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## **GENERAL DIRECTIONS IN CONTACT MODELLING DEVELOPMENT**

This paper presents a survey of works, selected from the period of the last twenty years, on deformations in the contact between rough surfaces. All the selected works use FEM. They deal with the modelling of individual contact asperities or the use of experiment to verify contact models. First, research directions connected with the modelling of single asperities, whose shape is usually approximated with that of a hemisphere or a half cylinder, are presented. Section 3 discusses research directions concerning models which include the layer under asperities, and models for small contact surfaces (about 1 mm<sup>2</sup>). Section 4 reviews directions in contact modelling which takes into account neighbouring asperities and laterally loaded asperities. Section 5 discusses directions in the development of models and experiments used or suitable for verifying models. Finally, conclusions concerning accurate contact deformation modelling are presented.

### **1. INTRODUCTION**

As part of the research into the phenomena which occur between contacting rough surfaces attempts are made to develop effective methods of modelling and analysing the deformation taking place between such surfaces. Because of the complicated geometry of their asperities the surfaces are difficult to analyse and describe, as evidenced by the small number of studies on real asperities, mentioned in the overview articles by Liu et al. [1], Adams and Nosonovsky [2] and Ghaednia et al. [3]. The deformation analyses presented in the studies are based on simple asperity shapes and include complex relations between the physical quantities. Since the deformations are difficult to describe analytically, numerical modelling based on the finite element method (FEM) can be very useful for this purpose.

In the last twenty years FE modelling has developed on different levels of detail depending on the needs, the computing system capacity and the hardware computational power. This paper discusses the directions in the development of the modelling of the

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contact between rough surfaces. The discussion is based on selected FEM studies of the deformations taking place in the contact between two rough bodies. Individual analytical solutions of the contact problem in practical applications resulted in the creation (by means of FEM) of 3D models for complicated strongly anisotropic surfaces. Analytical and FE contact models for individual applications differ in their assumptions due to, among other things, the shape of the asperities or the surface roughness profile. Generally, through calculations based on such models one can determine the dependence between the displacement and the contact surface and the displacement-force dependence. Using the dependences one can compare, check and verify models.

According to the generally available literature on the subject, most models are verified by means of other models and much less often by experiment as in paper [4] by Jamari and Schipper. The contact is difficult to observe since its changes under loading are “hidden” from external view. One can expect the number of experimental studies to increase, in comparison with model studies, when a breakthrough in ways of measuring and observing hidden areas happens. Currently, researchers developing various contact models use both analytical and FEM solutions almost at the same time in their analyses. Generally, in each proposed contact model one can see its uniqueness, partial universality and suitability for the specific problem being solved. Immediately, however, the question suggests itself why the model lacks any experimental support. The need for the latter increases with contact complexity. Regrettably, because of the differences between models it is difficult to compare them unequivocally and create universal models.

The experimental and theoretical research conducted by Makodonski [5], Maciolka [6], and Goerke and Willner [7] meets the accurate contact modelling needs and aims at explaining the phenomena occurring during the deformation of individual asperities. It also leads to the refinement of the assumptions for modelling highly significant deformations under loading. The deformations have a significant effect on the precision of the mutual positioning of elements. The investigations described in Ito [8], indicate that contact deformations affect the mutual position of elements and the accuracy of the (e.g. machining) processes being realized.

It is observed that the development of different contact models is connected with the development of FEM. The latter development is based on the choice of appropriate finite elements, their arrangement and density and checking what effect material characteristics, boundary conditions and the geometry have on the results of the numerical simulation. This direction in modelling development is intertwined with other directions and occurs simultaneously with them, as is the case in, e.g., parametric studies of contact properties. The multidirectionality in the development of contact research is something natural and generally follows from application-specific contact investigations. It can be noticed that contact models using the sphere as the shape of asperities are developed less intensively than 3D models of asperities with complicated shapes. It is interesting to note that newer models do not replace or completely supersede the previous application-specific models.

In order to explain more broadly the essence of the development of modelling based on the phenomena occurring in the contact between individual asperities, selected important cases of modelling are discussed in the further part of this paper.

## 2. MODELLING OF INDIVIDUAL ASPERITIES

The contact between individual asperities is difficult to describe for their actual shapes (Fig. 1). The simplest geometry which can be used for this purpose is a circle, a sphere, a spherical surface or a cylinder. It is quite common to replace the contact between individual asperities with the contact between two spheres (Fig. 1).

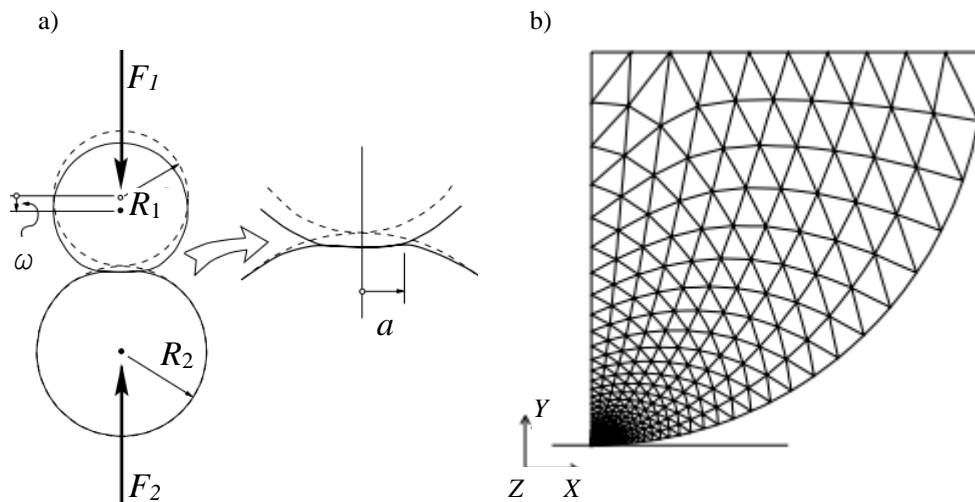


Fig. 1. Hertzian contact model: a) contact between two spheres, b) model of quarter sphere in contact with flat surface [10]

The mathematical functions, including the constitutive equations, used to describe the behaviour of such a contact come from Hertz theory [9]. Using them one can calculate the pressure in the contact, the deformation of the contact and the contact area, depending on the external load (normal force) applied to the contacting spheres, the radius of curvature of the latter and the elastic modulus of their material. Figure 1 shows that each of the spheres is symmetric relative to the loading direction. Consequently, the very small and flat contact surface is also symmetric relative to the loading direction. Hence the FE model representing such a case can be a quarter sphere contacting a flat surface, as shown by Vu-Quoc and Zhang [10]. Another simplification found in the literature on the subject consists in replacing the contact between two spheres with one sphere and a flat surface. This makes it necessary to determine the equivalent material properties (Young's modulus) and geometry (the equivalent radius). Such simplifications and appropriate boundary conditions are a compromise between model quality and computation time.

The constitutive Hertz equations are intelligible and readily used in the plastic strain range in practice. The equations are the starting point for many analyses and continue to be developed, considering the need for their use in many fields of science, e.g. when the strain range is to be extended to cover elastic-plastic strains. Such expanded and checked relations are used to verify more complex models of, e.g., rough surfaces with numerous asperities, in which individual asperities are approximated with spheres or hemispheres, depending on the spatial dimension of the problem.

