 & 0 & 0.0000001 & 0.0000076 & -0.000001 & 0 & 0.0000001 & 0.0000002 & 0.0000004 \\ 0.0000004 & 0.0000011 & 0.0000001 & 0.0000003 & -0.000001 & 0.0000066 & 0.0000001 & 0.0000003 & 0.0000005 & 0.0000012 \\ 0.0000001 & 0.0000004 & 0 & 0 & 0 & 0.0000001 & 0.0000052 & 0 & 0.0000002 & -0.0000004 \\ 0.0000005 & 0.0000002 & 0 & 0 & 0.0000001 & 0.0000003 & 0 & 0.0000063 & -0.0000006 & 0.0000014 \\ 0 & 0.0000001 & 0.0000014 & 0.0000007 & 0.0000002 & 0.0000005 & 0.0000002 & -0.0000006 & 0.0000065 & -0.0000002 \\ 0.0000001 & 0.0000002 & 0.0000005 & 0.0000003 & 0.0000004 & 0.0000012 & -0.0000004 & 0.0000014 & -0.0000002 & 0.0000064 \end{bmatrix} \quad (11)$$



Figure 6. Adjustment positions of five GNSS antennas on the first MPP

Figure 6 shows results of adjusting the coordinates of the GNSS receiver position on the first MPP using the correction data from GPS+GLONASS+

GALILEO on the tram loop described in section *Surveying campaign*. 2313 measuring intervals were obtained for the 5 GNSS receivers ($n = 11,565$).

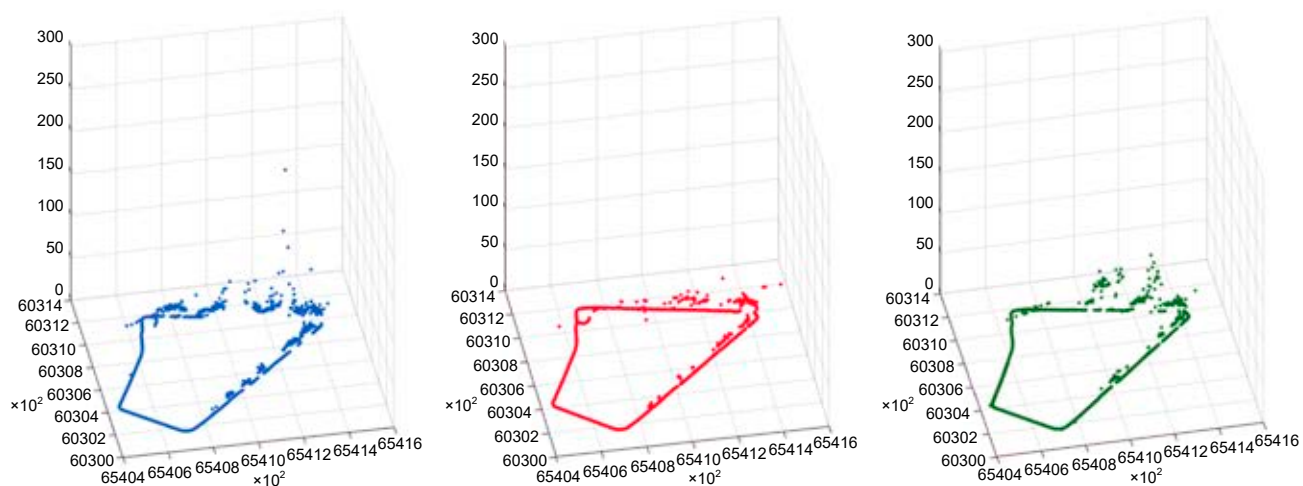


Figure 7. Distribution of average errors of the adjustment positions of antenna No. 5 on the tram loop (blue – using correction data from the GPS, red – using correction data from the GPS + GLONASS, green – using correction data from the GPS + GLONASS + GALILEO)

Figure 7 shows the mean error distribution when determining the position of the middle antenna on platform No. 1 (Figure 4) over the entire test tram loop depending on the type of correction.

Conclusions

This article proposes a non-standard application of the parametric method for adjusting observations in classical land surveying to process satellite survey data in mobile surveying campaigns. The constant development of satellite techniques, supported by the data processing methods used in land surveying, introduces a new type of mobile surveys that can be used in many research areas and for the practical implementation of technical tasks that require highly accurate coordinate determination.

Analysis of the values of mean errors of antenna positions on the measurement platform (Table 3) and the mean error values obtained based on the covariance matrix $\hat{C}_{\hat{X}_R}$, allowed it to be concluded that the accuracy of determinations was increased, which shows that the applied method met expectations. The method was as accurate as the collected measurement data allowed. Depending on the place of registration, the data may be more or less accurate, which is confirmed by Figure 6. In the northern part of the test tram loop, there is a densely built-up area, while the southern and eastern areas are exposed.

Analyzing the error distributions in Figure 7 allows it to be concluded that the most accurate data was obtained using the GPS + GLONASS correction data. Analyzing the use of measurement data as a function of the type of correction data will be the subject of future research.

In order to verify the validity of the adopted theoretical assumptions, we used the recordings obtained during the surveying campaign. The possibility of using the proposed adjustment method must be confirmed using data from other surveying campaigns, which will be the subject of research in the subsequent stages of project implementation. Moreover, the team will attempt to adapt other observation adjustment methods used in typical land surveying operations. The effects of the subsequent stages of work under this project will be the subject of subsequent publications to present the effects of research work obtained as part of project No. POIR.04.01.01-00-0017/17.

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