Investigation of effectiveness of α-constrained simplex method applied to design of optimal induction motors

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The paper presents a modified nonlinear simplex algorithm with lexicographic order comparison of solutions and its application to the design of optimal induction motors. In the comparison of the solutions generated in the optimization process, both the objective function value and the additional parameter, called the satisfaction level of constraints, have been taken into account. The comparison method assigns some advantage degree to feasible solutions, thus allows for the control of this advantage degree during the optimization process. Special attention has been paid to the choice of the algorithm parameters and to the kind of the mutation operator. The presented algorithm has been implemented in the object-oriented software. Calculation results of the selected double-cage induction motors have been compared with the results obtained with the evolution strategy $(\mu+\lambda)$-ES and with the hybrid algorithm assembled with the modified Price algorithm. An additional calculation experiment allows for the comparison of exploitation properties between the α-Constrained Simplex Method and the Modified Price Algorithm. As the investigations showed, the presented algorithm can be successively used for the optimization of the induction motors, however, with constraints, which are not very restrictive concerning respective functional parameters.

1. Introduction

Optimization of electrical machines, particularly of induction motors, consists in a search of a solution determined by the vector of independent variables x, which leads to minimization of the value of the assumed objective function but at the same time to fulfillment of all constraints $g_i(x)$ determining the functional machine parameters. Methods of the mathematical programming as well as non-deterministic optimization methods concern tasks in the non-constrained space. In order to take the constraints into consideration several methods have been applied, between others, the methods based on the internal or external penalty function. In the case of the non-deterministic methods usually external penalty function should be applied through adding a penalty component to the objective function value. Various ways of penalty component determination have been described among other works in [7].

In [8] a new method for optimization in the constrained search space was proposed. It converts the constrained optimization problem into an unconstrained one. It consists in the use of the modified nonlinear simplex method by entering: first, the special order for comparing solutions and second, the mutation operator. The main modification consists in this that the solutions obtained in the optimization process are judged not only using a sum of the objective function and the penalty component but on the basis of two quantities: a new parameter, which is the constraints satisfaction level, and the objective function value, with respect to variable influence grade of one of these values on the comparison result.

Optimization procedure proposed in work [8] has been tested on several analytically expressed test functions. Very good test results in comparison to the results obtained with other optimization procedures, encouraged authors of this work to the application and examination of the described method for optimization of the induction motors. In this case the objective function is expressed using complex algorithms with many loops and recurrence-iterative procedures.

2. Comparison of solutions using lexicographic order

Comparison of the value of the objective function enlarged by the penalty component for constraints violation in [8] has been replaced by the lexicographic assessment of solutions generated in the optimization process. These solutions have been assessed on the basis of the objective function values and on an additional parameter, namely the satisfaction level of constraints μ, calculated from the formula:

$$\mu(\boldsymbol{x}) = \max\left\{0, 1 - \frac{P(\boldsymbol{x})}{B}\right\} \tag{1}$$

where $P(\boldsymbol{x})$ is the penalty component for constraints violation; B is the positive constant (an algorithm parameter).

The value of the penalty component $P(\boldsymbol{x})$ may be expressed in various ways. Several strategies of its determination are presented in work [7].

In the minimization case, assessment of solutions is carried out on the basis of the relationship:

$$(f_1, \mu_1) <_\alpha (f_2, \mu_2) \Leftrightarrow \begin{cases} f_1 < f_2, \text{for} \quad \mu_1, \mu_2 \geq \alpha \\ f_1 < f_2, \text{for} \quad \mu_1 = \mu_2 \\ \mu_1 > \mu_2 \text{ otherwise} \end{cases} \tag{2}$$

where f_1, f_2 are the objective function values; μ_1, μ_2 are the satisfaction levels of constraints; α is a variable algorithm parameter.