Contact modelling assumptions quite often concern the strain range, but one can also assume contact surface propagation, load magnitude and its multiplication, load direction, material surface layer thickness, surface layer and bulk material physical characteristics, surface roughness and specimen type.

When using Hertz theory-based models with different assumptions one should bear in mind that the models also differ in their load-displacement characteristics. The use of characteristics, especially the ones based on constitutive equations, is correct exclusively for specific cases, which may differ significantly. Figure 2 shows exemplary calculation results for the contact between two spheres for respectively a material with elastic-range strains (the Hertz model) and a material with elastic-plastic strains (the FE model). The displacement determined using the respective models differs by about 3  $\mu\text{m}$  for the load of 1.5 kN.

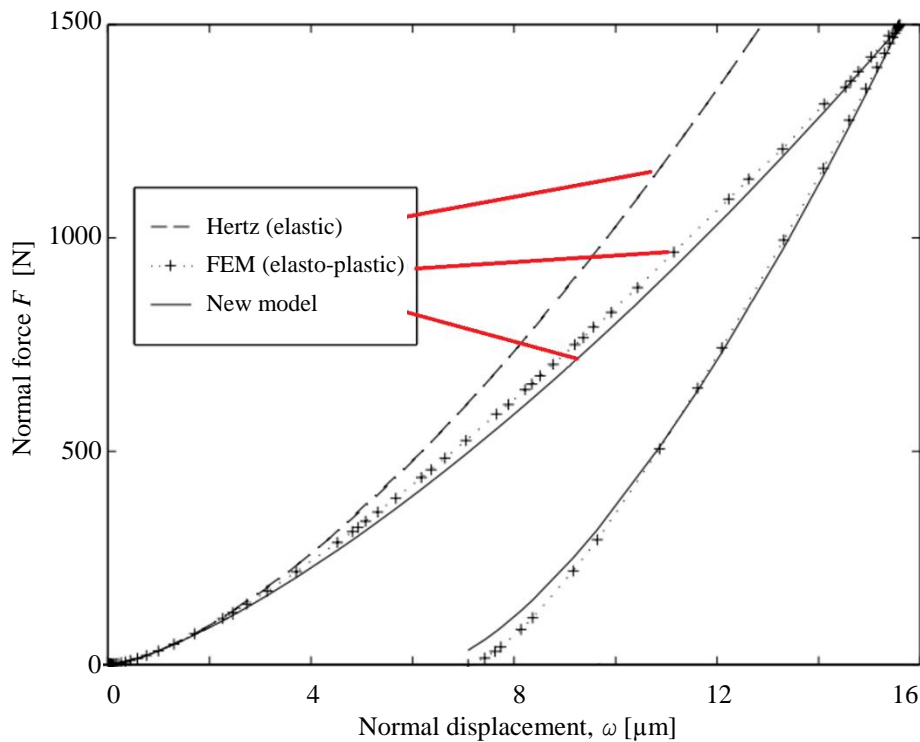


Fig. 2. Load-displacement characteristics of contact between two spheres made of respectively elastic material and elastic-plastic material [10]

Besides using the shape of a sphere or a hemisphere, one can use the shapes of other geometrical figures. But then the analyses become more complicated. The work by Néder and Váradi [11] deals with such analyses and the technique of approximating the shape of asperities. The authors used a special procedure to identify the actual shapes of the asperities, selected the highest asperities and using hemispherical, ellipsoidal and parabolic shapes created substitutes of the asperities (Fig. 3).

In the case of each of the substitutes, the location of the actual contact surface and the distribution of pressure in the contact were determined and compared with the results for

the measured original surfaces. Even though no FEM was used in the analyses, a very high convergence was obtained using the shape of a paraboloid (much less popular than that of a hemisphere).

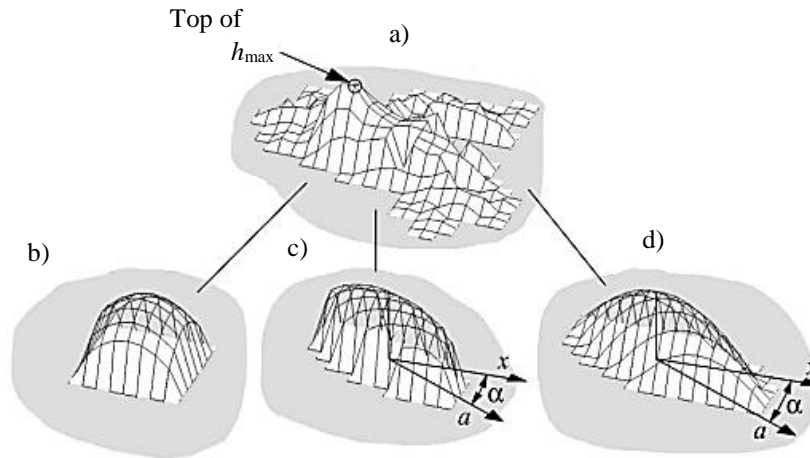


Fig. 3. Shape of actual asperity (a), its approximation with hemisphere (b), ellipsoid (c) and parabola (d) [11]

Instead of a surface with many individual asperities (Fig. 4a), one can use a homogenous layer (Fig. 4b) whose stiffness corresponds to the actual stiffness of the layer with asperities. Figure 4 shows a hemisphere in contact with a rough surface (Fig. 4a) with maximum asperity  $h_{max}$ , which has been replaced with a homogenous layer with thickness  $h_{surface}$  and equivalent elasticity  $E_{surface}$ , dependent on strain  $\varepsilon$ , placed on a material characterized by different elasticity  $E_{bulk}$  (Fig. 4 b).

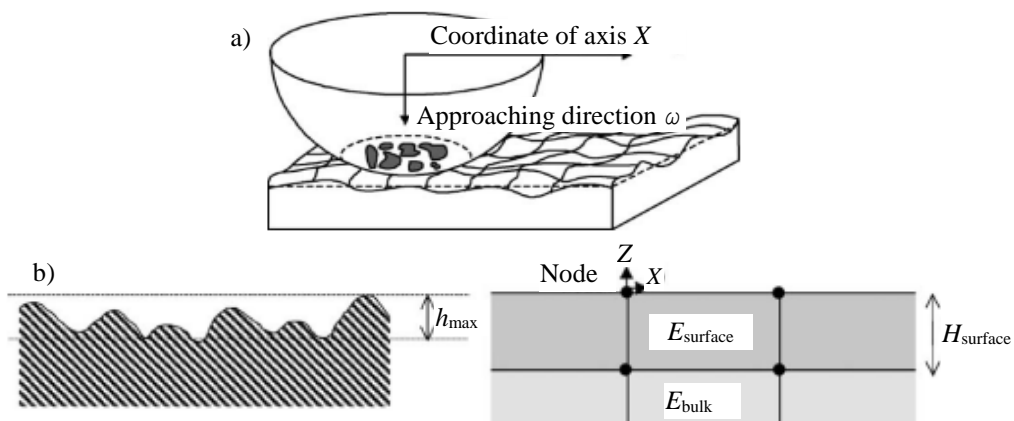


Fig. 4. Hemisphere in contact with rough surface (a), layered modelling of roughness profile (b) [12]

Using the above approach to contact modelling one must determine the thickness and characteristics of the material of such a homogenous contact layer. In Sellgren et al. [12] the authors presented a method of determining the elasticity of a homogenous material layer representing the rough layer of asperities (Fig. 5).

