Article citation info:
MŁYNARSKI S, PILCH R, SMOLNIK M, SZYBKA J. Methodology of network systems reliability assessment on the example of urban transport. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2018; 20 (2): 278–283, http://dx.doi.org/10.17531/ein.2018.2.14.

Stanisław MŁYNARSKI
Robert PILCH
Maksymilian SMOLNIK
Jan SZYBKA

METHODOLOGY OF NETWORK SYSTEMS RELIABILITY ASSESSMENT ON THE EXAMPLE OF URBAN TRANSPORT

METODYKA SZACOWANIA NIEZAWODNOŚCI UKŁADÓW SIECIOWYCH NA PRZYKŁADZIE KOMUNIKACJI MIEJSKIEJ*

Apart from reliability evaluation, the methodology of network systems reliability assessment presented in the article enables the design of modernisation of such systems targeted mainly at ensuring their required reliability. In practice the methodology can be applied for various network systems, e.g. computer, power, gas, water distribution, telecommunications and transport networks. A reliability analysis of a transport network in urban public transport is presented. Calculations were performed for selected criteria of network availability which actually conditions the quality of transport services provided. The basic calculation tool used was the factoring algorithm that enabled the assessment of the impact of individual connections failure (in particular those caused by physical factors) on the reliability of the whole network. The feasibility of modernisation of the network analysed is discussed and the results are presented in diagrams.

Keywords: *transport network, reliability, factoring algorithm.*

W artykule zaprezentowano opracowaną metodykę szacowania niezawodności układów sieciowych. Rozwiązanie to umożliwia dokonywanie oceny niezawodności oraz projektowanie modernizacji rozpatrywanej sieci przede wszystkim w aspekcie zapewnienia jej wymaganej niezawodności. Praktyczne wykorzystanie omawianej metodyki może mieć miejsce w odniesieniu do różnych układów sieciowych, np. sieci komputerowych, energetycznych, gazowych, wodociągowych, telekomunikacyjnych i transportowych. W artykule przedstawiono analizę niezawodności sieci komunikacyjnej w miejskim transporcie zbiorowym. Obliczenia przeprowadzono dla wybranych kryteriów zdatności sieci, które praktycznie warunkują jakość świadczonych usług transportowych. Podstawowe narzędzie obliczeniowe stanowił algorytm faktoryzacji, który umożliwia ocenę wpływu uszkodzeń poszczególnych połączeń (spowodowanych w szczególności czynnikami fizycznymi) na niezawodność całej sieci. W opracowaniu uwzględniono możliwość modernizacji analizowanej sieci, a uzyskane wyniki przedstawiono na wykresach.

Słowa kluczowe: *sieć komunikacyjna, niezawodność, algorytm faktoryzacji.*

1. Introduction

In general terms, the paper deals with issues of network reliability assessment, which due to the specificity of such networks cannot be done with traditional methods used in the theory of reliability. In the paper various types of systems are characterised, a methodology of reliability calculations procedure for network systems is presented. Moreover, using a case of urban transport the usefulness of the factorisation method applied is proved.

The activities of municipal transport companies are closely connected with the management of fixed assets whose components include transportation means and infrastructure, i.e. transport routes and networks. An important element of transportation means use is constituted by the maintenance-repair management, which is an indispensable condition for ensuring the continuity of operation and utilisation of the assets potential [2, 6, 27].

This management covers all the undertakings aimed at continuing the serviceability of transport networks. These undertakings involve the necessity of temporary closure of some lines and negatively affect the effects of company's operation. The aim of companies is to design the structure of connections in such a way that in the case of emergency or pre-planned repair of traction lines the transport system should guarantee users reaching their destination point, (cf. [10, 14, 31]).

Meeting the increasing demands in this area makes it necessary to look for methods of modelling and evaluating transportation systems taking into account emergencies and economic criteria. To achieve this aim the probability calculus is used (cf. [11, 27, 28, 33]). For instance, the probabilistic index of trips irregularity defined as the difference (in minutes) between the real and scheduled time in the conditions of ensuring stable and regular traffic is analysed. This index is affected by the transport network shape. Its favourable configuration is significant for accomplishing the transportation task, (cf. [9, 18]).

To solve this task the factoring algorithm proposed in the paper can be applied [15, 24]. It enables determining the reliability for complex network systems used in municipal transport systems and is an alternative to simulation methods applied in such cases, which, however, are less precise [17, 19, 20].

