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A COMPARATIVE STUDY OF VARIOUS MODELS OF EQUIVALENT PLASTIC STRAIN TO FRACTURE

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Abstract. *For more then half a centre just the same approach to the simulation of the ductile crack formation was developed independently by the scientific communities of foreign and native researchers. The importance at these studies drastically increased. A set of the characteristics, according to which it is recommendly to perform the detail comparison of the existing fracture models is developed. The examples of the analysis of a number of the most popular models by means of obtaining and study their analytical expressions regarding the conditions of the plane state are given. The generalized relations of the know models and a number of separate relations are obtained.*

Keywords: ductile fracture criteria, fracture graph, equivalent plastic strain to fracture, stress triaxiality, plane strain

BADANIE PORÓWNAWCZE RÓŻNYCH MODELI RÓWNOWAŻNEGO ODKSZTAŁCENIA PLASTYCZNEGO DO PĘKANIA

Streszczenie. *Od ponad pół wieku to samo podejście do modelowania uszkodzeń podczas odkształceń plastycznych jest opracowywane niezależnie przez zespoły naukowe złożone z naukowców zagranicznych i krajowych. W ostatnich dziesięcioleciach znaczenie tych badań dramatycznie wzrosło. Opracowano zestaw cech, zgodnie z którymi proponuje się przeprowadzenie szczegółowego porównania istniejących modeli zniszczenia. Podano przykłady analizy szeregu najpopularniejszych modeli poprzez uzyskanie i badanie ich wyrażeń analitycznych dla warunków płaskiego stanu naprężenia. Otrzymano uogólnione wskaźniki znanych modeli oraz szereg wskaźników indywidualnych.*

Słowa kluczowe: kryteria powstawania pęknięć ciągliwych, wykres pękania, równoważne odkształcenie plastyczne do pękania, trójosiowość naprężeń, naprężenie płaskie

Introduction

In [1, 6, 8, 11, 12, 20, 23, 27, 28] ductile crack formation theory is constructed. In contrast to the approach [31, 33, 36], devoted to the study of crack growth, the approach considered in this paper is based on the models of summation of continuous material damage. In [20] fracture model, the base of the model being linear damage hypothesis and a notion of the fracture diagram which gives the equivalent plastic strain to fracture as a function of the stress state, is suggested.

In [12, 28] a fracture model, based on nonlinear damage hypothesis , is developed. Approximation of the fracture diagram is proposed. The technique of studying the equivalent plastic strain to fracture on conditions of common torsion and tension of the cylindrical samples, according to various trajectories of the plastic strain dependence on the stress triaxiality, is suggested.

Tensor – linear model of the initially isotropic body is developed in [11]. Generalization of the given theory for the case the initial anisotropy of the equivalent plastic strain to fracture is given in [26], and for the case of non-linear dependence between the increments of the damage and deformation tensors is presented in [27]. In [23] on the base of the tensor–nonlinear model the effects of change of the equivalent plastic strain to fracture are determined analytically and proved experimentally at two-stage deformation, which are not within the frame of the tensor–linear model. The tensor–nonlinear model, which represents the healing of damages, applying for a hot-forming method, is developed and investigation in [27]. On the base of this model in [21], modes were determined, at which the material transforms into the super plasticity state, and the optimization problem is considered. The formulation of a new problem within the framework of this approach and interesting results of its solution were obtained in [24].

One of the key problems of the phenomenological theory of ductile crack formation is the analytical presentation of the dependence of the equivalent plastic strain to fracture on the invariants of the stress and strain states.

The studies of the foreign researchers are devoted to the solution of this problem [3, 4, 5, 9, 10, 29, 32], were published approximately at the same historic period. Judging by the published materials, these directions were developing independently from each other. Therefore, it is advisable to conduct a comparative study of these results. It should be noted, that the popularity of the given foreign researches reached a high level and continues to grow, as it is seen from our studies, presented in table 1 and diagrams in Fig 1. In our opinion, certain numerical data of this popularity could be considered as an indirect index of the relevance of the corresponding studies.

In any of the modern studies [7, 19, 22, 34, 35], an attempt is made to draw at least a fragmentary comparisons of the ductile crack formation theories. The comparison results shows that the scientific community does not completely realize that these research are referred to the same phenomenological approach to the construction of the limit state theory, general statements of which are proposed in [18] and specified in [27].