For this purpose the authors used the dependence between the ratio of the actual contact surface to the nominal surface ( $A_r/A_n$ ) and the asperity height coordinate (Fig. 5a). The ratio was assumed to be equal to the ratio of the elasticity of the surface to the elasticity of the material bulk ( $E_{\text{surface}}/E_{\text{bulk}}$ ), depending on strain  $\varepsilon$  (Fig. 5b). As a result, strain  $\varepsilon$  changed from zero to (overall)  $\varepsilon_c$  at which actual surface  $A_r$  is equal to nominal surface  $A_n$ .

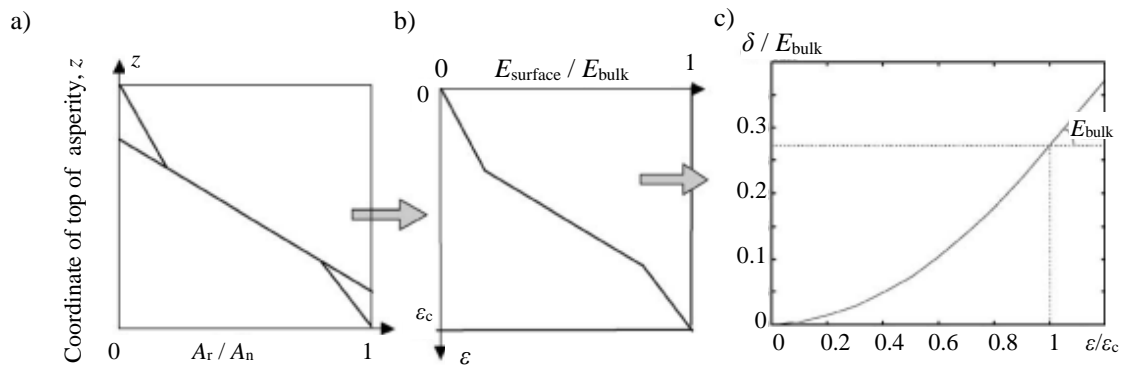


Fig. 5. Method of determining elasticity of rough layer of asperities [12]

In this range the elasticity of the surface is nonlinear and further deformation above  $\varepsilon_c$  is possible when bulk elasticity  $E_{\text{bulk}}$  is engaged (Fig. 5c). What is missing in this work is a study of thickness selection for such a homogenous layer, which could provide other researchers with valuable clues. The paper also presents a division of models according to the degree of detail of rough surface modelling and their behaviour. According to this division, modelling can be considered at three different levels: a highly detailed model with the actual geometry of the asperities, a less detailed topographic layer with a nonlinear constitutive model, and an abstract model with a specified nonlinear contact thickness. The first of the above levels, i.e. the highly detailed one with the actual geometry of the asperities, is the most suitable for the contact model developed by the present authors Maciolka and Jędrzejewski [13].

In comparison with less detailed models, a certain difficulty in FE analyses performed using models of individual asperities approximated, e.g., by a hemisphere is posed by the modelling of the fixing conditions (boundary conditions). This is due to material inhomogeneity which limits the application of half or quarter models to a symmetrical geometry. Hence the advantage of less detailed models is that owing to the low ratio of deformation to the thickness of the layer representing asperities it is easier to select proper boundary conditions and as the layer is being loaded, its geometry along the whole height as well as along the force direction and the transverse direction is proportionally deformed by a constant value. In the case of models of individual asperities, the ratio of deformation to asperity height is high. Consequently, the maximum loading of the hemisphere and the boundary conditions significantly affect the accuracy of the obtained results. Figure 6 shows the course of the deformation of a hemispherical asperity under very heavy loading at a constant asperity volume.

The geometry of an asperity and its change during loading are significant factors in the analysis and if they are not sufficiently accurately represented, this can result in

calculation errors. Jackson and Green [14] observed that material hardness changes, depending on the evolving contact geometry, with the ratio of the contact surface radius to the hemisphere radius ( $a/R$ ). Therefore for theoretical analysis purposes they determined three ranges of ratio  $a/R$ , as stages in contact geometry evolution. One range for a very small contact when  $a/R$  is close to 0 and the limit ratio of the average pressure to the yield point is  $H_G/Re = 3$ , another (intermediate) range for the contact between  $0 < a/R < 1$  when  $3 > H_G/Re > 1$  and a third range for a very large contact when ratio  $a/R$  is close 1 and  $H_G/Re = 1$  (a case similar as for a member in compression).

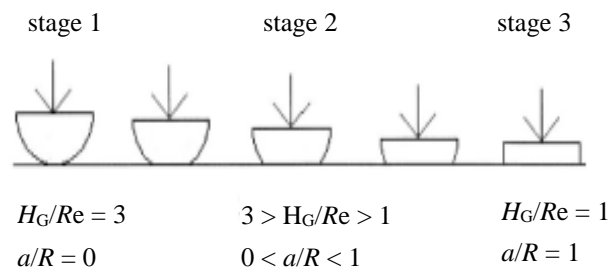


Fig. 6. Stages in deformation of hemisphere model at constant asperity volume [14]

In their investigations Jackson and Green [14] focused on contact analysis by means of a real rough surface model. The model had real material characteristics, i.e. Young's modulus, yield strength ( $Re$ ) and Poisson's ratio, assigned to asperities approximated with a hemisphere. The model did not cover the layer under the asperities and the displacements of the bottom of the asperities were strongly constrained in all directions. Such assumptions are a great simplification since in reality the material situated under the asperities transfers stresses, whereby the asperities interact. Besides, the derived empirical equations were verified with FE modelling results instead of experimental results which are a prerequisite for high reality mapping accuracy. Despite this, the work is interesting, abounding in analyses in which normalized results are proposed for both macrocontacts (e.g. rolling bearings) and microcontacts (e.g. the contact between surface asperities for positioning some machine bodies).

### 3. MODELS WITH LAYER UNDER ASPERITIES AND SMALL ASPERITIES INCLUDING SURFACE

Many studies do not take into account the material under asperities and the actual shape of the latter. This is done on purpose to simplify the beginning of the contact modelling process, though researchers rather aim at developing an accurate model. It is more correct and closer to reality to model asperities on a material layer thicker than they are. Figure 7 shows a 3D model taking into account the layer of asperities and the material below which also undergoes deformation under loading. In this case, the influence of the boundary

conditions on the results is reduced, but another problem, connected with the layer's material characteristics and its thickness, arises.

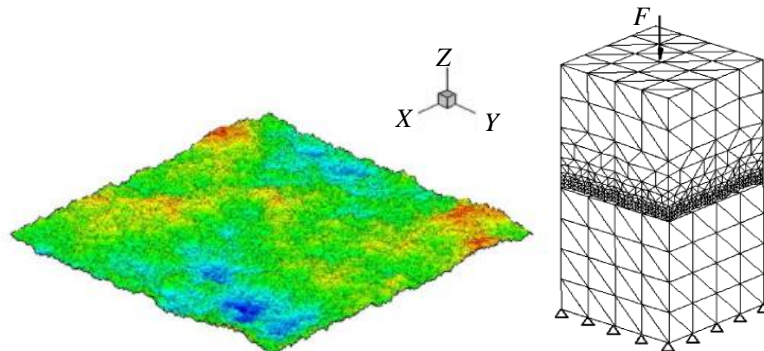


Fig. 7. 3D model taking into account layer of asperities and material below which undergoes deformation [15]

The paper by Pei et al. [15] deals with contact analyses taking into account the rough surface, consisting of numerous numerically generated asperities, and the subsurface. The 3D model presented there was created especially for parametric analyses in which the material under the asperities was considered and the effect of the material characteristics on the contact's parameters, i.e. its surface area and pressure, was determined. The conventional model of isotropic plasticity and the isotropic hardening law were used and plasticity  $J_2$  (the second deviatoric stress invariant) was assumed. The properties of the material were the same along the whole height of the modelled layer and the numerically generated surface roughness characteristics applied to polished surfaces. Even though the paper does not cover experimental investigations, the parametric studies showed general relationships between the properties of the contact and the key parameters of the material. In comparison with experiments designed for a similar purpose, the advantage of parametric studies is that many parameters are analysed separately or jointly at no excessive cost.