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl***

2. Characteristics of network systems

Network systems include power supply, IT, water supply, transportation that ensure the operational continuity of a company and product distribution as well as other systems of similar type.

All the above belong to technical infrastructure which covers facilities, transmission networks and related technical objects performing tasks of supply of heat, gas and water, wastewater and waste disposal, transportation, etc. Infrastructure denotes essential facilities and service institutions indispensable for the operation of economy and society. Linear infrastructure in a network system is characterised by the linear connection between the technical means (components) that perform certain functions.

To evaluate network systems block diagram based methods can be used. These can be applied for reliability module structures and extensive transportation systems in urban agglomerations that operate on a large area and are composed of numerous nodes that contain mapped internal subsystems [12].

Network structures are frequently modelled using graphs composed of nodes and sections connecting the nodes. The reliability of structures mapped as graphs can be effectively calculated using factorisation methods [17, 19, 23]. The reliability structures in municipal transport systems can be subject to continuous evolution. This may be because the area covered by the operation may change, or the demand for transportation services may change. The correct identification of real-life structures of technical systems together with the method of calculating reliability indices are of key significance for the proper reliability analysis and assessment of threat to the effective operation of a municipal transport system.

What makes the assessment of network system reliability difficult is the fact that they do not have reliability structure generally considered classic, there may occur interdependencies of failures between parts of the system, and the application of typical reliability indices and characteristics is limited. This is why the reliability assessment of this type of structures can be done using the factorisation algorithm making use of the graph theory and the method of network reduction [4, 23, 24]. The shaping and optimisation of the structure of connections of a network type system as early as the design stage is an important aspect of ensuring its operational reliability [5, 13, 26]. The credibility of network reliability assessment can be improved by taking into account the failures and recoveries in the operational process of all the elements of the network [25].

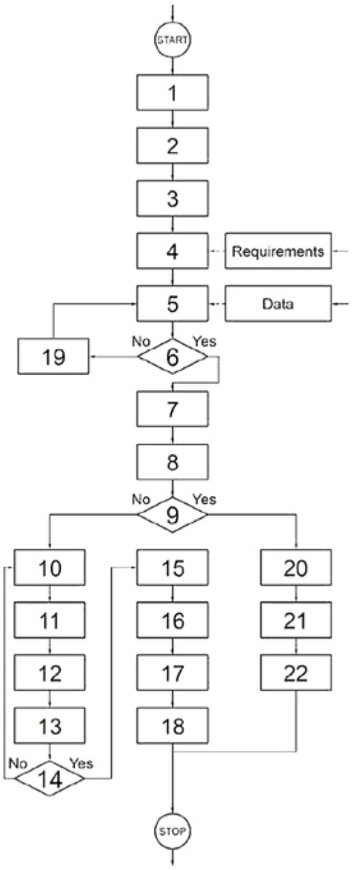
3. Methodology of network systems reliability testing

The assessment of network systems reliability requires the use of mathematical models and methods involved in their application. This activity, however, should be preceded by preliminary works and followed by relevant processing of the obtained results [30]. All the activities that may accompany network system reliability assessment have been systematised in the framework of the network assessment and modernisation method developed. Its scheme is presented in figure 1.

The immediate goal of a network system analysis may be the specification of the indices characterising its reliability, but this should merely support more complex activities of more significant impact. These include, in particular, scheduling of preventive maintenance of the network and preparation of network modernisation (especially by modification of its reliability structure).

Preventive maintenance scheduling is outside the scope of the method proposed, however, it does point out that modernisation projects should be preceded by thorough evaluation of the network system reliability.

As shown in figure 1, reliability assessment requires preliminary steps and precisely formulated requirements as to the correct operation of the network system analysed together with data on its operation and maintenance history so far. When the quality of the available data is not sufficient, equivalent data sources on the operation process can be used, for instance accounting records, records of utilisation of spare parts and materials, etc (cf. [30]).



1. Formulate the aim of analysis.
2. Model the network in question as a system of nodes and links.
3. Name individual nodes and links.
4. Specify the function of the network and criterion of its availability.
5. Verify network reliability data.
6. Is the quality of network reliability data sufficient?
7. Process data on individual nodes and links.
8. Evaluate network reliability according to criterion (cf. 4), using mathematical models.
9. Is network reliability evaluation the sole aim of the analysis (cf. 1)?
10. Identify the "weak links" of the network.
11. Develop network modernisation variant.
12. Adopt expected reliability of network new sections.
13. Assess reliability $R_P(t)$ of modernised network.
14. Is $R_P(t) \geq R_W(t)$?
15. Check the entire solution.
16. Develop relevant technical documentation.
17. Verify documentation accuracy.
18. Approve the entire solution and documentation.
19. Plan and perform relevant operational tests or use data equivalent sources.
20. Process analysis results.
21. Verify analysis accuracy.
22. Save analysis results.

