Engineering Transactions, **70**(2): 123–139, 2022, doi: 10.24423/EngTrans.1387.20220609
Institute of Fundamental Technological Research Polish Academy of Sciences (IPPT PAN)
Université de Lorraine • Poznan University of Technology

Research Paper

On the Onset of Thermal Instability of a Porous Medium Layer Saturating a Jeffrey Nanofluid

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The onset of stationary convection in thermal instability of porous layer saturating a Jeffrey nanofluid is studied. The behaviour of nanofluid is described by a Jeffrey fluid model and the porous layer is assumed to follow Darcy's law. Due to the presence of the Jeffrey parameter and nanoparticles, the momentum-balance equation of fluid is modified. The linear stability analysis and normal modes analysis method are utilised to derive the dispersion relation for the Rayleigh number in terms of various parameters for free-free boundaries. The effects of the Jeffrey parameter, Lewis number, modified diffusivity ratio, nanoparticles' Rayleigh number and medium porosity on the physical system are discussed analytically and graphically.

Key words: nanofluid; Jeffrey model; Rayleigh number; porous medium; convection.

Notations

a – wave number,
d – thickness of the horizontal layer,
Da – Darcy number,
D_{B} – Brownian diffusion coefficient [m^2/s],
D_T – thermophoretic diffusion coefficient,
$\mathbf{g}$ – gravitational acceleration vector,
k_1 – medium permeability,
k_f – thermal conductivity of the fluid,
k_m – thermal conductivity of porous medium,
k_p – thermal conductivity of the nanoparticle,
Le – Lewis number,
N_A – modified diffusivity ratio,
N_B – modified particle density increment,
p – pressure,
Pr – Prandtl number,
$\mathbf{q}_{\mathrm{D}}$ – Darcy velocity vector [m/s],

Ra – Rayleigh number,
Rm – basic density Rayleigh number,
Rn – nanoparticles Rayleigh number,
T – temperature,
Va – Vadasz number.

Greek symbols

α – coefficient of thermal expansion,
ε – medium porosity,
κ_m – thermal diffusivity,
λ_1 – stress relaxation-time parameter,
λ_2 – strain relaxation-time parameter,
λ_3 – Jeffrey parameter,
μ – fluid viscosity,
ρ_f – density of fluid,
ρ_p – density of nanoparticle,
σ – thermal capacity ratio,
φ – nanoparticles volume fraction,
∇ – Laplacian operator,
∇_H^2 – horizontal Laplacian operator.

Superscripts

' – non-dimensional variables,
$*$ – perturbed quantity.

Subscripts

f – fluid,
p – particle,
0 – lower boundary,
1 – upper boundary.

1. Introduction

Non-Newtonian fluids are widely used in various industries and find important applications in various fields of science and technology such as manufacturing of plastic, polymer industries, dying of papers and textiles, food processing, geophysics, chemical and biological industries, etc. Engine oil, soap solution, sauce, foam, paints, lubricants and biological liquids such as blood are some examples of non-Newtonian fluids. With the relevance of non-Newtonian fluids in modern technology and industry, various constitutive relations have been derived in the modelling of non-Newtonian fluids. One such type of constitutive relations is the Jeffrey non-Newtonian fluid model. The Jeffrey fluid model is a linear model that uses time derivatives instead of convective derivatives. Jeffreys [1] examined the stability of a layer of fluid heated from below. He derived a numerical solution to some problems related to the stability of a layer of incompressible fluid when temperature decreases upwards. A detailed literature review on thermal instability in a Newtonian fluid has been given by Chandrasekhar [2]. Several researchers have studied the Jeffrey fluid model and now it is considered as the best fluid model to depict the behaviour of physiological and industrial fluids [3–10].

The flow of a fluid through a homogeneous and isotropic porous medium is governed by Darcy's law that states that the usual viscous term in the momentum-balance equations is replaced by the resistance term $\left[-\frac{\mu}{k_1(1+\lambda_3)}\mathbf{q}_{\mathrm{D}}\right]$, where μ is the viscosity, k_1 is the medium permeability, λ_3 is the Jeffrey parameter, and $\mathbf{q}_{\mathrm{D}}$ is the Darcian (filter) velocity of the Jeffrey fluid. The research on flow through the porous layer has various practical applications, viz., flow in earth's molten cores, petroleum reservoirs, tyres, ropes, cushions, seats, flow in sand-beds, etc. Sandstones, limestones, the human lungs, bile duct, and gallbladder with stones in blood vessels are some examples of natural porous media. Lapwood [11] studied the convective flow in a porous medium, whereas Wooding [12] studied the Rayleigh's instability of a thermal boundary layer in a flow through a porous medium. They found that the layer is stable provided that the Rayleigh number for the system does not exceed a critical positive value and that the wave-number of the critical neutral disturbance is finite. A detailed study of convection in a porous media was given by Nield and Bejan [13].