The lack of mutual references in the above-mentioned publications disorients the scientific community, that uses fracture models. Many researchers have a distorted notion regarding the above-mentioned scientific directions fracture simulation, as those, based on basically different concepts. However, a similar situation is not natural and slows down the intensity of the process of obtaining new results in the limit states theory of the materials.

Table 1. Number of citations of the papers scientometric bases Google Schoolar, Web of Science and Scopus

Reference links to the paper from the list of references	Number of citations Google Schoolar	Number of citations Web of Science			Number of citations Scopus		
		Totally	In 2018	Percentage of citations in 2018, %	Totally	In 2018	Percentage of citations in 2018, %
[32]	4288	2528	-	-	2904	-	-
[10]	1649	793	-	-	-	-	-
[2]	831	497	91	18	584	93	16
[4]	1169	700	108	15	829	118	14

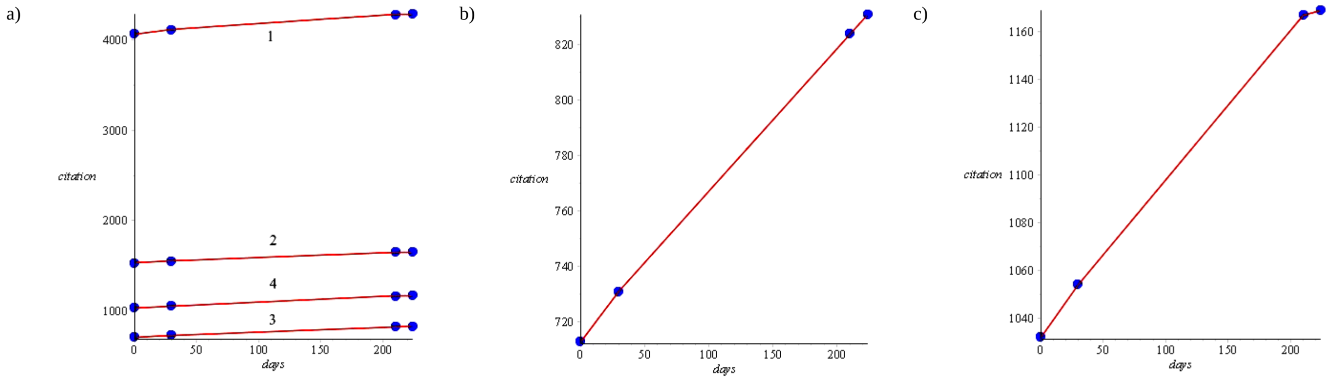

Fig. 1. Results of the publication citation according to scientometric bases: a) Google schoolar: curves 1 – 4 are constructed on the base of the relations [2, 4, 10, 32]; b), c) Scopus: on the base of the relations [2, 4], correspondingly

1. Aim of the research

The aim of the paper is to develop and substantiate of the classification characteristics, according to which the comparative study of the fracture models can be performed and carry out the corresponding analysis of a number of the most popular models, suggested both by foreign and native scientists.

2. Modeling

In foreign studies fracture models are mainly suggested in the form of the integral models, these models fake into consideration the impact on limit values of the equivalent plastic strain to fracture the change of the stress state during the deformation. In such form these models are shown in table 2.

An important characteristic of any model is the number of the parameters, to be determined on the base of the experimental data. Experiments, aimed at the determined on the equivalent plastic strain to fracture in case of different stress states are rather labor intensive and expensive. Thus, the desire to obtain models with an as small number of parameters as possible (one-, two-, three parametric) is quite reasonable.

Depending on the stress-state, when limit equivalent plastic strain to fracture is experimentally determined, the general model obtains the separate form. Corresponding relations are given in table 2.

The approach to the considered fracture simulation is based on the notion of the surface of limit equivalent plastic strain to fracture $\bar{\varepsilon}_{fs}$ depending on the indices, characterizing the no-changeable during the study of one sample stress state of the material. A greater part of the experimental data is obtained on the condition of the plane stress. In this case, the surface of the limit equivalent plastic strain to fracture is converted into the curve

$$\bar{\varepsilon}_{fs} = \bar{\varepsilon}_{fs}(\eta), \ -2 \le \eta \le 2 \qquad (1)$$

where the stress triaxiality η

$$\eta = \frac{3 \cdot \sigma_m}{\bar{\sigma}} \qquad (2)$$

σ_m, $\bar{\sigma}$ – mean stress and equivalent stress (von Mises) respectively.