Fig. 1. Procedure of the method of network system reliability assessment and modernisation

The activities and questions shown in the scheme by means of blocks with corresponding numbers (steps 1 to 22) are given in the figure. The presented block diagram describes the procedure in relation to which the reliability analysis of the tram network characterised in Chapter 4 was carried out. The descriptions in the individual blocks shown in figure 1 are general in nature which is a consequence of the universality of the developed method that can be used during various analyses and design works concerning the reliability of network systems.

In the method proposed it is assumed that network will be modernised by introducing changes in its reliability structure until its estimated reliability $R_P(t)$ becomes not less than required $R_W(t)$ – this is done in the iteration loop covering steps from 10 to 14. This method of reliability formation (for technical objects of a different class) was presented in [7, 8], (cf. [29]).

4. An example of performing a reliability analysis of a transport network in urban public transport

The requirements for urban transport services quality are highly dependent on the organisation system, technical means of transport and structure of transport connections network together with the possibility of using it. The quality and reliability of urban public transport are determined by the criteria of high significance for both the user and operator, who must also take into account factors related to the economy of use.

From among the quality and economy criteria in urban transport those of strategic importance include the infrastructure of connections network and vehicles performing the transportation services. The most significant criteria include [1, 3, 16, 22]:

- route availability resulting from the number and distribution of stops,
- spatial and functional integration with other public transport means (national, regional and local, *Park and Ride, Bike and Ride*, etc.),
- direct connections and functional flexibility understood as the possibility of choice of an alternative connection,
- functional reliability whose criteria include: punctuality, regularity, probability of reaching the end point of route.

The transport network structure determines the possibility of connections between nodes, which in turn affect directly the transportation potential of the entire system and the formation of users' opinions of the quality of the transport system. The term of node in a transport network structure denotes the location of tracks intersections and junctions and stops enabling changing to another means of transport, as well as the start and end stops. Rationally arranged infrastructure in the form of effective location of strategic transport nodes offers the possibility to optimise the connections network and the resulting high reliability of the system. What is important for the effectiveness and quality of the entire system is also the high reliability of transport nodes and sections, with particular attention to nodes of strategic significance.

The transport connections structure of an agglomeration can be presented as a graph that shows the functional relations between the components of the system, and analysed in the reliability framework. It can be interpreted as a connections network in urban rail transport between the infrastructure nodes. The transportation process is performed between the initial nodes (depots) and end nodes (tram reversing loops). The other components are the system nodes as partial execution of tasks (tram stops), while the lines are the rail infrastructure between nodes. In such interpretation the availability or unavailability of a section account for the transport service execution by one or more lines or failure to execute the service by the subordinate (any of them) line from beginning to end. It was assumed that the system availability corresponds to the execution of the service by all lines, while unavailability is interpreted as lack of execution of service even by one line [10, 16].

The application of factorisation algorithm in the network structural analysis for the assessment of reliability of connections in urban public transport is proposed.

The factorisation method is based on the following assumptions [4, 21, 25, 32]:

- analysed network is represented by undirected graph $G=(V, E)$, in which $V=\{v_1, v_2,\dots, v_n\}$ represents set of vertices (nodes in network) and $E=\{e_1, e_2,\dots, e_m\}$ represents set of edges (links in network)

- all links e_i in the network can fail statistically independently of each other with known probability

- the measure of network reliability is the probability that all nodes from specified set K are connected and: $2 \le |K| \le |V|$

- - specified set K of nodes determines the measure of network reliability (K – terminal reliability network)
- - the reduction process is based on a well-known principle of contracting and deleting of links which is recursively applied

  for all edges e_i graph G, which can be rewritten as:

$$R_s = R(G_K) = R_{e_i} \cdot R(G_{K'} * e_i) + (1 - R_{e_i}) \cdot R(G_K - e_i) \qquad (1)$$

where:

e_i – i-th link in a network represents by graph G

R_{ei} – probability that link e_i is in an operating state

$$G_{K'} * e_i = (V - v_k - v_l + v_{kl}, E - e_i), \ \ v_{kl} = v_k \cup v_l,$$

$$K' = \begin{cases} K & if \ \ v_k \ \ or \ \ v_l \notin K, \\ K - v_k - v_l + v_{kl} & if \ \ v_k \in K \ \ and \ \ v_l \in K, \end{cases}$$

$$G_K - e_i = (V, E - e_i),$$

- as the effect of reduction process one can obtain the formula which represents analytical form of reliability structure and enables for computation the specified measure of network reliability.