For the past decade, various researchers have shown keen interest in studying hydrodynamic thermal convection problems in porous/non-porous medium saturated by a nanofluid layer based on the Buongiorno model [14]. Nanofluid has various applications in automotive industries, energy-saving, nuclear reactors, etc. Suspensions of nanoparticles flourish in medical applications, including cancer therapy. Porous media heat transfer problems have several engineering applications, such as geothermal energy recovery, crude oil extraction, groundwater pollution, thermal energy storage, etc. The natural convection of a nanofluid based on Buongiorno's model was studied by different authors [15–23], and they revealed that nanofluids are big coolants due to their enhanced thermal conductivities.

For the last few years, an extensive interest has been observed in the study of non-Newtonian nanofluids due to their applications in petroleum drilling, manufacturing of foods and biomedical products, etc. Thermal convection in a viscoelastic nanofluid layer saturating a porous medium has been studied by a few researchers [24–28], and they found that viscoelastic nanofluid has applications in various automotive industries and biomedical engineering. Recently, Shahzad *et al.* [29] studied the Jeffrey nanofluid flow in the presence of Joule heating and viscous dissipation over a stretching sheet, whereas an unsteady flow heat transfer of a Jeffrey nanofluid was studied by Sreelakshmi *et al.* [30].

Keeping in view various applications of viscoelastic nanofluids specified above, the main aim of this paper is to study the effect of the Jeffrey parameter and other parameters in a porous layer saturating a nanofluid heated from below. An analytical/graphical investigation of thermal instability of porous layer saturating a Jeffrey nanofluid is carried out for free-free boundaries on the onset of stationary convection. To the best of the author's knowledge, this problem has not been studied yet.

2. Mathematical model

Here, an infinitely extending horizontal porous layer of Jeffrey nanofluid of thickness d heated from below and restricted by two impermeable and perfectly thermal conducting parallel planes $z = 0$ and $z = d$ is considered (see Fig. 1). Let T_0 be the temperature of lower planes and φ_0 be the volumetric fraction of nanoparticles at $z = 0$ while T_1 and φ_1 at $z = d$ ($T_0 > T_1$ and $\varphi_1 > \varphi_0$). The physical system is permeated by the gravity force $\mathbf{g} = \mathbf{g}(0, 0, -g)$.

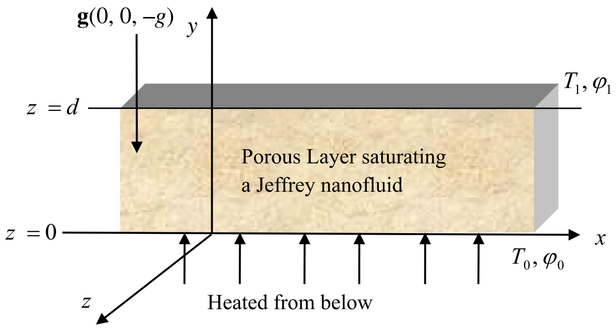

Fig. 1. Physical sketch of the problem.

2.1. Governing equations

For an incompressible fluid, the mass-balance equation is

$$\nabla \cdot \mathbf{q}_\mathrm{D} = 0, \tag{2.1}$$

where $\mathbf{q}_\mathrm{D}$ is the flow velocity of nanofluid.

The modified momentum-balance equation of Jeffrey nanofluid in a porous layer after applying the Boussinesq approximation (see [15–28]) is:

$$\frac{\rho_f}{\varepsilon}\left(\frac{\partial \mathbf{q}_\mathrm{D}}{\partial t} + \frac{1}{\varepsilon}\left(\mathbf{q}_\mathrm{D}\cdot\nabla\right)\mathbf{q}_\mathrm{D}\right) = -\nabla p - \frac{\mu}{k_1\left(1+\lambda_3\right)}\mathbf{q}_\mathrm{D} + \left[\varphi\rho_p + (1-\varphi)\,\rho_f\left\{1-\alpha\left(T-T_1\right)\right\}\right]\mathbf{g}, \tag{2.2}$$

where $\lambda_3 = \frac{\lambda_1}{\lambda_2}$, λ_1, λ_2, ρ_f, ρ_p, p, T, μ, k_1, and ε denote the Jeffrey parameter (accounting for viscoelasticity), the stress relaxation-time parameter, the strain relaxation-time parameter, the fluid density, the fluid pressure, the fluid temperature, the fluid viscosity, the medium permeability, and the medium porosity, respectively.