From formula (2) appears that the solutions with the same level of constraints satisfaction and these, which have bigger or equal satisfaction level of constraints than the varying value α, are assessed only on the basis of the objective function

value. Whereas in the case, in which the constraints satisfaction levels of both compared solutions are smaller than α, the solution characterized by the bigger value of parameter μ. is taken as the better one. In this way the comparisons provide a possibility to favor feasible and nearly feasible solutions and to control this privilege in subsequent algorithm iterations. This guaranties support of the equilibrium between the exploration and exploitation capabilities of the optimization procedure. Through appropriate strategy of selection of the parameter μ during the optimization process the algorithm convergence is achieved together with assurance of the solutions feasibility.

In this work a strategy according to the following relationships has been applied:

$$\alpha(t) = \begin{cases} \frac{1}{2}\left(\max_i \mu(x_i) + \frac{\sum_i \mu(x_i)}{N}\right), & \text{if } t = 0 \\ (1-\beta)\cdot\alpha(t-1) + \beta, & \text{if } 0 < t \leq \frac{T_{\max}}{2} \text{ and } (t \bmod T_\alpha) = 0 \\ \alpha(t-1), & \text{if } 0 < t \leq \frac{T_{\max}}{2} \text{ and } (t \bmod T_\alpha) \neq 0 \\ 1, & \text{if } t > \frac{T_{\max}}{2} \end{cases} \quad (3)$$

where N is the number of solutions in the set; t is the subsequent number of iteration; $T_{\max}$ is the maximal number of iterations (algorithm stop criterion); β, T_α are the algorithm parameters.

From relationship (3) it appears that in the first half of the optimization process a value of parameter α is corrected every T_α iterations, while in the second half of the process the solutions are assessed only on the basis of the objective function values. By such strategy of changing parameter α it is desirable to obtain the maximum number of feasible solutions in the first half of the optimization process.

3. Nonlinear simplex algorithm

Algorithm applied in work [8] is a modification of nonlinear simplex algorithm described in paper [5]. Its essence relies on seeking new solutions inside an $n+1$-dimensional simplex (n is the number of the problem dimensions) created on randomly selected solutions from the processed set containing N solutions.

During every iteration three unique solutions of the vector of independent variables x: the best x_b, the worst x_w and the second worst x_{sw} have been defined as follows:

$$x_b = \arg\min_k f(x_k)$$

$$x_w = \arg\max_k f(x_k) \qquad k = 1, 2, ..., N \qquad (4)$$

$$x_{sw} = \arg\max_{k \neq w} f(x_k)$$

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By selection of solutions corresponding to the simplex vertices often a selective pressure is applied in favor to better solutions. After each iteration the set of solutions is sorted in a non-growing order respectively to criterion (2). The solutions are therefore arranged from the best to the worst. In this work a selection of solutions with indices i expressed by

$$i = N\left(2^r - 1\right) \tag{5}$$

relationship has been applied, where r is the random number with uniform distribution in range [0, 1].

The following points are created in every algorithm iteration: the reflection point x_r relative to centroid center x_0 of the simplex, the expansion point x_e, the contraction point x_c. The worst solution is replaced by one of them.

Coordinates of the centroid center x_0 have been described by formula:

$$x_0 = \frac{1}{n} \sum_{k \neq w} x_k \tag{6}$$

where k is the number of subsequent centroid vertex.

During calculations of the centroid center coordinates, according to formula (6), a point corresponding to the worst solution is omitted ($k \neq w$).

Reflection point x_r, expansion point x_e, and contraction point x_c are created according to formulas:

$$
\begin{aligned}
x_r &= \left(1 + a\right) x_0 - a\, x_w & a &> 0 \\
x_c &= b\, x_w + \left(1 - b\right) x_0 & 0 &< b < 1 \\
x_e &= c\, x_r + \left(1 - c\right) x_0 & c &> 1
\end{aligned} \tag{7}
$$

where x_w is the worst solution; a, b, c are the constant algorithm parameters.