In different studies the indices of the type (2) are used with different numerical coefficients, it is not essential. All the models, shown in table 2, we converted into (1) and accumulated in table 3. Other known analytical relations are also given in this table.

Table 2. Limit state models

Limit state model's name		Model	
Cockcroft and Latham–Oh crack formation criterion	Mathematical model in general form [4, 10, 29]	$\int_0^{\bar{\varepsilon}_f} \frac{\langle \sigma_1 \rangle}{\bar{\sigma}} \cdot d\bar{\varepsilon} = C$	(3)
	Determination of the model parameter by the results of the equivalent plastic strain to fracture in shear	$\int_0^{\bar{\varepsilon}_f} \frac{\langle \sigma_1 \rangle}{\bar{\sigma}} \cdot d\bar{\varepsilon} = \varepsilon_k^*$	(4)
	Determination of the model parameter by the results of the equivalent plastic strain to fracture in tension	$\int_0^{\bar{\varepsilon}_f} \frac{\langle \sigma_1 \rangle}{\bar{\sigma}} \cdot d\bar{\varepsilon} = \varepsilon_p^*$	(5)
	Determination of the model parameter by the results of the equivalent plastic strain to fracture in equibiaxial tension	$\int_0^{\bar{\varepsilon}_f} \frac{\langle \sigma_1 \rangle}{\bar{\sigma}} \cdot d\bar{\varepsilon} = \bar{\varepsilon}_{fs}(2)$	(6)
	Determination of the model parameter by the results of the equivalent plastic strain to fracture in nonequibiaxial tension	$\int_0^{\bar{\varepsilon}_f} \frac{\langle \sigma_1 \rangle}{\bar{\sigma}} \cdot d\bar{\varepsilon} = \bar{\varepsilon}_{fs}(1,5)$	(7)
Hydrostatic stress criterion		$\int_0^{\bar{\varepsilon}_f} \frac{\sigma_m}{\bar{\sigma}} \cdot d\bar{\varepsilon} = C$	(8)
Clift criterion [9]		$\int_0^{\bar{\varepsilon}_f} \bar{\sigma} \cdot d\bar{\varepsilon} = C$	(9)

Table 2 (continuation). Limit state models

Limit state model's name		Model	
Rice-Tracey (high stress triaxiality approximation)	Mathematical model in general form [32]	$\int_0^{\bar{\varepsilon}_f} \exp\left(\frac{\eta}{2}\right) \cdot d\bar{\varepsilon} = C$	(10)
	Determination of the model parameter by the results of the equivalent plastic strain to fracture in compression	$\int_0^{\bar{\varepsilon}_f} \exp\left(\frac{\eta}{2}\right) \cdot d\bar{\varepsilon} = \varepsilon_c^*$	(11)
	Determination of the model parameter by the results of the equivalent plastic strain to fracture in shear	$\int_0^{\bar{\varepsilon}_f} \exp\left(\frac{\eta}{2}\right) \cdot d\bar{\varepsilon} = \varepsilon_k^*$	(12)
	Determination of the model parameter by the results of the equivalent plastic strain to fracture in tension	$\int_0^{\bar{\varepsilon}_f} \exp\left(\frac{\eta}{2}\right) \cdot d\bar{\varepsilon} = \varepsilon_p^*$	(13)
	Determination of the model parameter by the results of the equivalent plastic strain to fracture in equibiaxial compression	$\int_0^{\bar{\varepsilon}_f} \exp\left(\frac{\eta}{2}\right) \cdot d\bar{\varepsilon} = \bar{\varepsilon}_{fs}(-2)$	(14)
	Determination of the model parameter by the results of the equivalent plastic strain to fracture in nonequibiaxial compression	$\int_0^{\bar{\varepsilon}_f} \exp\left(\frac{\eta}{2}\right) \cdot d\bar{\varepsilon} = \bar{\varepsilon}_{fs}\left(-\frac{3}{2}\right)$	(15)
	Determination of the model parameter by the results of the equivalent plastic strain to fracture in equibiaxial tension	$\int_0^{\bar{\varepsilon}_f} \exp\left(\frac{\eta}{2}\right) \cdot d\bar{\varepsilon} = \bar{\varepsilon}_{fs}(2)$	(16)
	Determination of the model parameter by the results of the equivalent plastic strain to fracture in nonequibiaxial tension	$\int_0^{\bar{\varepsilon}_f} \exp\left(\frac{\eta}{2}\right) \cdot d\bar{\varepsilon} = \bar{\varepsilon}_{fs}\left(\frac{3}{2}\right)$	(17)