Let k_{B}, k_f, k_p, ρ_f, μ_f, and d_p denote the Boltzmann's constant, the thermal conductivities of the base fluid, the thermal conductivities of nanoparticles, the base fluid density, the base fluid viscosity, and the nanoparticles' diameter, respectively. The Brownian diffusion coefficient D_{B} and the thermophoretic diffusion coefficient D_T are defined, respectively, as:

$$D_{\mathrm{B}} = \frac{k_{\mathrm{B}} T}{3\pi \mu_f d_p} \qquad \text{and} \qquad D_T = \frac{\mu_f}{\rho_f} \frac{0.26 k_f}{(2k_f + k_p)} \varphi.$$

The momentum-balance equation of nanoparticles [13, 17, 26–28] is given by

$$\left[\frac{\partial}{\partial t} + \frac{\mathbf{q}_{\mathrm{D}} \cdot \nabla}{\varepsilon}\right] \varphi = D_{\mathrm{B}} \nabla^2 \varphi + \frac{D_T}{T_0} \nabla^2 T. \tag{2.3}$$

The energy-balance equation [13, 17, 26–28] is given by

$$(\rho c)_f \left\{\frac{\partial T}{\partial t} + \mathbf{q}_{\mathrm{D}} \cdot \nabla T\right\} = \kappa_m \nabla^2 T + \varepsilon (\rho c)_p \left\{D_{\mathrm{B}} \nabla \varphi \cdot \nabla T + \frac{D_T}{T_0} \nabla T \cdot \nabla T\right\}, \tag{2.4}$$

where κ_m is the thermal conductivity of porous medium and $(\rho c)_f$ is the heat capacity of fluid.

In non-dimensional form, Eqs. (2.1)–(2.4) can be written by omitting the dashes ($'$) for convenience as:

$$\nabla \cdot q_D = 0, \tag{2.5}$$

$$\left(\frac{1}{\mathrm{Va}} \frac{\partial}{\partial t} + \frac{1}{1+\lambda_3}\right) \nabla^2 w = -\nabla p - \mathrm{Rm}\, \widehat{e_z} + \mathrm{Ra}\, T\, \widehat{e_z} - \mathrm{Rn}\, \varphi\, \widehat{e_z}, \tag{2.6}$$

$$\frac{1}{\sigma} \frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon} q_D \cdot \nabla \varphi = \frac{1}{\mathrm{Le}} \nabla^2 \varphi + \frac{N_A}{\mathrm{Le}} \nabla^2 T, \tag{2.7}$$

$$\frac{\partial T}{\partial t} + q_D \cdot \nabla T = \nabla^2 T + \frac{N_A}{\mathrm{Le}} \nabla \varphi \cdot \nabla T + \frac{N_A N_B}{\mathrm{Le}} \nabla T \cdot \nabla T. \tag{2.8}$$

Here, we have used the non-dimensional variables:

$$(x', y', z') = \left(\frac{x, y, z}{d}\right), \qquad (u', v', w') = \left(\frac{u, v, w}{\kappa_m}\right) d,$$

$$t' = \frac{t \kappa_m}{\sigma d^2}, \qquad p' = \frac{p k_1}{\mu \kappa_m} d^2,$$

$$\varphi' = \frac{\varphi - \varphi_0}{\varphi_1 - \varphi_0}, \qquad T' = \frac{T - T_1}{T_0 - T_1},$$

$$K' = \frac{K}{\gamma_0 E_0 \Delta T d}, \qquad V = \gamma_0 E_0 \beta d V',$$

where $\kappa_m = \frac{k_m}{(\rho c)_f}$ is the thermal diffusivity of the base fluid, $\sigma = \frac{(\rho c)_p}{(\rho c)_f}$ is the thermal capacity ratio, the Prandtl number is $\mathrm{Pr} = \frac{\mu}{\rho_f \kappa_m}$, Darcy's number is $\mathrm{Da} = \frac{k_1}{d^2}$, the Vadasz number is $\mathrm{Va} = \frac{\varepsilon \mathrm{Pr}}{\mathrm{Da}}$, the Rayleigh number is $\mathrm{Ra} = \frac{\rho_f g \beta d k (T_0 - T_1)}{\mu_f \kappa_m}$, nanoparticles' Rayleigh number $\mathrm{Rn} = \frac{(\rho_p - \rho_f)(\varphi_1 - \varphi_0) g k_1 d}{\mu \kappa_m}$, the modified particle density increment is $N_B = \frac{(\rho c)_p}{(\rho c)_f}(\varphi_1 - \varphi_0)$, the Lewis number is $\mathrm{Le} = \frac{\kappa_m}{D_\mathrm{B}}$, the modified diffusivity ratio is $N_A = \frac{D_T (T_0 - T_1)}{D_\mathrm{B} T_1 (\varphi_1 - \varphi_0)}$, the basic density Rayleigh number is $\mathrm{Rm} = \frac{\{\rho_p \varphi + \rho_f + (1 - \varphi_0)\} g k_1 d}{\mu \kappa_m}$, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is a Laplacian operator, and $\nabla_H^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is a horizontal Laplacian operator.