Also Stupkiewicz and Sadowski [16] presented a 3D contact model which took asperities and the material under them into account (Fig. 8). The model was used to investigate the contact, whose surface area amounted to  $1 \text{ mm}^2$ , between steel specimens. Such a small surface area facilitates analyses as regards the creation of a detailed FE model and its experimental verification. It was assumed that an experiment consisting in pressing three rams into one specimen could be compared to a model in which one ram was pushed into a single specimen. This means that the model was not fully a representation of the experiment. The significance of asperities and the effect of their shape on the force and displacement measurement results were disregarded. It is not mentioned whether the shape of the asperities was the same in both cases and if so, to what degree this shape affected the research results and what else and how affected the results. However, it is known that sandblasted surfaces, where numerous asperities occur, were investigated and perhaps this was the reason for neglecting the asperity shape problem.

The comparison of the contact pressure versus approach, obtained from respectively the model and the experiment for the load of over 600 MPa (Fig. 8) deserves attention. In the diagram one can notice discrepancies between the experimental results and the



calculation results, which were attributed to macroscopic deformations. But there is no information about the influence of the test stand or the measuring system on the results. Additional analyses on the basis of which one could draw more detailed conclusions are missing.

Similarly as in the above paper, Yeo et al. [17] as part of parametric studies verified model results with experimental results, but this time a 2D model was used (Fig. 9a). However, the differences were so large that no satisfactory convergence was obtained. Fig. 9a shows the 2D model which takes into account the layer under one asperity. The model represents a superficially hardened material, i.e. hard asperities on a soft substrate.

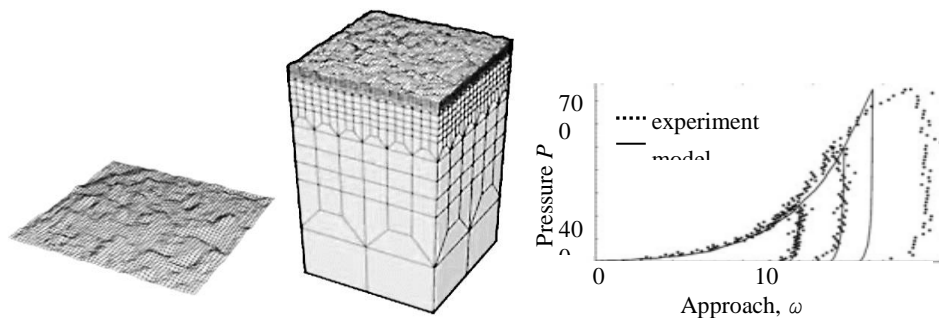


Fig. 8. 3D model of rough surface, comparison of model and experimental contact pressure results depending on approach [16]

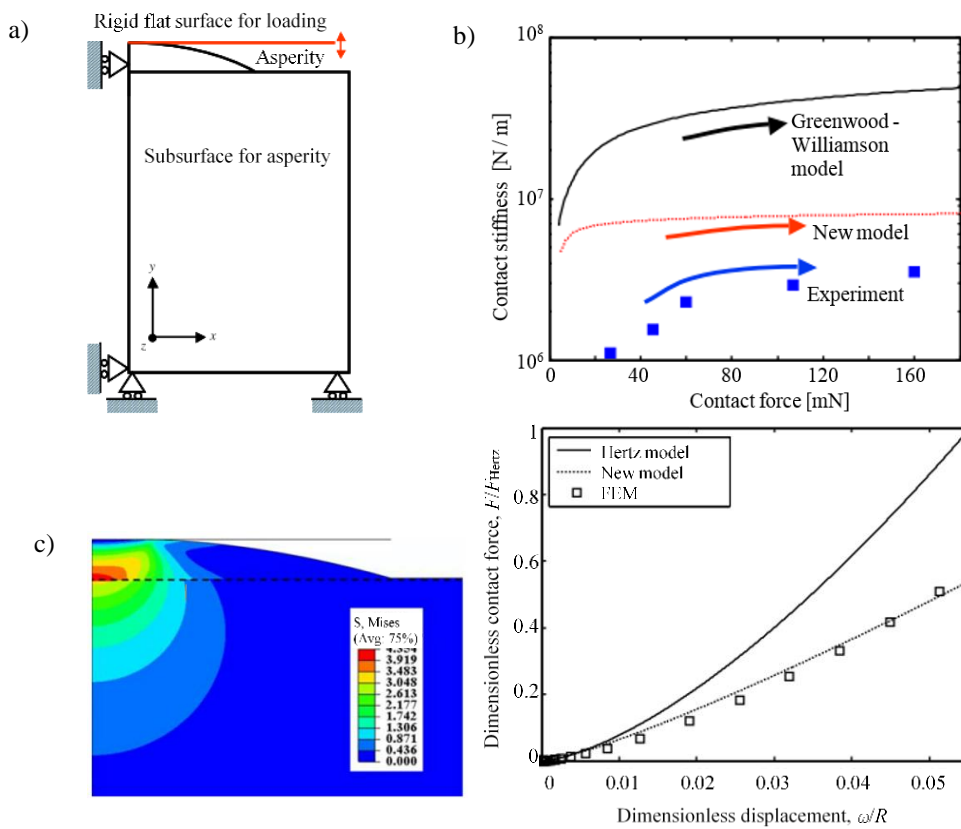


Fig. 9. 2D Polycarpou model of asperities with subsurface layer (a), comparison of contact stiffness with results obtained by Greenwood-Williamson (b), comparison of normalized force-displacement dependence with results obtained by Hertz (c) [17]

It is valuable that owing to the investigations the range of validity of the proposed model was determined and the model was compared with the Greenwood-Williamson model [18] (Fig. 9b) and the Hertzian contact model [9] (Fig. 9c). What is missing in this study is an analysis taking into account neighbouring asperities and their shape, which could explain the causes of the discrepancy between the calculated stiffness and the measured one, which can be seen in Fig. 9b.

The common feature of the two models shown in Fig. 9b is the presence of numerous asperities in the contact. The asperities were described using statistical methods. The use of a proper distribution of asperities and other roughness parameters, i.e. skewness and load capacity, is a challenge for further research. Statistical parameters are highly practical and they are used to generate synthetic maps of rough surfaces for deformation models.