In work [8] besides the mentioned modifications, i.e.: the application of lexicographic order for comparison of solutions and the use of mutations, repeatedly created simplex has been introduced. The simplex is created in every algorithm iteration. In this process $n+2$ points participate, $n+1$ as simplex vertices and the worst point x_w from all set of the processed solutions. Such procedure reduces a risk of obtaining wrong optimal solutions as an effect of the lack of affine independence of points corresponding to simplex vertices.

A flow-chart of the nonlinear simplex algorithm with the lexicographic order for comparison of solutions is presented in Fig. 1.
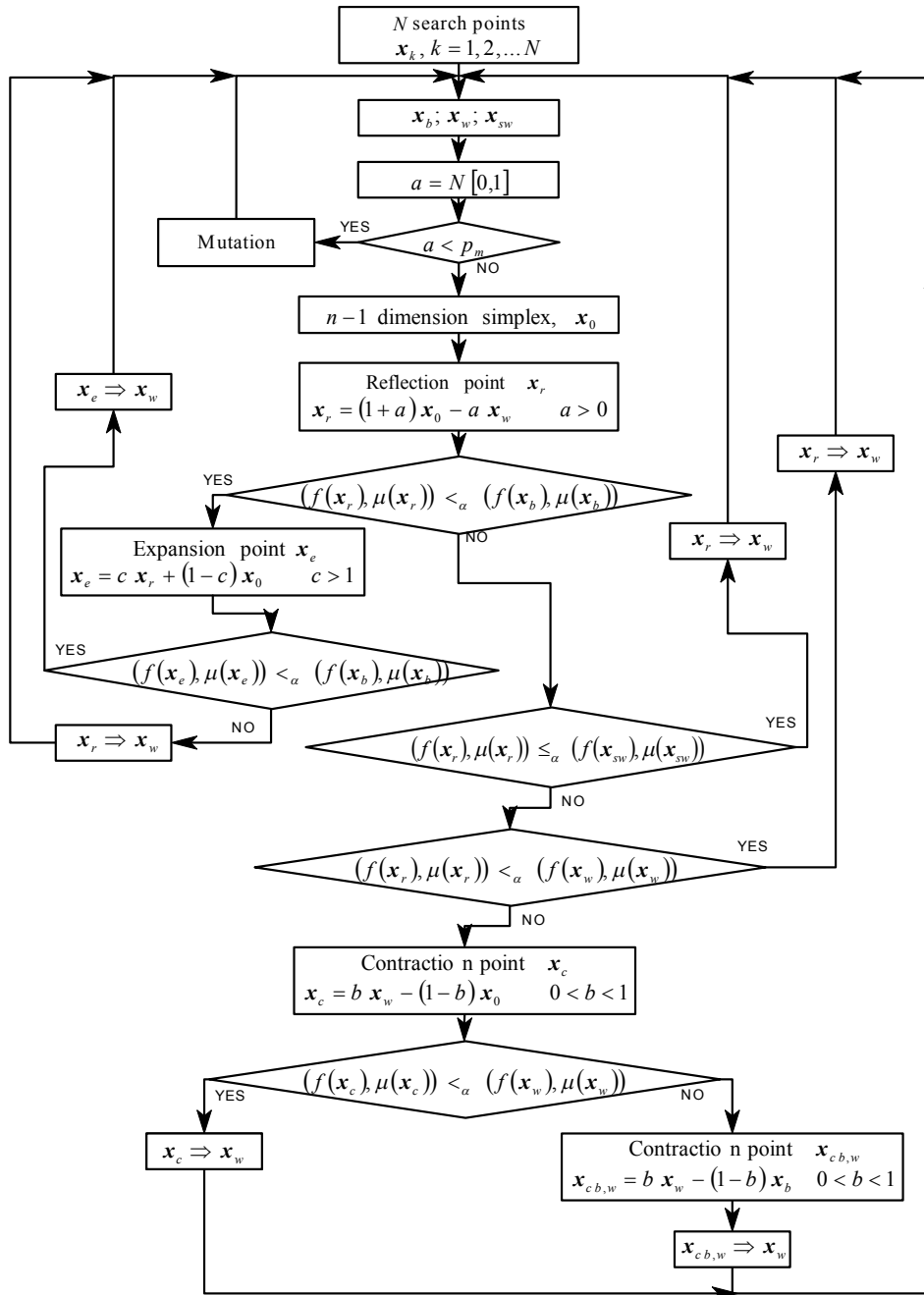
Fig. 1. A flow-chart of the modified nonlinear simplex algorithm with lexicographic order
of solutions comparison