The dimensionless boundary conditions are

$$\begin{aligned} &w = 0, \quad T = T_0, \quad \varphi = \varphi_0 \quad \text{at} \quad z = 0, \\ \text{and} \quad &w = 0, \quad T = T_1, \quad \varphi = \varphi_1 \quad \text{at} \quad z = 1. \end{aligned} \tag{2.9}$$

2.2. Perturbation solutions

Let the physical system be slightly perturbed/disturbed from the equilibrium position. We suppose that the velocity, pressure, temperature, medium porosity, volumetric fraction of nanoparticle perturbed as

$$q_D = 0 + q^*, \quad p = p + p^*, \quad T = T_b + T^*, \quad \varepsilon = \varepsilon + \varepsilon^*, \quad \varphi = \varphi + \varphi^*, \tag{2.10}$$

where q_D^*, p^*, T^*, ε^*, and φ^* are the perturbations overlapped into the physical quantities of the equilibrium state.

Using the perturbation Eq. (2.10) in Eqs. (2.5)–(2.8) and linearising, we obtain the non-dimensional perturbed equations as

$$\nabla \cdot q_D^* = 0, \tag{2.11}$$

$$\left(\frac{1}{\mathrm{Va}}\frac{\partial}{\partial t} + \frac{1}{1 + \lambda_3}\right)\nabla^2 w^* = \mathrm{Ra}\,\nabla_1^2 T^* - \mathrm{Rn}\,\nabla_1^2 \varphi^*, \tag{2.12}$$

$$\frac{1}{\sigma}\frac{\partial \varphi^*}{\partial t} + \frac{q_D^*}{\varepsilon} = \frac{1}{\mathrm{Le}}\nabla^2 \varphi^* + \frac{N_A}{\mathrm{Le}}\nabla^2 T^*, \tag{2.13}$$

$$\frac{\partial T^*}{\partial t} - q_D^* = \nabla^2 T^* + \frac{N_B}{\mathrm{Le}}\left(\nabla T^* - \nabla \varphi^*\right) - \frac{2 N_A N_B}{\mathrm{Le}}\nabla T^*, \tag{2.14}$$

and boundary conditions are

$$w^* = 0, \quad T^* = T_0^*, \quad \varphi^* = \varphi_0^*, \quad \text{at} \quad z = 0,$$
$$\text{and} \quad w^* = 0, \quad T^* = T_1^*, \quad \varphi^* = \varphi_1^*, \quad \text{at} \quad z = 1. \tag{2.15}$$

3. Normal mode analysis

The infinitesimal perturbation is analysed into a complete set of normal modes and then the stability of every mode is examined separately. Thus, we describe the quantities dependence on xy and t of the form $\exp(ilx+imy+\omega t)$, where l and m are the wave numbers in the x and y-direction, respectively, and ω is the growth rate of the disturbances, which is, in general, a complex constant. The perturbations quantities w^*, T^*, φ^*, and V^* are supposed to be of the form

$$w^*, T^*, \phi^* \left(x, y, z, t\right) = \left[W(z), \Theta(z), \Phi(z)\right] \exp\left(ilx + imy + \omega t\right). \tag{3.1}$$

Using expression (3.1), the set of partial differential Eqs. (2.11)–(2.14) reduces to ordinary differential equations:

$$\left(\frac{\omega}{\mathrm{Va}} + \frac{1}{1+\lambda_3}\right)\left(\mathrm{D}^2 - a^2\right) W + a^2 \mathrm{Ra}\, \Theta - a^2 \mathrm{Rn}\, \Phi = 0, \tag{3.2}$$

$$\frac{W}{\varepsilon} - \frac{N_A}{\mathrm{Le}}\left(\mathrm{D}^2 - a^2\right)\Theta - \left\{\frac{1}{\mathrm{Le}}\left(\mathrm{D}^2 - a^2\right) - \frac{\omega}{\sigma}\right\}\Phi = 0, \tag{3.3}$$

$$W + \left\{\frac{N_B}{\mathrm{Le}}\mathrm{D} + \left(\mathrm{D}^2 - a^2\right) - 2\frac{N_A N_B}{\mathrm{Le}} - \omega\right\}\Theta - \frac{N_B}{\mathrm{Le}}\mathrm{D}\,\Phi = 0, \tag{3.4}$$

where $\mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}z}$ and $a^2 = l^2 + m^2$ is the dimensionless resultant wave number.

The set of differential Eqs. (3.2)–(3.4) together with the boundary conditions (2.15) constitute a characteristic value problem for Rayleigh number Ra and given values of the other parameters λ_3, Rn, ε, Le, N_A, N_B, Va whose solutions have to be obtained.