Table 3. Limit state models of the material relatively the stress triaxiality index

	Models of the equivalent plastic strain to fracture in the plane stress condition	
(3)	$\bar{\varepsilon}_{fs}(\eta) = \frac{C}{\eta + 2 \cdot \cos\left[\frac{1}{3} \cdot \arccos\left(0,5 \cdot \eta \cdot \left(3 - \eta^2\right)\right)\right]}, \quad -1 < \eta \leq 2$	(18)
(4)	$\bar{\varepsilon}_{fs}(\eta) = \frac{\sqrt{3} \cdot \varepsilon_k^*}{\eta + 2 \cdot \cos\left[\frac{1}{3} \cdot \arccos\left(0,5 \cdot \eta \cdot \left(3 - \eta^2\right)\right)\right]}, \quad -1 < \eta \leq 2$	(19)
(5)	$\bar{\varepsilon}_{fs}(\eta) = \frac{3 \cdot \varepsilon_p^*}{\eta + 2 \cdot \cos\left[\frac{1}{3} \cdot \arccos\left(0,5 \cdot \eta \cdot \left(3 - \eta^2\right)\right)\right]}, \quad -1 < \eta \leq 2$	(20)
(6)	$\bar{\varepsilon}_{fs}(\eta) = \frac{3 \cdot \bar{\varepsilon}_{fs}(2)}{\eta + 2 \cdot \cos\left[\frac{1}{3} \cdot \arccos\left(0,5 \cdot \eta \cdot \left(3 - \eta^2\right)\right)\right]}, \quad -1 < \eta \leq 2$	(21)
(7)	$\bar{\varepsilon}_{fs}(\eta) = \frac{2 \cdot \sqrt{3} \cdot \bar{\varepsilon}_{fs}(\sqrt{3})}{\eta + 2 \cdot \cos\left[\frac{1}{3} \cdot \arccos\left(0,5 \cdot \eta \cdot \left(3 - \eta^2\right)\right)\right]}, \quad -1 < \eta \leq 2$	(22)
(8)	$\bar{\varepsilon}_{fs}(\eta) = \frac{3 \cdot C}{\eta}, \quad \eta > 0$	(23)
(9)	$\bar{\varepsilon}_{fs}(\eta) = \frac{3 \cdot \sigma_m \cdot C}{\eta}, \quad -2 \leq \eta \leq 2$	(24)
Dell models [2]	$\bar{\varepsilon}_{fs}(\eta) = \frac{\varepsilon_c \cdot \varepsilon_k^*}{\varepsilon_c + \eta \cdot \left(\varepsilon_c - e \cdot \varepsilon_k^*\right)} \cdot e^{-\eta}, \quad -1 \leq \eta \leq 1$	(25)
	$\bar{\varepsilon}_{fs}(\eta) = \frac{\varepsilon_k^*}{1 + \eta} \cdot e^{-\eta}$	(26)
(10)	$\bar{\varepsilon}_{fs}(\eta) = C \cdot \exp\left(-\frac{\eta}{2}\right), \quad -2 \leq \eta \leq 2$	(27)
(11)	$\bar{\varepsilon}_{fs}(\eta) = \frac{\varepsilon_c^*}{\sqrt{e}} \cdot \exp\left(-\frac{\eta}{2}\right), \quad -2 \leq \eta \leq 2$	(28)

Table 3(continuation). Limit state models of the material relatively the stress triaxiality index