The factorisation method makes it possible to calculate the reliability of network systems, which is not possible with the use of classical methods of reliability evaluation. The effect of these calculations is, of course, obtaining typical reliability characteristics of the entire system, however the path leading to their determination is not within the standard procedure for the classical approach to estimating the reliability of technical objects.

Urban transport systems can be mapped as network structures for which many inputs z and outputs w_k should be distinguished, while in the graph representation we have to do with graphs or networks as shown in figure 1. As the availability condition of such a graph the existence of a connection from each initial node (input)) $z \in \boldsymbol{Z}$ to any final node (output) $w \in \boldsymbol{W_k}$, is generally adopted. It is also assumed that some nodes of the graph may be both input nodes and out nodes (i.e. $\boldsymbol{Z} \cap \boldsymbol{W_k} \neq \emptyset$). For the case discussed in the paper it was assumed that the service is executed when from node $\boldsymbol{z_1}$vehicles reach the predetermined nodes from $\boldsymbol{w_{k1}}$ to $\boldsymbol{w_{k4}}$, and, similarly, from $\boldsymbol{z_2}$ to corresponding from $\boldsymbol{w_{k2}}$ to $\boldsymbol{w_{k6}}$.

For the needs of research and to present the calculation possibilities a model of network connections structure in the form of an undirected graph is proposed, as shown in figure 2. For the analysis, estimated values of reliability indices of the connections of this structure, on condition of nodes availability, have been adopted.

The values of these reliability characteristics reflect the effects of all kinds of external influences on the system considered. External factors may be random or may be controlled by system operators. The estimation of the values of the characteristics is statistical and is the result of observation of the actual operating systems. Therefore, the article does not systematise the difficulties occurring within the transport network in the urban area, treating the obtained characteristics as a comprehensive representation of all external interactions.

Next, modifications in the structure were introduced by increasing the number of connections between the nodes, marked with dotted lines. The scheme of the structure with modifications is shown in figure 3.

For the modified structure, calculations were done again and the resulting changes in the network reliability were identified. In the analysis it was assumed that the network reliability measure, resulting from the factorisation algorithm, is the probability of the existence of a connection between a selected depot (Z1 or Z2) and all the end points of routes starting from the given depot. For depot Z1 the end points are WK1-WK4, and for depot Z2 the end points are WK2-WK6). In other words, it is the probability of executing all the scheduled trips from the given depot (reaching the destination point).

Figure 4 presents the network reliability measures discussed above, calculated for a standard network (Fig. 2) and modified one (Fig. 3), at various reliability values of connections between nodes: R=0,998; R=0,95; R=0,92 and R=0,72. To carry out calculations a