We have applied stress-free conditions for a free surface. Now, the vanishing of the shear stresses tangent to the surface and continuity equation give the boundary conditions for free-free boundary as follows:

$$W = 0, \quad \mathrm{D}^2 W = 0, \quad \Theta = 0, \quad \Phi = 0 \quad \text{at} \quad z = 0$$
$$\text{and} \quad W = 0, \quad \mathrm{D}^2 W = 0, \quad \Theta = 0, \quad \Phi = 0 \quad \text{at} \quad z = 1. \tag{3.5}$$

We assume the solution to W, Θ, and Φ is of the form

$$W = W_0 \sin \pi z, \quad \Theta = \Theta_0 \sin \pi z, \quad \Phi = \varphi_0 \sin \pi z, \tag{3.6}$$

which satisfies boundary conditions (3.5).

Substituting solution (3.6) into Eqs. (3.2)–(3.4), integrating each equation from $z = 0$ to $z = 1$, and performing some integrations by parts, we obtain the following matrix equation:

(3.7)
$$\begin{bmatrix} \left(\frac{1}{1+\lambda_3} + \frac{\omega}{\mathrm{Va}}\right)(a^2+\pi^2) & -a^2\mathrm{Ra} & a^2\mathrm{Rn} \\ 1 & -\left(a^2+\pi^2\right) - \omega & 0 \\ \frac{1}{\varepsilon} & \frac{N_A\left(a^2+\pi^2\right)}{\mathrm{Le}} & \frac{a^2+\pi^2}{\mathrm{Le}} + \frac{\omega}{\sigma} \end{bmatrix} \begin{bmatrix} W \\ \Theta \\ \Phi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

The above matrix equation has a non-trivial solution if

$$\begin{vmatrix} \left(\frac{1}{1+\lambda_3} + \frac{\omega}{\mathrm{Va}}\right)(a^2+\pi^2) & -a^2\mathrm{Ra} & a^2\mathrm{Rn} \\ 1 & -\left(a^2+\pi^2\right) - \omega & 0 \\ \frac{1}{\varepsilon} & \frac{N_A\left(a^2+\pi^2\right)}{\mathrm{Le}} & \frac{a^2+\pi^2}{\mathrm{Le}} + \frac{\omega}{\sigma} \end{vmatrix} = 0, \tag{3.8}$$

which implies

$$\mathrm{Ra} = \frac{\left(\frac{1}{1+\lambda_3} + \frac{\omega}{\mathrm{Va}}\right)\left(\pi^2+a^2\right)\left(\pi^2+a^2+\omega\right)}{a^2} - \frac{\varepsilon N_A\left(\pi^2+a^2\right) + \mathrm{Le}\left(\pi^2+a^2+\omega\right)}{\left(\pi^2+a^2\right)\sigma + \omega\mathrm{Le}} \frac{\sigma}{\varepsilon}\mathrm{Rn}. \tag{3.9}$$

Equation (3.9) is the required dispersion relation accounting for the effect of the Jeffrey parameter, Lewis number, nanoparticle's Rayleigh number, modified diffusivity ratio and medium porosity on the onset of thermal instability in a porous layer saturating a Jeffrey nanofluid.

4. Stationary convection

For the case of steady-state (i.e., the principle of exchange of stability), we put $\omega = 0$ in Eq. (3.9), and obtain

$$\mathrm{Ra}_S = \frac{1}{1+\lambda_3}\frac{\left(a^2+\pi^2\right)^2}{a^2} - \left(N_A + \frac{\mathrm{Le}}{\varepsilon}\right)\mathrm{Rn}. \tag{4.1}$$

Equation (4.1) expresses the Rayleigh number as a function of the dimensionless resultant wave number a, the Jeffrey parameter λ_3, the medium porosity ε, the nanoparticle Rayleigh number Rn, the Lewis number Le, and modified-diffusivity ratio N_A.

Since Eq. (4.1) does not contain the particle increment parameter N_B but contains the diffusivity ratio parameter N_A in association with the nanoparticle Rayleigh number Rn. This indicates that the nanofluid's cross-diffusion terms approach is dominated by the regular cross-diffusion term.

In the absence of the Jeffrey parameter, that is, $\lambda_3 = 0$, Eq. (4.1) becomes

$$\mathrm{Ra}_S = \frac{\left(a^2+\pi^2\right)^2}{a^2} - \left(N_A + \frac{\mathrm{Le}}{\varepsilon}\right)\mathrm{Rn}, \tag{4.2}$$

which is in good agreement with the previous result of Nield and Kuznetsov [16]. In the absence of nanoparticles, that is, $\mathrm{Rn} = 0$ and $N_A = 0$, Eq. (4.1) diminishes to

$$\mathrm{Ra}_S = \frac{\left(a^2+\pi^2\right)^2}{a^2}, \tag{4.3}$$

which is in good agreement with the earlier result of Chandrasekhar [2] for ordinary regular fluids.