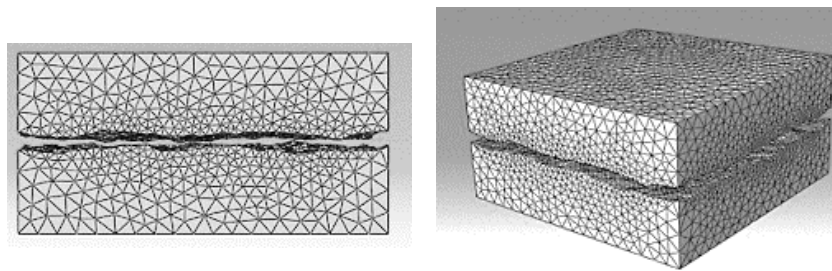


Fig. 10. Model of contact between numerically generated surfaces [19]

Ardito et al. [19] focused on the analysis of a contact with adhesion for a 3D rough surface (Fig. 10) with asperities distributed according to the Gaussian distribution. The authors emphasize that when creating a model of the contact between rough surfaces “It is necessary to include the substrate until a critical depth, after which the mechanical effects are negligible, because of the dependence of the adhesive phenomena on the deformation of the surfaces”. But they do not give any methods of determining the thickness of this layer. However, they mention the problem of and the need for increasing computational power. The experimental verification of the results could be the direction in which such analysis should develop.

It is equally important to take the actual material characteristics and the parameters of the asperities and the subsurface layer into account when creating a contact model.

Sahoo et al. [20] took the bilinear property of the material (Fig. 11a) of hemispherical asperities into account in their model and the model results were compared (Figs 11b and c) with the ones yielded by the model described in Kogut-Etsion [21]. The elastic model developed by Sahoo was used to test model quality in the ANSYS 10.0 environment. The proposed model was described in detail and the effect of mesh density on the results of calculations of, i.a., the displacements, the force and the contact surface area, was checked. Ultimately, the FE model had the form of a quarter sphere in contact with a rigid flat surface. The FE model consisted of 12986 PLANE82 elements and 112 ONTA172 elements. Various hardening of the material with tangent elasticity modulus  $E_t$  ranging from 0 to 0.33 (Fig. 11c), including the case when the material was perfectly elastic-plastic (Fig. 11b,  $E_t = 0$ , the Kogut-Etsion model), was used. The differences between the results

yielded by this model and the ones obtained from the Kogut-Etsion model, visible in Fig. 11b, amount to maximally 3%. This was ascribed to the radius size of  $0.1 \leq R \leq 10$  mm, which in the Kogut-Etsion model is much larger than in the model proposed by Sahoo (0.01 mm).

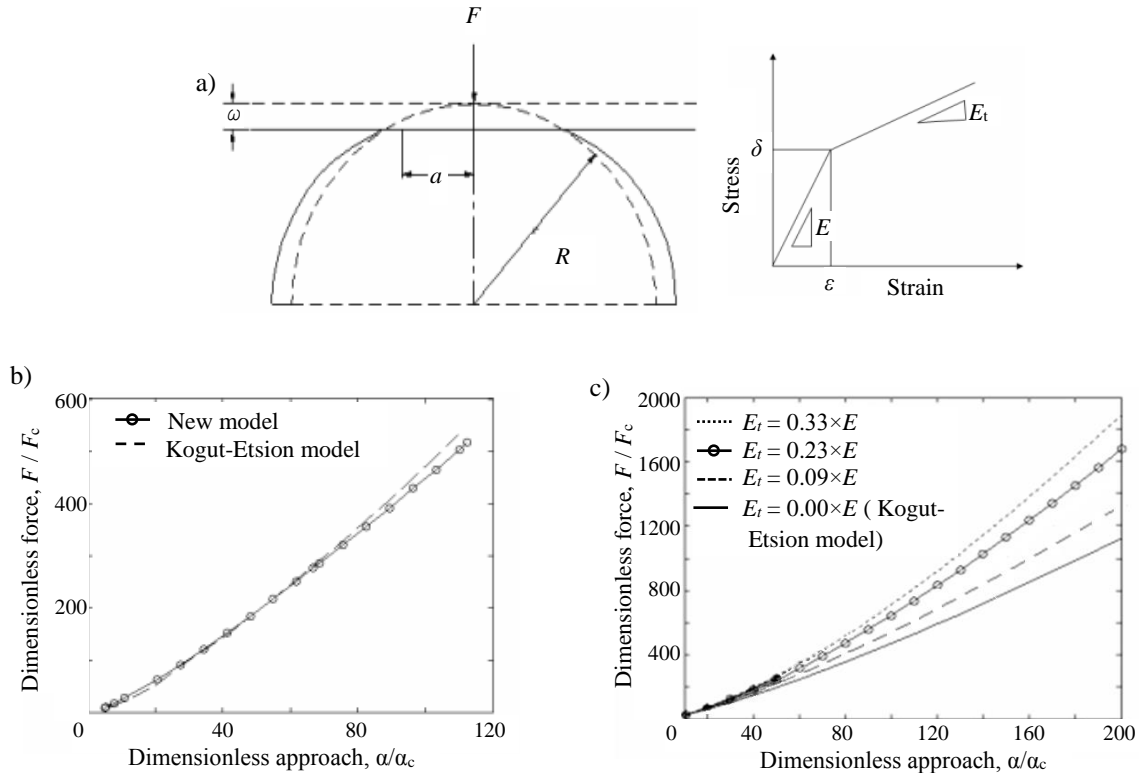


Fig. 11. Model of contact with (bilinear) property of asperities layer material different than that of substrate (a), comparison of Kogut-Etsion model’s dimensionless force versus dimensionless approach with results yielded by model with perfectly elastic-plastic material (b), and cases with different elasticity modulus  $E_t$  (c) [20]

One can expect that the development of the research aimed at taking into account the two properties of contact layer material will also cover the size and shape of its asperities.

#### 4. MODELLING INTERACTION BETWEEN NEIGHBOURING ASPERITIES IN CONTACT ENVIRONMENT AND MODELS WITH LATERAL LOADING OF ASPERITIES

The analyses presented so far did not take into account the interactions between asperities. This is another important factor which needs to be included in contact layer deformation analyses. Although when discussing some of the models described earlier the interactions between asperities were mentioned, they were not properly analysed. Bryant et al. [22] using a 2D contact model analysed the effect of neighbouring asperities on the contact layer’s deformations and its stress field. Three models of the contact between a plane and respectively a cylinder, a regular sinusoid and the actual profile of a surface

obtained after lapping, with truncated tops of the asperities (Fig. 12), were created. In other words, the tops of the asperities of the surface after lapping looked similarly as after preloading. In practice this means that if appropriate local loads are not exceeded, it is possible to load such a surface in the elastic strain range.