4. Mutation operator

In work [8] a mutation has been introduced to the nonlinear simplex algorithm. In every iteration with probability p_m the worst solution $\boldsymbol{x}_w$ from the processed set undergoes a mutation. If the mutated solution is better as the worst one in processed set, this worst solution is replaced by a new one. In opposite situation, i.e. if the mutation is not applied, normal iteration of the simplex algorithm is carried out. Applied mutation operator is similar to the boundary mutation proposed by Michalewicz in work [4]. That algorithm searched for a feasible solution for the maximal far-distant value of the randomly selected variable in a direction to either the lower or the upper limit of its variation range. In result a solution was created, which lay on the boundary of the feasible space. Whereas in this work a special mutation operator has been applied taking a small extend of feasible space into account as compared to the search space – what is distinctive in optimization of induction motors. For a randomly selected independent variable a feasible solution in the range of its variability has been searched for either bigger or smaller values than a value before mutation. If ten subsequent trials bring no success, i.e. if the feasible solution is not obtained, then as a mutated solution is accepted as the best among them. Certainly, the solutions are assessed according to relationship (2).

5. Results of computational experiments

The presented algorithm, characterized by a big number of parameters, has been realized in an object oriented programming form. Preliminary computational experiments have been aimed to the selection of appropriate algorithm parameter values. The following aspects have been analyzed: first, the number of successful iterations, i.e. those leading to improvement of the worst solution in the processed set of N elements as a result of replacement the worst solution by a solution corresponding to a reflection, second, expansion and contraction points, third, the number of mutations, which caused improvement of the worst solution, fourth the graph of a curve, which depicted the relative number of feasible solutions in respect to all solutions in the processed set in subsequent iterations (the curve marked with arrow in fig. 2). As it turned out, the algorithm parameter values and the assumed strategy of selection of parameter α according to relationship (3), have essential influence on the graph character of this curve, i.e. on the speed of increasing the number of feasible solutions participating in the optimization process.

As a result of this analysis the following parameter values have been assumed in the calculations: $B = 1000$; $\beta = 0{,}08$; number of solutions in the processed set $N = 1000$; maximal number of iterations $T_{\max} = 20000$; period of change of the parameter α every $T_\alpha = 50$ iterations; parameters for reflection, expansion, and

contraction $a = 1$; $b = 0,75$; $c = 2$, respectively; mutation probability $p_m = 0,1$.

Using the elaborated software six 3-phase induction double-cage motors have been optimized. In table 1 calculation results are compared obtained by application of: the investigated α-constrained simplex algorithm (upper values); the evolution strategy $(\mu + \lambda)$-ES with the number of parents $\mu = 200$, offspring $\lambda = 400$ and the generations number $g = 100$ (middle values); the hybrid algorithm [1, 2, 6] compounded with the evolution strategy $(\mu + \lambda)$-ES, and the modified Price algorithm (lower values). The fields in lower rows in table 1 which corresponds to the average fitness and standard deviations remain empty, because the results obtained with the hybrid algorithm are received only simple solution.