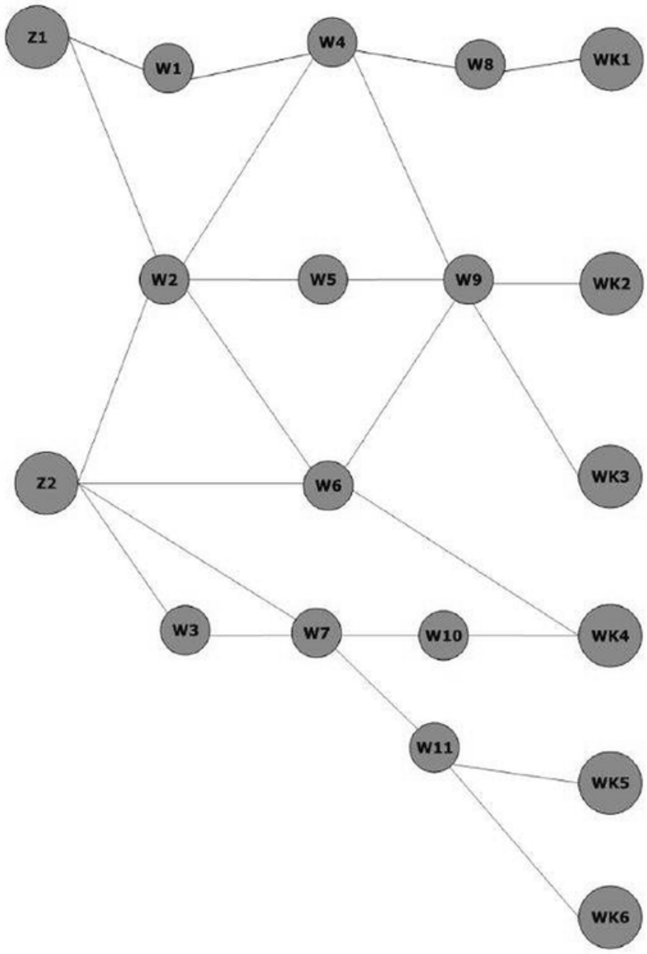


Fig. 2. Scheme of urban transport network structure

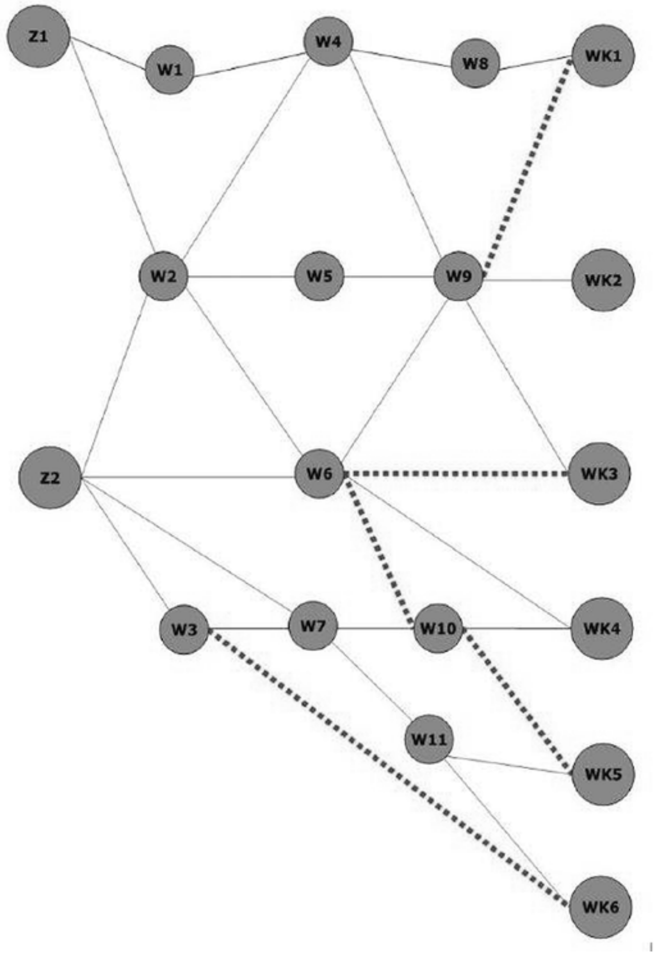


Fig. 3. Scheme of urban transport network structure modified

computer program was used, written in C language, which performs calculations in accordance with the presented procedure of the factorisation algorithm.

As can be noticed, only at high reliability of connections between nodes (R=0,998) the introduced modifications do not change significantly the network reliability (relative change for Z1 is 0,8% and for Z2 is also 0,8%). In the other cases, the lower the reliability of the connections the larger the change of network reliability resulting from modification. In the extreme case (R=0,72) the relative changes are, respectively, for Z1 65,2% and for Z2 69,8%.

Additionally, an analysis of the reliability of a network with failed nodes was performed. Node failure is treated as a failure of all the connections associated with the given node.

Consequently, the so-called critical nodes can be distinguished, i.e. nodes whose failure results in the lack of possibility of performing at least one of the scheduled trips from the given depot to end points of routes. In the case of a standard network (Fig. 2) these nodes are W4, W8, W9, W6, W7 and W11. In the case of a modified network (Fig. 3) the number of critical nodes is significantly reduced, and they are only W8 and W6.

Taking into account the failures of selected nodes (W4, W2, W10), which apart from W4 in the analysed network are not critical nodes, for a standard network the results obtained are shown as histograms (Figs 5 and 6).