	Models of the equivalent plastic strain to fracture in the plane stress condition	
(12)	$\bar{\varepsilon}_{fs}(\eta)=\varepsilon_k^*\cdot\exp\left(-\frac{\eta}{2}\right),\quad -2\le\eta\le 2$	(29)
(13)	$\bar{\varepsilon}_{fs}(\eta)=\sqrt{e}\cdot\varepsilon_p^*\cdot\exp\left(-\frac{\eta}{2}\right),\quad -2\le\eta\le 2$	(30)
(14)	$\bar{\varepsilon}_{fs}(\eta)=\frac{\bar{\varepsilon}_{fs}(-2)}{e}\cdot\exp\left(-\frac{\eta}{2}\right),\quad -2\le\eta\le 2$	(31)
(15)	$\bar{\varepsilon}_{fs}(\eta)=\frac{\bar{\varepsilon}_{fs}\left(-\frac{3}{2}\right)}{\sqrt[4]{e^3}}\cdot\exp\left(-\frac{\eta}{2}\right),\quad -2\le\eta\le 2$	(32)
(16)	$\bar{\varepsilon}_{fs}(\eta)=e\cdot\bar{\varepsilon}_{fs}(2)\cdot\exp\left(-\frac{\eta}{2}\right),\quad -2\le\eta\le 2$	(33)
(17)	$\bar{\varepsilon}_{fs}(\eta)=\sqrt[4]{e^3}\cdot\bar{\varepsilon}_{fs}\left(\frac{3}{2}\right)\cdot\exp\left(-\frac{\eta}{2}\right),\quad -2\le\eta\le 2$	(34)
[26]	$\bar{\varepsilon}_{fs}(\eta)=\varepsilon_k^*\cdot\exp\left(-\eta\cdot\ln\left(\frac{(1-\eta)\cdot\varepsilon_c^*}{2\cdot\varepsilon_k^*}+\frac{(1+\eta)\cdot\varepsilon_k^*}{2\cdot\varepsilon_p^*}\right)\right),\quad -2\le\eta\le 2$	(35)
	$\bar{\varepsilon}_{fs}(\eta)=\varepsilon_k\cdot\left(\frac{\varepsilon_p}{\varepsilon_c}\right)^{\frac{\eta}{2}}\cdot\left(\frac{\varepsilon_p\cdot\varepsilon_c}{\varepsilon_k^2}\right)^{\frac{\eta^2}{2}},\quad -2\le\eta\le 2$	(36)

Designations:
σ_1 – the maximum principal tensile stress; $\bar{\varepsilon}_f$ – equivalent plastic strain to fracture on conditions an arbitrary process of the deformation (*η= const* or *η≠ const*);
C = const; $\langle\sigma_1\rangle=\begin{cases}\sigma_1, \sigma_1\ge 0\\ 0, \sigma_1<0\end{cases}$; ε_c^* – equivalent plastic strain to fracture in compression; ε_k^* – equivalent plastic strain to fracture in shear; ε_p^* – equivalent plastic strain to fracture in tension

3. Analyses of the obtained relations enables

The analysis of the obtained relations enables to obtain a number of regularities in the variations of the limit equivalent plastic strain to fracture with the increase of the stress triaxiality, represented by the corresponding models. From the relations (18)–(24), (27)–(34) it follows that the form of the given dependence in invariant relatively the value of the parameter for one-parametric Cockcroft and Latham–Oh models, hydrostatic stress, Rice-Tracey and Dell models. All the dependencies are concave and monotone decreasing functions (except Cockcroft and Latham–Oh models) on the domain of definition. Cockcroft and Latham–Oh model reaches the minimal value

$$\left(\bar{\varepsilon}_{fs}\right)_{\min}=\bar{\varepsilon}_{fs}(\eta=\sqrt{3})=\frac{\sqrt{3}}{6}\cdot C \tag{37}$$

We will build the fracture model, based on the relation (25)

$$\int_0^{\bar{\varepsilon}_f}\frac{\varepsilon_c+\eta\cdot(\varepsilon_c-e\cdot\varepsilon_k)}{\varepsilon_c\cdot\varepsilon_k}\cdot e^{\eta}\cdot d\bar{\varepsilon}=1 \tag{38}$$

This model refers to the family of two-parametric models. To make use of the model it is necessary to have the value of the equivalent plastic strain to fracture on conditions of the compression strain ε_c^* and shear strain ε_k^*. We will generalize this model for the case when limiting values of plastic deformation are known for the arbitrary values of stress triaxiality η:

$$\bar{\varepsilon}_{fs}=\bar{\varepsilon}_{f1}, \bar{\varepsilon}_{fs}=\bar{\varepsilon}_{f2} \text{ correspondingly if } \eta=\eta_1, \eta=\eta_2 \tag{39}$$

On conditions of the stationary deformation for two different values of η_1,η_2 on the base of the model (38) we obtain:

$$\bar{\varepsilon}_{f1}=\frac{\varepsilon_c\cdot\varepsilon_k\cdot e^{-\eta_1}}{\varepsilon_c+\eta_1\cdot(\varepsilon_c-e\cdot\varepsilon_k)};\quad \bar{\varepsilon}_{f2}=\frac{\varepsilon_c\cdot\varepsilon_k\cdot e^{-\eta_2}}{\varepsilon_c+\eta_2\cdot(\varepsilon_c-e\cdot\varepsilon_k)} \tag{40}$$