Table 1. Results of optimization calculations obtained using: α-constrained simplex algorithm, evolution strategy $(\mu + \lambda)$-ES, and hybrid algorithm

Motor	Best fitness f_{min} [zł]	Average fitness f_{av} from 20 runs f_{av} [zł]	Standard deviation σ [%]	Average calculation time t [s]
$P_N = 7,5$ kW $2p = 2$	3297,25 3295,59 3292.19	3304,39 3301,20 –	0,15 0,08 –	55,3 21,9 21,9+11,9
$P_N = 18,5$ kW $2p = 4$	8102,51 8065,00 8048.61	8141,30 8078,69 –	0,29 0,12 –	83,7 50,1 50,1+25,8
$P_N = 22$ kW $2p = 2$	8889,45 8881,52 8860,66	8902,70 8886,75 –	0,11 0,04 –	72,4 34,3 34,3+21,3
$P_N = 22$ kW $2p = 4$	9185,01 9186,94 9171.64	9234,68 9230,93 –	0,24 0,30 –	80,6 49,2 49,2+15,7
$P_N = 75$ kW $2p = 4$	– 23832,07 23740.08	– 23936,99 –	– 0,18 –	– 44,9 44,9+75,4
$P_N = 90$ kW $2p = 4$	27415,11 27414,69 27351.58	27492,13 27482,63 –	0,20 0,19 –	66,3 40,2 40,2+49,4

For a motor of the rated power $P_N = 75$ kW the examined algorithm did not find feasible solutions also with other values of the algorithm parameters.
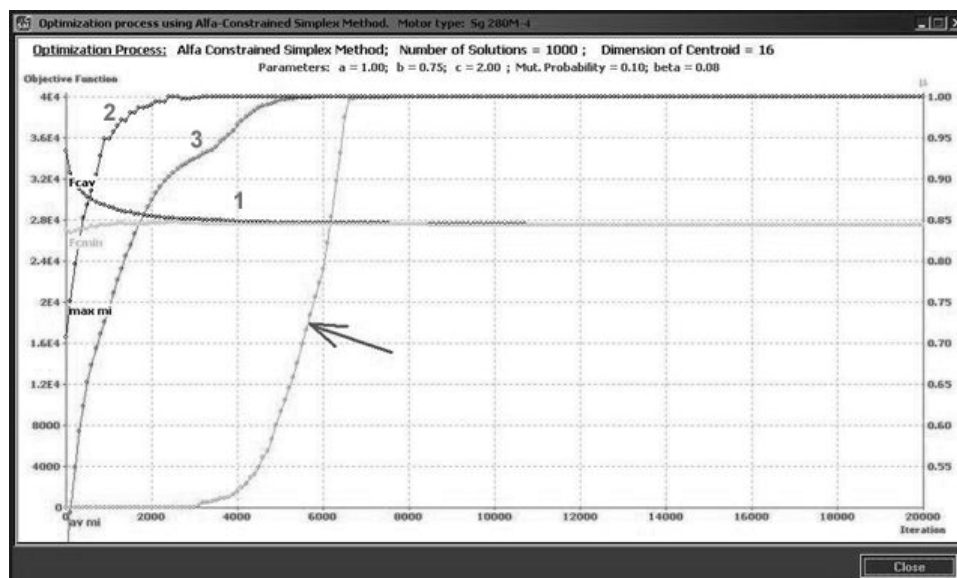
Fig. 2. An example of optimization process with application of α-CSM algorithm.
Curve 1 –objective function, Curve 2 – constraints satisfaction level for best solution,
Curve 3 – average constraints satisfaction level and relative number of feasible solutions
(a curve depicted with an arrow)

6. Exploitation properties comparison of α-CSM and MPA algorithms

In order to compare exploitation properties of the examined algorithm (α-CSM) and the modified Price algorithm (MPA) expressed according to [1, 2, 6], an additional computational experiment has been executed. It consisted in:

– the recording of the full set of solutions ($N = 1000$), which has been processed with the algorithm α-CSM to the disc file after the feasibility of all solutions has been reached. The recordings came in the nearest iterations with the indices divisible by T_{a};

– an introduction of this solution set to the MPA procedure, execution of calculation and comparison of the results with those obtained with the α-CSM procedure after the end of the work. Objective function values and differences between the objective function values of the best and the worst solutions in the obtained sets have been compared.

Additional calculations have been executed only for five motors. The motor of rated power $P_{N} = 75$ kW was omitted, while for this motor α-CSM algorithm cannot find the feasible solutions.