The limit values in the form of relative percentage are:

- for Z1 failure of node W4 reduces the reliability by 100% in all cases (it is a critical node),

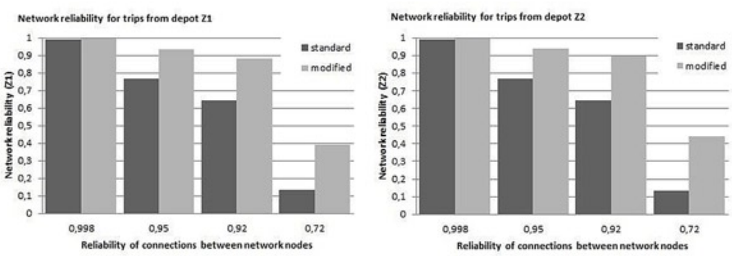


Fig. 4. Network reliability for trips from depots Z1 and Z2

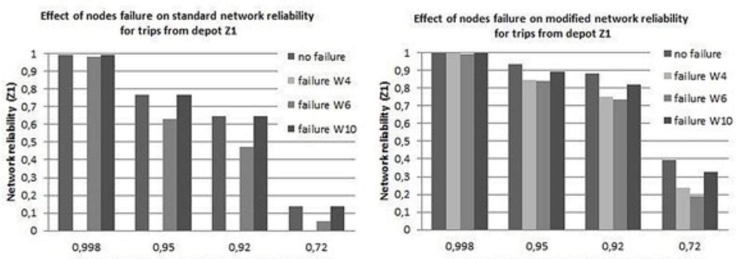


Fig. 5. Effect of nodes damage on network reliability for trips from depot Z1

- for Z2 failure of node W4 reduces the reliability in extreme cases by ≈0% for R=0,998 and by 7,7% for R=0,72,
- for Z1 failure of node W2 reduces the reliability in extreme cases by 0,8% for R=0,998 and by 61,9% for R=0,72,

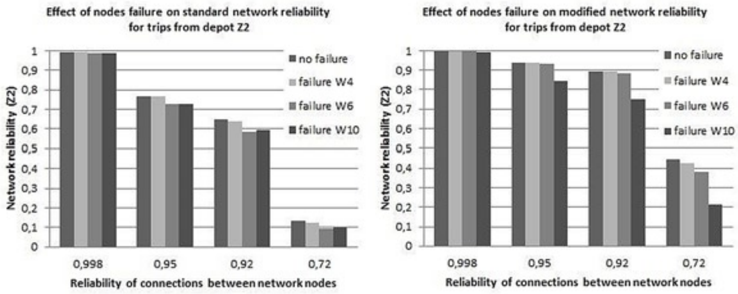


Fig. 6. Effect of nodes damage on network reliability for trips from depot Z2

- for Z2 failure of node W2 reduces the reliability in extreme cases by 0,2% for R=0,998 and by 30,7% for R=0,72,
- for Z1 failure of node W10 does not reduce reliability,
- for Z2 failure of node W10 reduces the reliability in extreme cases by 0,2% for R=0,998 and by 25,5% for R=0,72.

In an analogous way, the results for a modified network are (Figs 4 and 5):

- for Z1 failure of node W4 reduces the reliability in extreme cases by 0,4% for R=0,998 and by 38,9% for R=0,72,
- for Z2 failure of node W4 reduces the reliability in extreme cases by ≈0% for R=0,998 and by 3,7% for R=0,72,
- for Z1 failure of node W2 reduces the reliability in extreme cases by 0,4% for R=0,998 and by 51,6% for R=0,72,
- for Z2 failure of node W2 reduces the reliability in extreme cases by ≈0% for R=0,998 and by 14,5% for R=0,72,
- for Z1 failure of node W10 reduces the reliability in extreme cases by 0,2% for R=0,998 and by 16,8% for R=0,72,
- for Z2 failure of node W10 reduces the reliability in extreme cases by 0,4% for R=0,998 and by 51,5% for R=0,72.

5. Conclusions

The assessment of network systems reliability requires the use of specialised calculation methods which are not covered by the classic theory of reliability.

These methods include the factorisation algorithm which is useful in fast assessment of changes of reserving effectiveness according to the number and location of power supply points, in the selection of an optimum number of power sources and their location so as to ensure the high reliability of power supply of the analysed part of the networks, in the design of network taking into account the reliability and cost effectiveness of reserving. Its application enables the monitoring of network reliability changes when the structure is modified (by e.g. extending the connections), which will be practically useful for the designers of new networks as well as in modernisation and development projects of the existing networks. The factorisation algorithm used in the above analysis enables specification of reliability of networks of any configuration.