The system of two linear equations (40) relatively ε_c^* and ε_k^* will be solved, as a result we obtain

$$\varepsilon_c^*=\frac{\bar{\varepsilon}_{f1}\cdot\bar{\varepsilon}_{f2}\cdot e\cdot(\eta_2-\eta_1)}{\bar{\varepsilon}_{f2}\cdot e^{-\eta_1}\cdot(1+\eta_2)-\bar{\varepsilon}_{f1}\cdot e^{-\eta_2}\cdot(1+\eta_1)};$$
$$\varepsilon_k^*=\frac{\bar{\varepsilon}_{f1}\cdot\bar{\varepsilon}_{f2}\cdot(\eta_2-\eta_1)}{\bar{\varepsilon}_{f2}\cdot\eta_2\cdot e^{-\eta_1}-\bar{\varepsilon}_{f1}\cdot\eta_1\cdot e^{-\eta_2}},\ \eta_1\ne\eta_2 \tag{41}$$

Taking into account the later relations, the model (38) will obtain the form

$$\int_0^{\bar{\varepsilon}_f}\frac{\bar{\varepsilon}_{f2}\cdot e^{-\eta_1}\cdot(\eta_2-\eta)-\bar{\varepsilon}_{f1}\cdot e^{-\eta_2}\cdot(\eta_1-\eta)}{\bar{\varepsilon}_{f1}\cdot\bar{\varepsilon}_{f2}\cdot(\eta_2-\eta_1)}\cdot e^{\eta}\cdot d\bar{\varepsilon} \tag{42}$$

Fracture diagram, that follows from the model (42) has the form

$$\bar{\varepsilon}_{fs}(\eta)=\frac{\bar{\varepsilon}_{f1}\cdot\bar{\varepsilon}_{f2}\cdot(\eta_2-\eta_1)}{\bar{\varepsilon}_{f1}\cdot e^{-\eta_2}\cdot(\eta-\eta_1)-\bar{\varepsilon}_{f2}\cdot e^{-\eta_1}\cdot(\eta-\eta_2)}\cdot e^{-\eta} \tag{43}$$

In separate case if

$$\bar{\varepsilon}_{f1}=\varepsilon_c^*, \bar{\varepsilon}_{f2}=\varepsilon_k^*, \eta_1=-1, \eta_2=0 \text{ or}$$
$$\bar{\varepsilon}_{f2}=\varepsilon_c^*, \bar{\varepsilon}_{f1}=\varepsilon_k^*, \eta_2=-1, \eta_1=0 \tag{44}$$

the relation (43) becomes identical to the model (26).

On the base of the model (43) it is easy to obtain many other relations, which use the values of the equivalent plastic strains at fracture in case of typical tests. In the values of the equivalent plastic strains at fractures are used as such tests according to five various values of stress triaxiality η={-2, -1, 0, 1, 2}, then we can obtain the number of the separate relations that are equal to the number of combinations of n=5 elements taken r=2 at a time, i. e.

$$C_5^2=\frac{5!}{2!\cdot(5-2)!}=10 \tag{45}$$

We will give these relations:

$$\bar{\varepsilon}_{fs}(\eta)=\frac{2\cdot\varepsilon_c^*\cdot\varepsilon_p^*\cdot e^{-\eta}}{(1+\eta)\cdot e^{-1}\cdot\varepsilon_c^*+(1-\eta)\cdot e\cdot\varepsilon_p^*}; \quad (46)$$

$$\left(\varepsilon_1=\varepsilon_c^*,\ \varepsilon_2=\varepsilon_p^*,\ \eta_1=-1,\ \eta_2=1\right)$$

$$\bar{\varepsilon}_{fs}(\eta)=\frac{\varepsilon_k^*\cdot\varepsilon_p^*\cdot e^{-\eta}}{\eta\cdot e^{-1}\cdot\varepsilon_k^*+(1-\eta)\cdot e^{0}\cdot\varepsilon_p^*}; \quad (47)$$

$$\left(\varepsilon_1=\varepsilon_k^*,\ \varepsilon_2=\varepsilon_p^*,\ \eta_1=0,\ \eta_2=1\right)$$

$$\bar{\varepsilon}_{fs}(\eta)=\frac{4\cdot\varepsilon_{\eta=-2}\cdot\varepsilon_{\eta=2}\cdot e^{-\eta}}{(2+\eta)\cdot e^{-2}\cdot\varepsilon_{\eta=-2}+(2-\eta)\cdot e^{2}\cdot\varepsilon_{\eta=2}}; \quad (48)$$