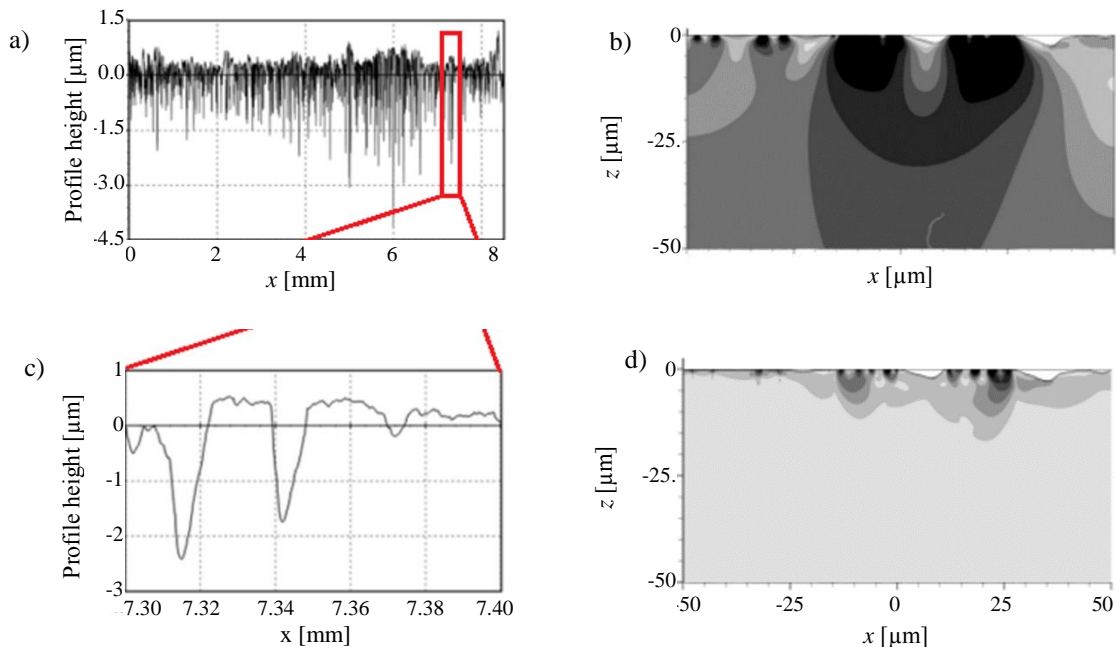


Fig. 12. Actual roughness profile of surface after lapping (a), map of its von Mises stresses during loading (b), sample taken for analysis (c), residual stresses after unloading (d) [22]

All the considered models of the contact between the plane and the cylinder, the sinusoid and the actual surface profile, respectively, were verified in the course of modelling. Regrettably, the final model with the actual rough surface was not verified. However, it is likely that this will be done in the next stage of the research. The verification of the final model is important since the model's assumption concerning the use of asperities in the shape of a hemisphere raises doubts, despite the fact the actual surface had been previously deformed by lapping.

Similar investigations as above were conducted by Yastrebov et al. [23] using a three-dimensional model (Figs 13b and d), consisting of a rough surface, described by the sinusoidal shape, and a subsurface layer. The interactions between the asperities were investigated. The 2D model included an asperity situated on the subsurface layer of various thickness:  $1L$ ,  $2L$  and  $4L$  (Fig. 13a). The 3D model was a segment of the surface on which each asperity neighbored four (Fig. 13b) or six other asperities (Fig. 13d). The considered cases of the 3D model differed in their geometry consistently with the repeatable blue areas of the sinusoidal surface with asperities arranged as the red circles (Figs 13c and e). In this way, by means of the 3D model as a segment of the surface together with appropriate assumptions concerning interactions, the layer under the asperities and the different distances between the crests of the asperities were taken into account in the analyses. However, the authors say that “the independence of the interaction remains questionable

and has to be further investigated...”. This continues to be an interesting current direction of research. Moreover, the structures of two other models, including a model with constitutive equations, proposed as an alternative to the FE model to reduce computation time, were presented. FE models were used to analyse the effect of the asperity shape, the mesh density of a single asperity and boundary conditions on the results in 2D and 3D. The work is interesting since it uses extensive knowledge on various models. But it lacks experimental verification, which would confirm the correctness of the analyses.

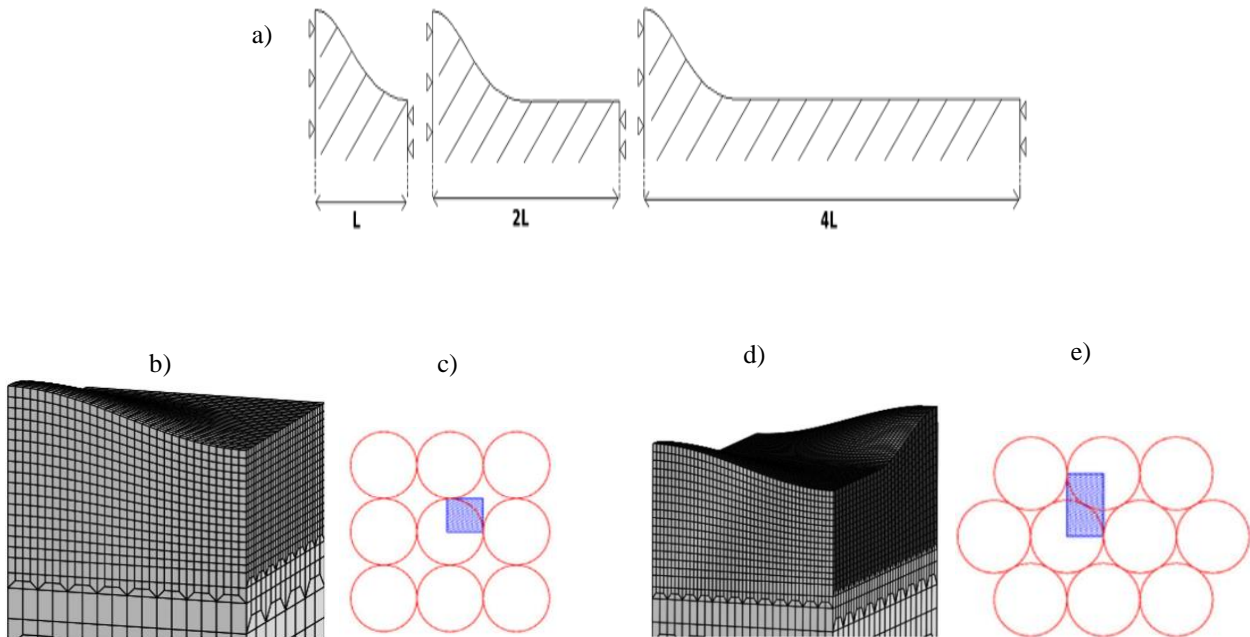


Fig. 13. Models of sinusoidal asperities: a) 2D model without neighbouring asperities, with subsurface layer of different thickness b), c) 3D model of asperities – slice of four contiguous asperities, d), e) 3D model of asperities – slice of six asperities [23]

In most of the studies, asperities, as they are in contact with a flat surface, are considered to be loaded on their crests. But in the case of the contact between two asperities the place of loading is no longer so obvious. Everything depends on the shape of the asperities and the relative position of the asperities' axes passing through their crests. The research devoted to the effect of the lateral loading of asperities on analytical results represents a rather unpopular area of the research on the contact loaded in the normal direction. This a complex problem from the field of tribology and concerns surfaces in motion.

Poulius and Klit [24] considered two cases of contact between two asperities. One of the cases concerns asperities contacting with their crests and the other concerns asperities contacting with their sides. In addition, the effect of the distribution of asperities and that of the kind of their material on the contact parameters was examined using a hard material, a soft material with yield and a combination of the two materials. Regrettably, the asperities had one shape, i.e. a hemisphere with a constant radius. However, owing to this the model could be compared the Greenwood-Williamson model (Fig. 14).

Figure 14 shows that in comparison with the Greenwood-Williamson model [8], the contact surface area values are underestimated while the contact pressures are overestimated. The model's peculiarity is that it takes into account the material under the asperities (i.e. the subsurface layer with the characteristics of the bulk material), but under the assumption that the subsurface layer and the asperities forming the surface layer have the same material properties. In addition, the way of generating a finite element grid for large-scale models is described and approximate computation times are given. When the computations were performed using 10 processors, the computation time amounted to 1–30 h, depending on the model's size and configuration.