Instead of the classic reliability indices and characteristics the application of failure stream parameter in reliability assessment, which not only does not reduce the possibility of modernisation of the examined systems, on the contrary, it extends the utilitarian character of reliability analyses performed.

Multiple and fast repetitions of calculations for different variants of network modification require the algorithm to be saved as a computer program.

Network reliability assessment extended by an analysis of economy and operation related risk analysis can be useful for transport network designers, in modernisation and development projects of the existing networks.

The article presents reliability evaluation of the selected parts of a transportation network, assuming that the possibility of completing journeys between selected network nodes is an important criterion for users (passengers) of public transport. The possibilities of occurrence of failures between connections in network nodes are observed in real operating conditions and constitute a serious impediment to the functioning of public transport. For this reason, the presented reliability analysis should be useful for urban transport companies in the planning and development of transportation systems.

The factorisation method is presented in the literature presented below and it was not characterised in a detailed way, presenting only the results of calculations because it would cause a significant increase in the volume of the article, which aims to present the methodology for estimating the reliability of network systems, and not the factorization method.

References

1. Borowiecki R, Kaczmarek J, Magiera J, Młynarski S. Eksploatacja taboru szynowego komunikacji miejskiej. Niezawodność, jakość, ekonomika. Kraków: Wydawnictwo Akademii Ekonomicznej w Krakowie, 2004.
2. Brons M, Nijkamp P, Pels E, Rietveld R. Efficiency of urban public transit: A meta analysis. Transportation 2005; 32: 1-21, https://doi.org/10.1007/s11116-004-0939-4.
3. Bryniarska Z. Ocena satysfakcji pasażerów z innowacyjnego rozwiązania w miejskim transporcie zbiorowym w Krakowie. Transport miejski i regionalny 2017; 11: 5-9.
4. Carlier J, Lucet C. A decomposition algorithm for network reliability evaluation. Discrete Applied Mathematics 1996; 65: 141-156, https://doi.org/10.1016/0166-218X(95)00032-M.
5. Cats O, Koppenol GJ, Warnier M. Robustness assessment of link capacity reduction for complex networks: Application for public transport systems. Reliability Engineering and System Safety 2017; 167, https://doi.org/10.1016/j.ress.2017.07.009
6. Crainic TG, Ricciardi N, Storchi G. Models for Evaluating and Planning City Logistics Systems. Transportation Science 2009; 43(4): 432-454, https://doi.org/10.1287/trsc.1090.0279.
7. Heinrich M, Lenkiewicz W. Erhöhung und Auswertung des Zuverlässigkeitsniveaus von tribologischenPaarungenimZeitraum der AnfangsnutzungeineseinzelnenObjekts. Tribologie und Schmierungstechnik 2002, 3.
8. Heinrich M. Badania eksploatacyjne jednostkowych złożonych obiektów technicznych w celu podniesienia ich niezawodności konstrukcyjnej. Zagadnienia Eksploatacji Maszyn 1993; 1-2 (93-94): 125-133.
9. Holmgren J. The efficiency of public transport operations - An evaluation using stochastic frontier analysis. Research in Transportation Economics 2013; 39 (1): 50-57, https://doi.org/10.1016/j.retrec.2012.05.023.
10. Jan R-H. Design of reliable networks. Computers & Operations Research 1993; 20: 25-34, https://doi.org/10.1016/0305-0548(93)90093-X.
11. Jong GDe, Cheung F. Stochastic frontier models for public transport. Meersman H, van de Voorde E, Winkelmans W eds. World Transport

Research: Selected Proceedings of the 8th World Conference on Transport Research. Transport Modes and Systems. New York: Pergamon, 1999; 1: 373-386.