$$\left(\varepsilon_1=\varepsilon_{\eta=-2},\ \varepsilon_2=\varepsilon_{\eta=2},\ \eta_1=-2,\ \eta_2=2\right)$$

$$\bar{\varepsilon}_{fs}(\eta)=\frac{\varepsilon_p^*\cdot\varepsilon_{\eta=2}\cdot e^{-\eta}}{(-1+\eta)\cdot e^{-2}\cdot\varepsilon_p^*+(2-\eta)\cdot e^{-1}\cdot\varepsilon_{\eta=2}}; \quad (49)$$

$$\left(\varepsilon_1=\varepsilon_p^*,\ \varepsilon_2=\varepsilon_{\eta=2},\ \eta_1=1,\ \eta_2=2\right)$$

$$\bar{\varepsilon}_{fs}(\eta)=\frac{\varepsilon_{\eta=-2}\cdot\varepsilon_c^*\cdot e^{-\eta}}{(2+\eta)\cdot e\cdot\varepsilon_{\eta=-2}+(-1-\eta)\cdot e^{2}\cdot\varepsilon_c^*}; \quad (50)$$

$$\left(\varepsilon_1=\varepsilon_{\eta=-2},\ \varepsilon_2=\varepsilon_c^*,\ \eta_1=-2,\ \eta_2=-1\right)$$

$$\bar{\varepsilon}_{fs}(\eta)=\frac{2\cdot\varepsilon_k^*\cdot\varepsilon_{\eta=2}\cdot e^{-\eta}}{\eta\cdot e^{-2}\cdot\varepsilon_k^*+(2-\eta)\cdot e^{0}\cdot\varepsilon_{\eta=2}}; \quad (51)$$

$$\left(\varepsilon_1=\varepsilon_k^*,\ \varepsilon_2=\varepsilon_{\eta=2},\ \eta_1=0,\ \eta_2=2\right)$$

$$\bar{\varepsilon}_{fs}(\eta)=\frac{3\cdot\varepsilon_c^*\cdot\varepsilon_{\eta=2}\cdot e^{-\eta}}{(1+\eta)\cdot e^{-2}\cdot\varepsilon_c^*+(2-\eta)\cdot e\cdot\varepsilon_{\eta=2}}; \quad (52)$$

$$\left(\varepsilon_1=\varepsilon_c^*,\ \varepsilon_2=\varepsilon_{\eta=2},\ \eta_1=-1,\ \eta_2=2\right)$$

$$\bar{\varepsilon}_{fs}(\eta)=\frac{3\cdot\varepsilon_{\eta=-2}\cdot\varepsilon_p\cdot e^{-\eta}}{(2+\eta)\cdot e^{-1}\cdot\varepsilon_{\eta=-2}+(1-\eta)\cdot e^{2}\cdot\varepsilon_p}; \quad (53)$$

$$\left(\varepsilon_1=\varepsilon_{\eta=-2},\ \varepsilon_2=\varepsilon_p,\ \eta_1=-2,\ \eta_2=1\right)$$

$$\bar{\varepsilon}_{fs}(\eta)=\frac{2\cdot\varepsilon_{\eta=-2}\cdot\varepsilon_k^*\cdot e^{-\eta}}{(2+\eta)\cdot\varepsilon_{\eta=-2}-\eta\cdot e^{2}\cdot\varepsilon_k^*}; \quad (54)$$

$$\left(\varepsilon_1=\varepsilon_{\eta=-2},\ \varepsilon_2=\varepsilon_k^*,\ \eta_1=-2,\ \eta_2=0\right)$$

The results of the calculations according to the obtained relations as compared with the experimental data are given in Fig. 2. It follows from these results that one-parametric models provide satisfactory quantitative and even qualitative matching only for the narrow range of stress triaxiality, η.

As it is shown in [12] the equation (26) is used only if $\varepsilon_c^* > e\cdot\varepsilon_k^*$.

For the steel 20A this condition is not fulfilled. Curve 3 in Fig. 2d demonstrates the anomalous behavior at changes of equivalent plastic strain to fracture if $\eta > 0$.