5. Analytical and numerical results

To study the effect of the Jeffrey parameter, Lewis number, modified diffusivity ratio of nanoparticle, nanoparticle's Rayleigh number and medium porosity, we examine the behaviour of $\frac{\partial \mathrm{Ra}_S}{\partial \mathrm{Le}}$, $\frac{\partial \mathrm{Ra}_S}{\partial N_A}$, $\frac{\partial \mathrm{Ra}\, a_S}{\partial \mathrm{Rn}}$, and $\frac{\partial \mathrm{Ra}_S}{\partial \varepsilon}$ analytically. For heavy nanoparticles ($\rho_p > \rho_f$), according to the definition, Rn has a negative value, and consequently N_A also has a negative value according to the definition of N_A.

In the following, values of Rn and N_A were set to be negative for most cases if not specified (i.e., bottom-heavy configuration).

From Eq. (4.1), we obtain

$$\frac{\partial \mathrm{Ra}_S}{\partial \lambda_3} = -\frac{1}{\left(1+\lambda_3\right)^2}\frac{\left(\pi^2+a^2\right)^2}{a^2}, \tag{5.1}$$

$$\frac{\partial \mathrm{Ra}_S}{\partial \mathrm{Le}} = -\frac{\mathrm{Rn}}{\varepsilon}, \tag{5.2}$$

$$\frac{\partial \mathrm{Ra}_S}{\partial N_A} = -\mathrm{Rn}, \tag{5.3}$$

$$\frac{\partial \mathrm{Ra}_S}{\partial \mathrm{Rn}} = -\left(N_A + \frac{\mathrm{Le}}{\varepsilon}\right), \tag{5.4}$$

$$\frac{\partial \mathrm{Ra}_S}{\partial \varepsilon} = \frac{\mathrm{Le}\,\mathrm{Rn}}{\varepsilon^2}. \tag{5.5}$$

From Eq. (5.1), we see that the partial derivative of Rayleigh number Ra_S with respect to the Jeffrey parameter λ_3 is negative, implying that the Jeffrey parameter postpones the stationary convection. Thus, the Jeffrey parameter has as destabilising effect on the system for both top-heavy and bottom-heavy nanoparticle distribution, which is in agreement with the result derived by SHEHZAD *et al.* [8], IMTIAZ *et al.* [9], HAYAT *et al.* [11] and IMRAN *et al.* [12].

The right-hand sides of Eqs. (5.2) and (5.3) are negative if nanoparticle's Rayleigh number Rn is positive, but for the bottom-heavy nanoparticle's distribution, Rn is negative. Thus, the Lewis number Le and modified diffusivity ratio N_A have stabilising effect on the system for the bottom-heavy nanoparticle distribution, which is in agreement with the result obtained by SHEU [24–25], CHAND and RANA [19–20], and YADAV *et al.* [21–23]. Equation (5.4) indicates that nanoparticles' Rayleigh number destabilises the system. The right-hand side of Eq. (5.5) is positive but it will be negative if Rn is negative, implying that the medium porosity has a destabilising effect on the system, which is in agreement with the results derived by NIELD and KUZNETSOV [13], CHAND and RANA [19, 20] and YADAV *et al.* [21–23].

The dispersion relation (4.1) is also analysed numerically. Graphs have been plotted by giving some numerical values to the parameters to depict the stability characteristics, e.g., the Lewis number ($10^2 \leq \mathrm{Le} \leq 10^4$), nanoparticles' Rayleigh number ($-1 \leq \mathrm{Rn} \leq 10$), and porosity parameter ($0.1 \leq \varepsilon \leq 1$) (RANA *et al.* [26–28]).

In Figs. 2, 5, and 6, it is observed that the Rayleigh number increases with the decrease in the Jeffrey parameter, nanoparticles' Rayleigh number and medium porosity, implying that the Jeffrey parameter, nanoparticles' Rayleigh number and medium porosity postpone the stationary convection. In Fig. 3, the variations of the thermal Rayleigh number with the wave number for three different values of the Lewis number are observed, which shows that the Rayleigh number increases with the increase of the Lewis number. Thus, the Lewis number advances the stationary convection, which verifies the analytical result. In Fig. 4, it is found that the Rayleigh number decreases very slightly with the decrease in the modified diffusivity ratio, implying thereby that the modified diffusivity ratio has a slightly stabilising effect on the system. Thus, the effect of increasing N_A is to stabilise the system when Rn is negative.

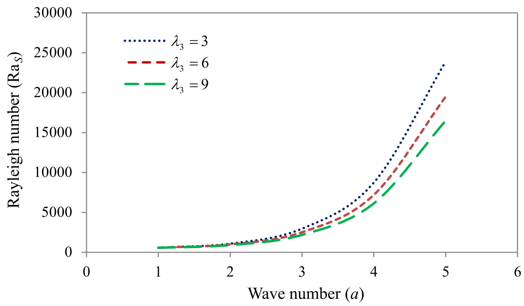

FIG. 2. The stationary Rayleigh number (Ra_S) versus wave number (a) for different values of the Jeffrey parameter (λ_3).