In table 2 a comparison of results obtained with both algorithms α-CSM and the modified Price algorithm (MPA) is presented. The following denotations have been applied: $fc_{\alpha CSM}$, fc_{MPA} –objective function values of the best solutions in sets

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obtained from the α-CSM algorithm and the modified Price algorithm, respectively; $\Delta fc_{\alpha CSM}$, Δfc_{MPA} –differences between the objective function values in the best and the worst solutions in these sets, respectively.

Values provided in the second column of table 2 are slight different than those in table 1. The reason is that during additional experiments single independently separated calculations have been executed. Table 1 contains the best and the average results from 20 calculations.

Table 2. Comparison of results obtained by using α-CSM and MPA algorithms applied to the sets of feasible solutions

Motor	$fc_{\alpha CSM}$ [zł]	$\Delta fc_{\alpha CSM}$ [zł]	fc_{MPA} [zł]	Δfc_{MPA} [zł]
$P_N = 7{,}5$ kW $2p = 2$	3302,15	2,14	3292,18	0,01
$P_N = 18{,}5$ kW $2p = 4$	8113,58	1,81	8083,73	0,01
$P_N = 22$ kW $2p = 2$	8903,73	9,46	8879,89	0,01
$P_N = 22$ kW $2p = 4$	9237,73	1,72	9219,54	0,01
$P_N = 90$ kW $2p = 4$	27453,53	12,55	27391,87	0,37

The stopping criterion in the modified Price algorithm is either achievement of a very small difference between the objective function values in the first half of the processed solution set, or achievement of the maximal assumed number of iterations. This second situation takes place in case of the motor of the rated power $P_N = 90$ kW. This explains bigger differences between objective function values in the last row of table 2.

7. Conclusions

Nonlinear simplex algorithm with lexicographic manner of solutions comparison may be applied to the design of the optimal induction motors but only with constraints concerning exploitation and starting parameters, which are not very restrictive. For motors with weaker constraints, calculation results are comparable with those obtained by applying evolution strategy $(\mu+\lambda)$-ES and only slightly poorer than those obtained with the hybrid algorithm compounded from the evolution strategy $(\mu+\lambda)$-ES and the modified Price algorithm. However, the calculation time using the examined algorithm is longer.

The algorithm did not find feasible solutions for a motor of the rated power P_N = 75 kW. This motor is characterized by very restrictive constraints concerning exploitation and starting parameters. This causes decrease of dimensions of the feasible region in comparison with the search region and its non-cohesivity. Research of feasible region structure is described in work [3]. The research results indicate that in the optimization induction motors by restrictive constraints the feasible region may be very small and non-cohesive. Probably the main reason of poorer results obtained by application of the simplex algorithm are their poor exploration abilities in comparison to the evolution strategy $(\mu+\lambda)$-ES, which causes smaller diversity of solutions in the processed set.

An additional computational experiment showed that the examined algorithm α-CSM has also poorer exploitation capability than the modified Price algorithm. This is an outcome of the applied strategy of parameter α change according to relationship (3). By such strategy in the second half of the optimization process, α-CSM algorithm behaves like a classical nonlinear simplex algorithm according to [4], while in the modified Price algorithm special heuristics in target to move creation trial points closer to better solutions from the processed set are applied. In the MPA algorithm the trial point is localized on the n-dimensional line between the center of centroid and a point with the worst objective function value. Its localization also depends on the worst and the best value of the objective function and on the degree of advance of the optimization process. The trial point position is defined by a coefficient, which depends on the distance between the worst point and the centroid center. The bigger the difference between the objective function value in worst point and its value in centroid center the bigger is the distance between the trial point and the worst point. In this manner the algorithm utilizes information encoded in coordinates of all points considered in calculations. By application of the α-CSM algorithm in second half of the optimization process, the trial point positions are independent on mutual positions of the best point and the centroid center and on the degree of advance of the optimization process.

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