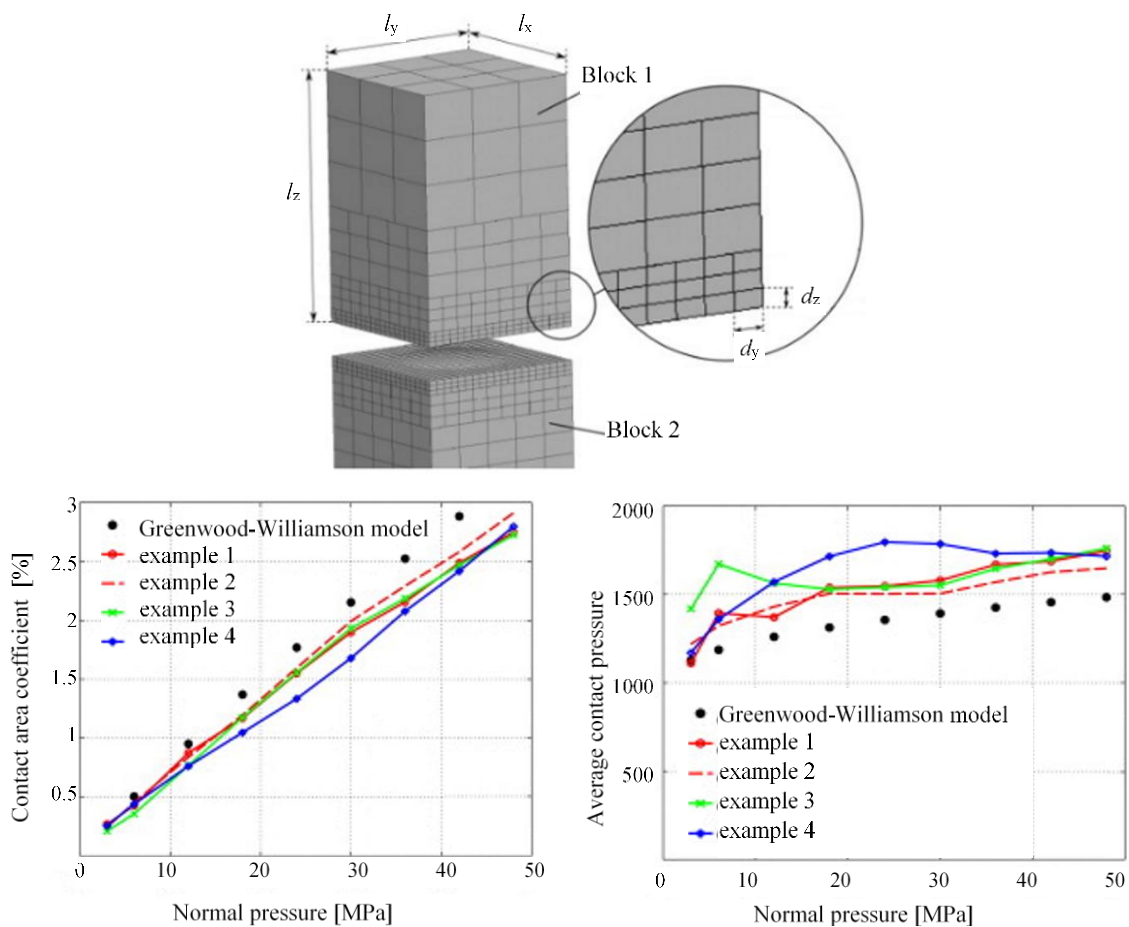


Fig. 14. 3D model with lateral load acting on asperities and analytical results against Greenwood-Williamson model [24]

## 5. CONTACT MODEL VERIFYING EXPERIMENTS

Experiments are rarely used in contact analyses because, as already mentioned, the contact is “hidden” from external view. Experimental examinations are much more difficult than the observations of the phenomena simulated by models. It emerges from the research in which experiments are used that it is much more difficult to measure the surface area of the contact or the deformation of the latter, in the form of separate elastic

and plastic components, than the displacement during the loading of samples with an external force.

Kucharski and Starzyński [25] presented a model of a single asperity and a model with constitutive formulas for the whole surface representing roughness after sandblasting. Three different yield points of the material and a model of a hemisphere with constant radius  $R = 30 \mu\text{m}$  (Fig. 15b) were considered. The way in which the model developed is schematically shown in Fig. 15a, where one can see a chord with ends  $X_1$ – $X_2$ , suggesting the radius to be used in the model shown in Fig. 15b. The model for this surface was verified by an experiment (Fig. 15d) carried out on the stand shown in Fig. 15c.

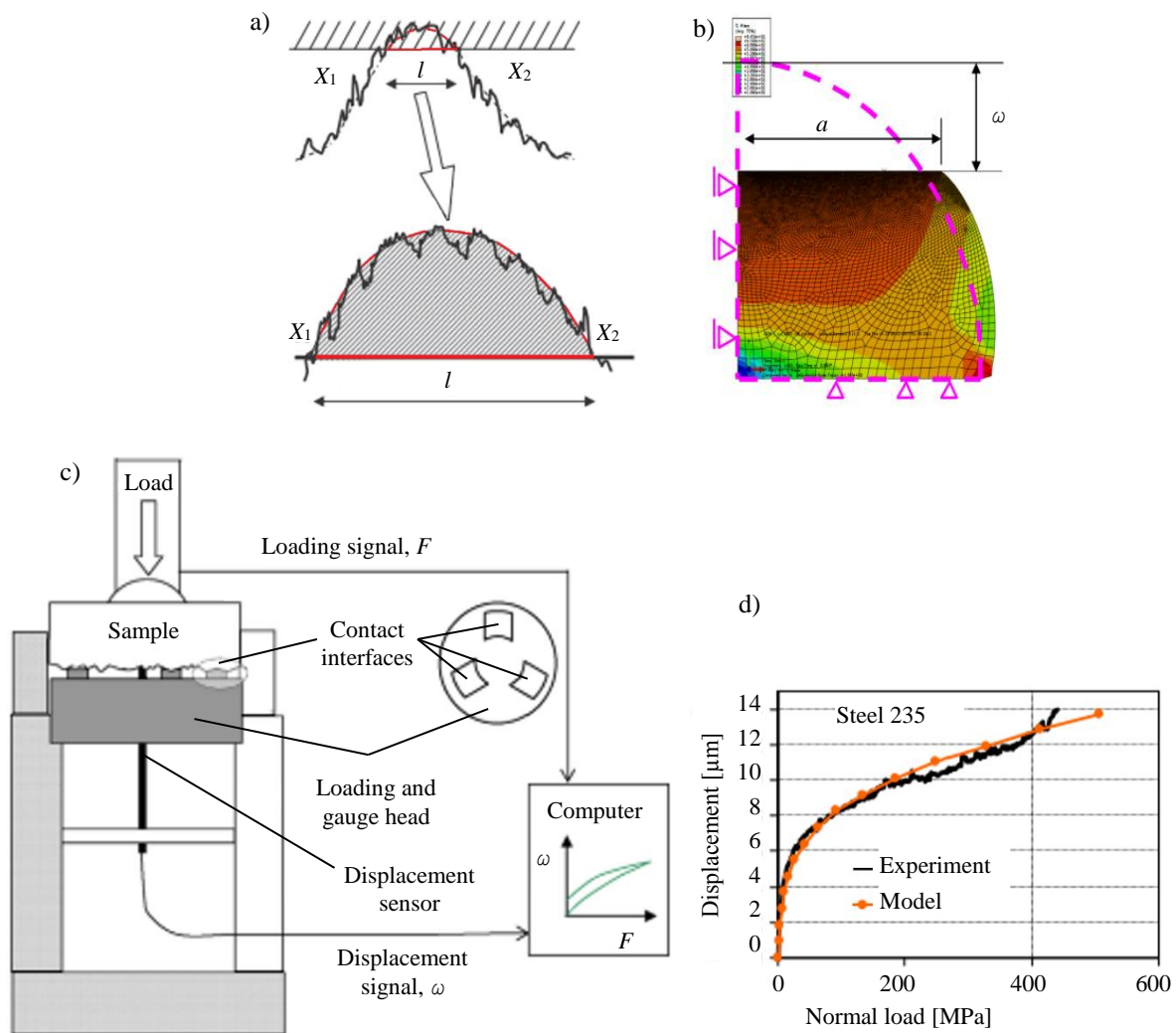


Fig. 15. Model of single asperity in contact with flat surface (a), equivalent model during loading (b), test stand with gauge head with three contact points (c), comparison of modelling and experimental results (d) [25]

Also the problem of the proper number of measurement points for the correct mapping of the asperities on the investigated surface was considered and it was checked how a change of the frequency of profilogram sampling affected the values of the model’s major parameters.