Table 1. Numerical values of the stationary Rayleigh number (Ra_S) with respect to wave number (a) for different values of the Jeffrey parameter (λ_3).

a	λ_3	ε	N_A	Rn	Le	Ra_S
1	0.3	0.2	−5	−0.1	1000	590.4261
2	0.3	0.2	−5	−0.1	1000	1091.614
3	0.3	0.2	−5	−0.1	1000	2965.213
4	0.3	0.2	−5	−0.1	1000	8737.886
5	0.3	0.2	−5	−0.1	1000	23885.42
1	0.6	0.2	−5	−0.1	1000	573.3775
2	0.6	0.2	−5	−0.1	1000	980.5923
3	0.6	0.2	−5	−0.1	1000	2502.892
4	0.6	0.2	−5	−0.1	1000	7193.189
5	0.6	0.2	−5	−0.1	1000	19500.56
1	0.9	0.2	−5	−0.1	1000	561.7126
2	0.9	0.2	−5	−0.1	1000	904.6304
3	0.9	0.2	−5	−0.1	1000	2186.567
4	0.9	0.2	−5	−0.1	1000	6136.29
5	0.9	0.2	−5	−0.1	1000	16500.39

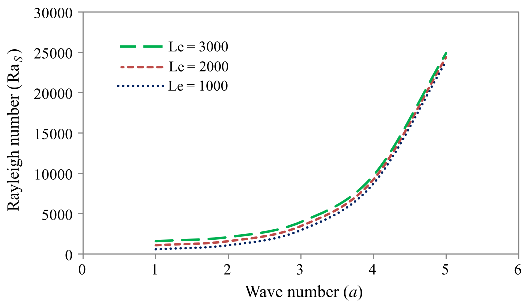

FIG. 3. The stationary Rayleigh number (Ra_S) versus wave number (a) for different values of the Lewis number (Le).

Table 2. Numerical values of the stationary Rayleigh number (Ra_S) with respect to wave number (a) for different values of the Lewis number (Le).

a	λ_3	ε	N_A	Rn	Le	Ra_S
1	0.3	0.2	−5	−0.1	1000	590.4261
2	0.3	0.2	−5	−0.1	1000	1091.614
3	0.3	0.2	−5	−0.1	1000	2965.213
4	0.3	0.2	−5	−0.1	1000	8737.886
5	0.3	0.2	−5	−0.1	1000	23885.42
1	0.3	0.2	−5	−0.1	2000	1090.426
2	0.3	0.2	−5	−0.1	2000	1591.614
3	0.3	0.2	−5	−0.1	2000	3465.213
4	0.3	0.2	−5	−0.1	2000	9237.886
5	0.3	0.2	−5	−0.1	2000	24385.42
1	0.3	0.2	−5	−0.1	3000	1590.426
2	0.3	0.2	−5	−0.1	3000	2091.614
3	0.3	0.2	−5	−0.1	3000	3965.213
4	0.3	0.2	−5	−0.1	3000	9737.886
5	0.3	0.2	−5	−0.1	3000	24885.42

Table 3. Numerical values of the stationary Rayleigh number (Ra_S) with respect to wave number (a) for different values of the modified diffusivity ratio (N_A).

a	λ_3	ε	N_A	Rn	Le	Ra_S
1	0.3	0.2	−5	−0.1	1000	1090.426
2	0.3	0.2	−5	−0.1	1000	1091.614
3	0.3	0.2	−5	−0.1	1000	2965.213
4	0.3	0.2	−5	−0.1	1000	8737.886
5	0.3	0.2	−5	−0.1	1000	23885.42
1	0.3	0.2	−50	−0.1	1000	1085.926
2	0.3	0.2	−50	−0.1	1000	1087.114
3	0.3	0.2	−50	−0.1	1000	2960.713
4	0.3	0.2	−50	−0.1	1000	8733.386
5	0.3	0.2	−50	−0.1	1000	23880.92
1	0.3	0.2	−95	−0.1	1000	1081.426
2	0.3	0.2	−95	−0.1	1000	1082.614
3	0.3	0.2	−95	−0.1	1000	2956.213
4	0.3	0.2	−95	−0.1	1000	8728.886
5	0.3	0.2	−95	−0.1	1000	23876.42

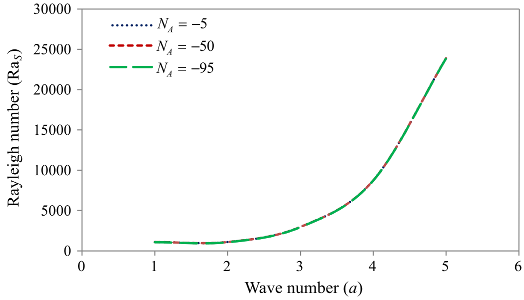

FIG. 4. The stationary Rayleigh number (Ra_S) versus wave number (a) for different values of the modified diffusivity ratio (N_A).