12. Karpiński J, Korczak E. Metody oceny niezawodności dwustanowych systemów technicznych. Omnitech Press, 1990.
13. Koide T, Shinmori S, Ishii H. Topological optimization with a network reliability constraint. Discrete Applied Mathematics 2001; 115: 135-149, https://doi.org/10.1016/S0166-218X(01)00221-9.
14. Levaggi R. Parametric and non-parametric approach to efficiency: The case of urban transport in Italy. StudiEconomici 1994; 49: 67-88.
15. Lucet C, Manouvrier J-F. Exact methods to compute network reliability, in Statistical and Probabilistic Models in Reliability. Ionescu DC and Limnios N eds. Boston: Birkhauser, 1999: 279-294, https://doi.org/10.1007/978-1-4612-1782-4_20.
16. Malasek J. Metoda oceny dostępności i atrakcyjności przystanków miejskiego transportu zbiorowego. Transport miejski i regionalny 2017; 9: 26-32.
17. Młynarski S. Niezawodność strukturalna w systemach logistycznych ratownictwa. Logistyka [CD] 2014; 4: 871-880.
18. Nolan JF. Determinants of productive efficiency in urban transit. Logistics and Transportation Review 1996; 32: 319-342.
19. Oprzędkiewicz J, Młynarski S. Problems of economic safety and intelligence system and reliability of global systems. In Enterprises in the face of 21st century challenges. Development - management - entrepreneurship. Borowiecki R & Jaki A eds. Cracow: Cracow University of Economics, 2008: 307-315.
20. Oprzędkiewicz J. Komputerowa metoda oceny niezawodności systemów technicznych. Lublin: LTN, 1997.
21. Page LB, Perry JE. A practical implementation of the factoring theorem for network reliability. IEEE Transactions on Reliability 1988; 37: 259-267, https://doi.org/10.1109/24.3752.
22. Paszkowski J, Kucharski R. Paradoksy przepustowości miejskiej sieci drogowej i sposoby ich odwzorowania w modelu czterostadiowym. Transport miejski i regionalny 2017; 10: 5-11.
23. Pilch R, Szybka J. Koncepcja zastosowania algorytmu faktoryzacji do oceny niezawodności ciągów komunikacyjnych - Application of factoring algorithms for estimation of road network reliability. Problemy Eksploatacji - Maintenance Problems 2007; 2: 129-136.
24. Pilch R. Factorisation algorithm-based metod used for the calculation of network system's reliability - Metodyka wyznaczania niezawodności układów sieciowych w oparciu o algorytm faktoryzacji. Zagadnienia Eksploatacji Maszyn - Scientific Problems of Machines Operation and Maintenance 2011; 4 (168): 45-57.
25. Pilch R. Reliability evaluation of networks with imperfect and repairable links and nodes. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2017; 19 (1): 19-25, https://doi.org/10.17531/ein.2017.1.3.
26. Ramirez-Marquez JE, Rocco CM. All-terminal network reliability optimization via probabilistic solution discovery. Reliability Engineering and System Safety 2008; 93: 1689-1697, https://doi.org/10.1016/j.ress.2008.01.001.
27. Rymarz J, Niewczas A, Krzyżak A. Comparison of operational availability of public city buses by analysis of variance. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2016; 18 (3): 373-378, https://doi.org/10.17531/ein.2016.3.8.
28. Smalko Z. Modelowanie eksploatacyjnych systemów transportowych. Radom: ITE, 1996.
29. Smolnik M. A conception of modernising LEMACH designing methods using TRIZ instruments. Souchkov V, Kässi T. eds. TRIZfest-2014. Theories and applications. The 10th international conference: September 4-6, 2014, Prague, Czech Republic. Conference proceedings. noxville: International TRIZ Association - MATRIZ, 2014.
30. Smolnik M. Koncepcja systemu informacyjnego wspomagającego kierowanie eksploatacją złożonych odnawialnych obiektów technicznych. Logistyka 2015; 5: 1307-1312.
31. Wiśniewski P. Projekt i implementacja systemu wspomagania decyzji dla zarządzania kryzysowego transportem miejskim. Transport miejski i regionalny 2017; 11: 10-16.
32. Wood RK. Factoring algorithms for computing K-terminal network reliability. IEEE Transactions on Reliability 1986; 35 (3): 269-278, https://doi.org/10.1109/TR.1986.4335431.
33. Zamojski W. Niezawodność i eksploatacja systemów. Wrocław: Politechnika Wrocławska, 1981.

Stanisław MŁYNARSKI
Cracow University of Technology
Institute of Rail Vehicles
al. Jana Pawla II 37, 31-864 Cracow, Poland

Robert PILCH
Maksymilian SMOLNIK
Jan SZYBKA
AGH University of Science and Technology
Faculty of Mechanical Engineering and Robotics
al. A. Mickiewicza 30, 30-059 Cracow, Poland

Emails: mlynarski_st@poczta.onet.pl, pilch@agh.edu.pl, smolnik@agh.edu.pl, szybja@agh.edu.pl,