Moreover, it was shown that the adopted assumption was not completely correct since the fixing of the asperity model's lower edge only slightly limited its deformations in the direction perpendicular to the height (dimension  $a$  in Fig. 15b). In reality, the widening (swelling) of the asperity is less intensive due to the presence of the other asperities in its neighbourhood. Unfortunately, the assumption turned out to be oversimplified and probably this was the reason why it was proposed to change this assumption to a more detailed one based on appropriate interactions for neighbouring asperities. Another matter is that the experiment was carried out for three places of contact on a single specimen simultaneously. Consequently, the model load had to be reduced three times so that the results could be compared with the experiment (Fig. 15d).

As part of their studies conducted in the years 2013–2017 Maciolka and Jędrzejewski designed and built a special test stand for investigating contact deformations. The stand was successfully used to model individual asperities and test their stiffness (Maciolka and Jędrzejewski [13]). The observation that new models had been usually verified using the already existing models and less by experiment (which often is difficult to carry out, but very reliable) was taken into account in the investigations. Another observation which was taken into account was that there had been many analyses of the contact with numerous asperities resulting from machining (e.g. sandblasting, grinding and polishing). Such surfaces can be statistically described. But few models had been created for single asperities. The latter cannot be statistically described, but they can provide one with a deep insight into the mechanism of generation of deformations. Therefore the authors undertook research into the deformation of asperities on a small surface, where it is impossible to determine the statistical distribution of asperities with the actual shape, and to investigate asperities situated on the layer being part of the surface layer and the bulk of the material.

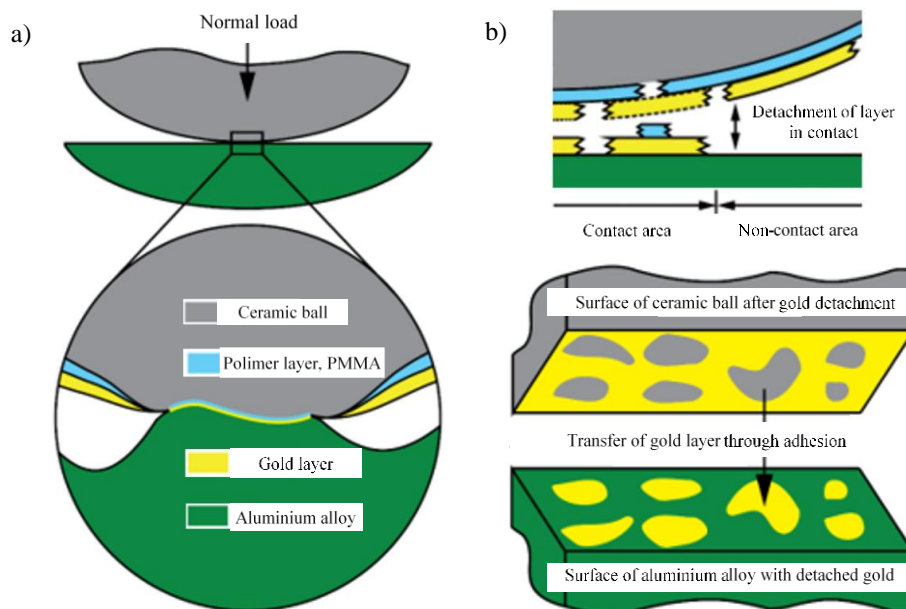


Fig. 16. Method of measuring actual surface of contact between flat sample and ball coated with polymer and gold: a) closeup of contact between layers characterized by different adhesion, i.e. between ceramic and polymer, polymer and gold, and gold and aluminium alloy, b) transfer of gold coating from ceramic ball onto aluminium alloy sample surface [27]



A significant achievement in experimental contact investigations is the work by Xu et al. [27], which presented the measurement of the actual surface of the contact between a ceramic ball and a flat surface (Fig. 16).

The method consists in coating a ceramic ball with polymer and gold layers which as a result of the pressure exerted by the rough surface made of aluminium alloy adhere to the latter in the places of contact. The places of contact between the ball and the rough surface after unloading can be viewed even with the naked eye. The work is interesting and innovative, but the measurement method requires that an additional layer be introduced between the contacting surfaces.

## 6. CONCLUSION

The reviewed works come from the last 20 years and are representative for contact modelling evaluation. The present paper focused on papers devoted to this subject, which contained any FE analyses. On this basis, methods of verifying models depending on the assumptions, the specific contact parameters (e.g. contact surface area, roughness, material, load, sample fixing) and the additional phenomena occurring in the contact, such as adhesion, friction and wear, were considered. So far in contact studies models have been usually verified with other existing models and less frequently by experiment, which often is difficult, but very reliable. Another observation is that a large number of analyses deal with the contact between surfaces with many asperities resulting from machining (e.g. sandblasting, grinding and polishing). Such surfaces can be statistically described. But few models have been developed for single asperities. The latter cannot be statistically described, but they can provide one with a deep insight into the mechanism of generation of deformations. The models in the studies in which an attempt was made to take irregularly shaped individual asperities into account using FEM are devoted to the modelling of the deformation of large surfaces. In most cases, the investigated individual asperities were approximated with the shape of the sphere, the cylinder, the ellipsoid or the sinusoid. The asperities were arranged on a surface in accordance with the statistical distribution. Their properties were elastic-plastic. Whereas, the connection between asperities and the bulk material was usually analysed separately and for thin material layers differing in their properties. Therefore there is a need to study asperities on a small surface, where it is impossible to determine the statistical distribution of asperities having the actual shape, and to investigate asperities situated on a layer being part of the surface layer and the material bulk. Such investigations should explain the complex deformation of asperities and lead to much higher contact modelling accuracy.

It was noticed that the works reviewed contain little information about the effect the material properties (Young's modulus, yield point and compressive strength) of the surface layer and those of the asperities, the subsurface layer and the material bulk on modelling. Moreover, the works under review contain little information about the effect of such factors as the degree of detail of the modelled rough surface geometry, including an elementary asperity, and the use of computer resources, such as the number of processors

and the size of the needed memory. The present authors concluded that the above factors have a significant effect on contact modelling accuracy and decided to take them into account in their research. It is worth noting that so far the number of comprehensive descriptions of all the factors having a bearing on contact conditions is small. The factors interact with one another and are decisive for elementary (actually shaped) asperity model accuracy. This is a challenge for the future research.

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