Table 4. Numerical values of the stationary thermal Darcy-Rayleigh number (Ra_S) with respect to wave number (a) for different values of the nanoparticles' Rayleigh number (Rn).

a	λ_3	ε	N_A	Rn	Le	Ra_S
1	0.3	0.2	−5	−0.1	1000	590.4261
2	0.3	0.2	−5	−0.1	1000	1091.614
3	0.3	0.2	−5	−0.1	1000	2965.213
4	0.3	0.2	−5	−0.1	1000	8737.886
5	0.3	0.2	−5	−0.1	1000	23885.42
1	0.3	0.2	−5	−0.3	1000	1589.426
2	0.3	0.2	−5	−0.3	1000	2090.614
3	0.3	0.2	−5	−0.3	1000	3964.213
4	0.3	0.2	−5	−0.3	1000	9736.886
5	0.3	0.2	−5	−0.3	1000	24884.42
1	0.3	0.2	−5	−0.5	1000	2588.426
2	0.3	0.2	−5	−0.5	1000	3089.614
3	0.3	0.2	−5	−0.5	1000	4963.213
4	0.3	0.2	−5	−0.5	1000	10735.89
5	0.3	0.2	−5	−0.5	1000	25883.42

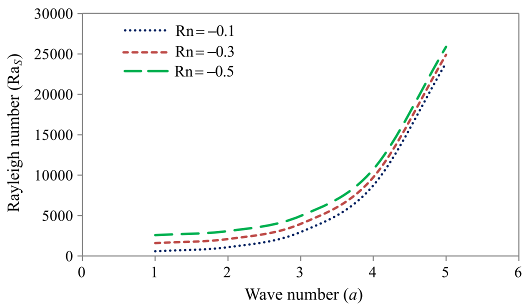

FIG. 5. The stationary Rayleigh number (Ra_S) versus wave number (a) for different values of the nanoparticles' Rayleigh number (Rn).

Table 5. Numerical values of the stationary Rayleigh number (Ra_S) with respect to wave number (a) for different values of the medium porosity (ε).

a	λ_3	ε	N_A	Rn	Le	Ra_S
1	0.3	0.2	−5	−0.1	1000	590.4261
2	0.3	0.2	−5	−0.1	1000	1091.614
3	0.3	0.2	−5	−0.1	1000	2965.213
4	0.3	0.2	−5	−0.1	1000	8737.886
5	0.3	0.2	−5	−0.1	1000	23885.42
1	0.3	0.2	−5	−0.3	1000	1589.426
2	0.3	0.2	−5	−0.3	1000	2090.614
3	0.3	0.2	−5	−0.3	1000	3964.213
4	0.3	0.2	−5	−0.3	1000	9736.886
5	0.3	0.2	−5	−0.3	1000	24884.42
1	0.3	0.2	−5	−0.5	1000	2588.426
2	0.3	0.2	−5	−0.5	1000	3089.614
3	0.3	0.2	−5	−0.5	1000	4963.213
4	0.3	0.2	−5	−0.5	1000	10735.89
5	0.3	0.2	−5	−0.5	1000	25883.42

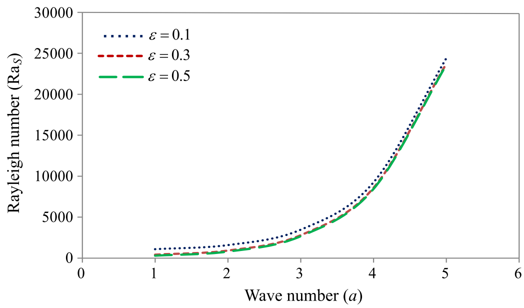

Fig. 6. The stationary Rayleigh number (Ra_S) versus wave number (a) for different values of the medium porosity (ε).

6. Conclusion

The onset of thermal instability in a layer of porous medium saturating a Jeffrey nanofluid was investigated analytically and numerically for free-free boundaries. The behaviour of various dimensionless parameters on the onset of stationary convection was analysed analytically and graphically. The main concluding remarks are as follows:

- The Jeffrey parameter stabilises the stationary convection for both top/bottom-heavy configurations. The Rayleigh number for a Jeffrey dielectric nanofluid saturated porous layer is always smaller than when a regular nanofluid is involved.

- The Lewis number and the modified diffusivity ratio destabilise the physical system slightly for both the top/bottom-heavy distribution.
- The nanoparticles' Rayleigh number destabilise the system for both top/bottom-heavy configurations.
- The medium porosity postponed/advanced the stationary convection for bottom/top-heavy nanoparticles' distribution.

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Received July 5, 2021; accepted version April 7, 2022.