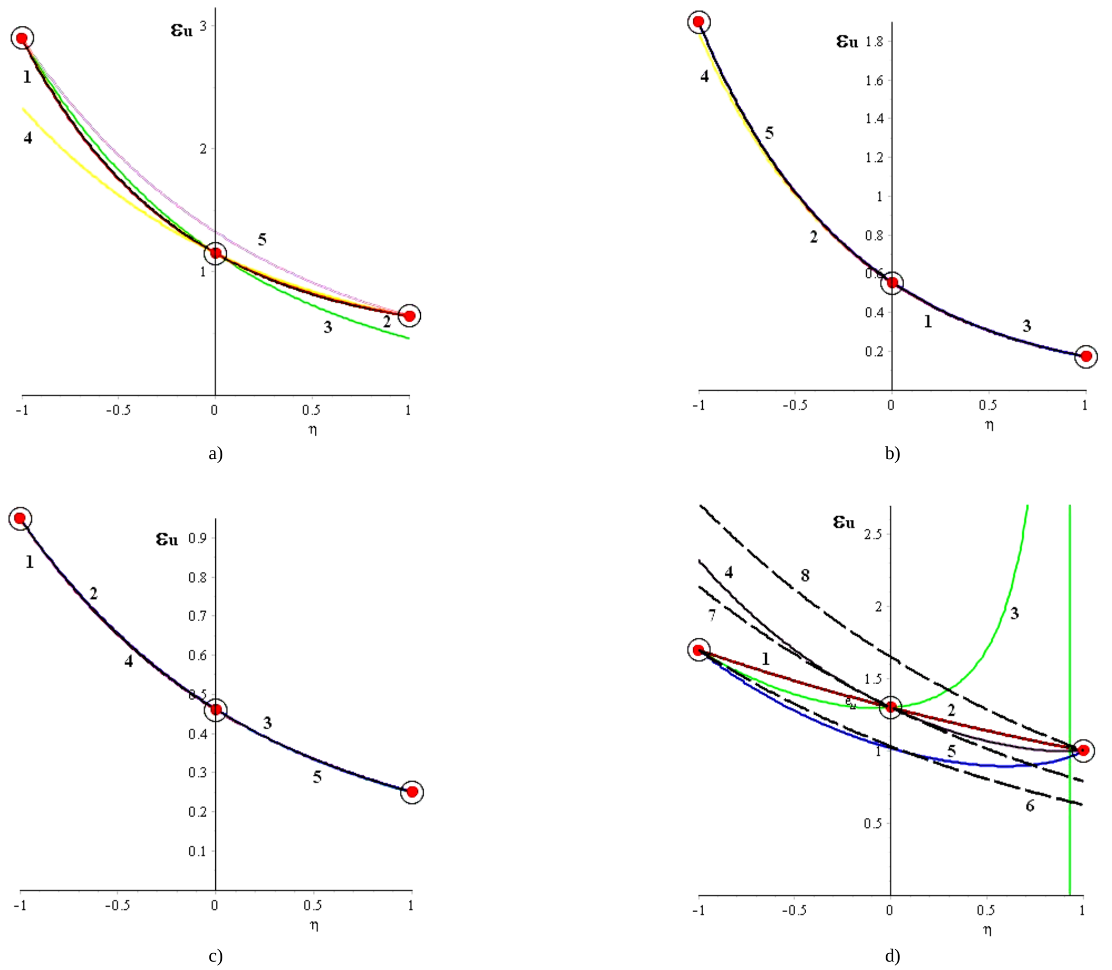

Fig. 2. Dependences of the equivalent plastic strain to fracture on the stress triaxiality of the alloy VT-1 (a), R12 (b), R6M5 (c), 20-A (d) according to the approximations of the curves of the equivalent plastic strains at fracture: 1 – 8 – according to the relations (35), (36), (25), (47), (46), (28), (29), (30), using the experimental data, presented in [30]

4. Conclusions

1. During the last decades the relevance of the studies, dealing with the equivalent plastic strains at fracture at different schemes of stress states drastically increased.
2. The suggested classification characteristics for the comparison of the fracture models are necessary for the realization of the systematization of the results of the research of these model and development of the substantiated recommendations regarding their usage.
3. Development of the generalized relations for the known fracture models regarding the plane stress enabled to simplify the process of obtaining of the great number of the original separate relations.
4. The obtained analytical dependences provided the possibility to obtain a number of general properties, represented by various models.
5. Models of the equivalent plastic strains at fracture of the sheet materials [2, 3, 4, 13, 14, 15, 16, 17] require separate studies.

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