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PRACTICAL APPLICATION OF THE PERCOLATION PHENOMENON IN SPATIAL STRUCTURES

PRAKTYCZNE ZASTOSOWANIE ZJAWISKA PERKOLACJI W STRUKTURACH PRZESTRZENNYCH

***Summary**: This study aimed to present the possibilities for the practical use of the percolation phenomenon. The percolation phenomenon and the related concepts were broadly described. 3D CAD models of spatial structures, 3D prints using L-PBF technology from AISI 316 austenitic steel, models of the hip joint endoprosthesis stem and microscopic photos of the prints were presented. A number of possibilities related to the practical use of percolation theory have been identified, but also several limitations that require in-depth scientific analysis. The main challenge is to create a methodology for determining the percolation threshold and a methodology for analysing the microstructure of polycrystalline materials in 3D.*

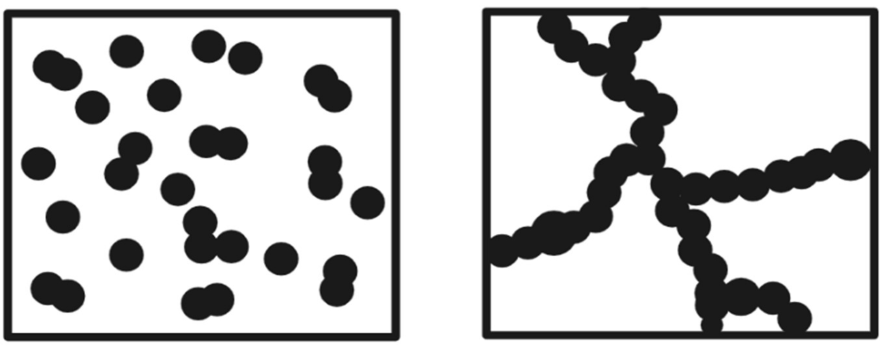
***Keywords**: percolation phenomenon, phase percolation, percolation threshold, porous structures, percolation application*

***Streszczenie**: Celem pracy było przedstawienie możliwości praktycznego wykorzystania zjawiska perkolacji. Dokonano szerokiego opisu zjawiska perkolacji i pojęć z nim związanych. Zaprezentowano modele 3D CAD struktur przestrzennych, wydruki 3D w technologii L-PBF ze stali austenitycznej AISI 316, modele trzpienia endoprotezy stawu biodrowego oraz zdjęcia mikroskopowe wydruków. Zidentyfikowano szereg możliwości związanych z praktycznym zastosowaniem teorii perkolacji, ale także kilka ograniczeń wymagających pogłębionej analizy naukowej. Głównym wyzwaniem jest stworzenie metodologii wyznaczania progu perkolacji oraz metodologii analizy mikrostruktury materiałów polikrystalicznych w 3D.*

***Słowa kluczowe**: zjawisko perkolacji, perkolacja fazowa, próg perkolacji, struktury porowate, zastosowanie perkolacji*

Introduction

The concept of percolation phenomenon refers to the movement or seepage of medium, initially focusing on the fluid dynamics within a network of obstructed channels. As scientific knowledge has advanced, the application of percolation theory has broadened, including areas such as materials sciences, mechanical engineering, chemical engineering, electrical engineering, or social sciences and architecture [1-4]. The phenomenon of percolation is also increasingly used in life sciences, such as epidemiology and virology to describe the phenomena related to the spread of infections. [5, 6].



a) b)

Fig. 1. Percolation of black phase in metal alloy microstructure – the same percentage share of black phase a) percolation does not occur b) percolation occurs

The term phase percolation exists in literature and describes the occurrence of percolation between various phases or components within a composite material. Composite materials consist of two or more separate phases mixed together to achieve specific desired characteristics [7]. Phase percolation happens when a continuous network or pathway develops among the phases (Fig. 1), facilitating the transfer of physical or chemical properties across the structure. The percolation phenomenon can be considered in the context of nano-, micro- and macro structures, because a multiphase network can be represented at the molecular level, e.g. for polymer chains, at the microstructural level [8], e.g. for austenite percolation in the ausferritic structure [9], and at the macrostructural level for the geometric shapes of physical models [10].

The percolation threshold is a key concept in the science of percolation phenomenon, marking the critical volume or concentration at which an interconnected single-phase lattice starts to form within a multiphase material [11]. Below this threshold, the phases remain isolated and do not notably influence the material's overall properties. Upon reaching the percolation threshold, there is a sudden shift in the material's behaviour as the interconnected network of the percolating phase starts to dominate the overall properties [12]. Determining the percolation threshold in material testing is, however, a complex activity that requires the development of an adequate methodology [13].

Currently, due to the rapid progress in introducing new technologies in the automotive, shipbuilding, aviation and space industries, in parallel with the need to discover new materials, there is a growing need to develop classic materials with wide availability and an established supply chain. With the development of mechanical and material engineering, it is possible to modify them appropriately to achieve better expected technical parameters. This can take place at the micro and nano levels, but the study focuses primarily on the application of an innovative research approach at the macro scale. The authors indicate that applying the percolation phenomenon at the stage of modeling the structure's connection network can contribute to the production of elements with desired mechanical properties, and above all, designed for specific applications.

Research in porous materials focuses on controlling the size, shape, and uniformity of the pores, as well as the atoms and molecules that create them. There is a considerable demand for creating new porous solids with organized structures from a variety of materials. This effort has led to manufacturing materials with unique properties, expanding their applications beyond traditional uses. Porous materials are becoming increasingly significant in the fields like microelectronics and medical implantology [14, 15].

Noting the great interest in the development of the applicability of porous structures, as well as the wide range of possibilities of using the percolation phenomenon in research on materials and mechanical properties of structures, the authors of the publication undertook actions aimed at exploiting the potential of an innovative approach in this field of science.

Percolation theory and mechanical properties

Typically, the properties of materials are determined by examining the concentrations of specific phases or chemical elements. Yet, identical chemical content of the material does not guarantee the same microscale structure. Thus, the properties of these materials can vary significantly [16]. This happens because the spatial arrangement of phases in the material in three dimensions is not considered. When a physical model is considered on a macro scale, researchers analyse the entire network of nodes between individual points. The same should be done with spatial structures at the micro and nano levels. However, the obstacles here are the widespread adoption of the classical materials research procedure, the lack of an appropriate methodology for the analysis of phase percolation in materials and equipment limitations. In order to fully determine the morphology of phases in a material, it must be analysed in three dimensions, while the most common form of analysis of polycrystalline materials remains two-dimensional analysis. The main difficulty of the issue, i.e. 3D imaging, for this type of materials results from the nature of the interaction of electromagnetic radiation with matter. In the visible spectrum, radiation is absorbed and scattered by crystals, making most construction materials de facto opaque. Considering this, the appearance of the structure in 3D should be approximated based on stereological relationships [17-19].

Percolation theory helps in understanding how cracks propagate through materials. This is critical in predicting failure in structures and in designing materials that can better resist different forms of mechanical failure. In cyclic loading conditions, materials can also develop networks of micro-cracks. Understanding the percolation phenomenon of the materials phase structures can help in predicting the point at which these micro-cracks form a connected network leading to failure.

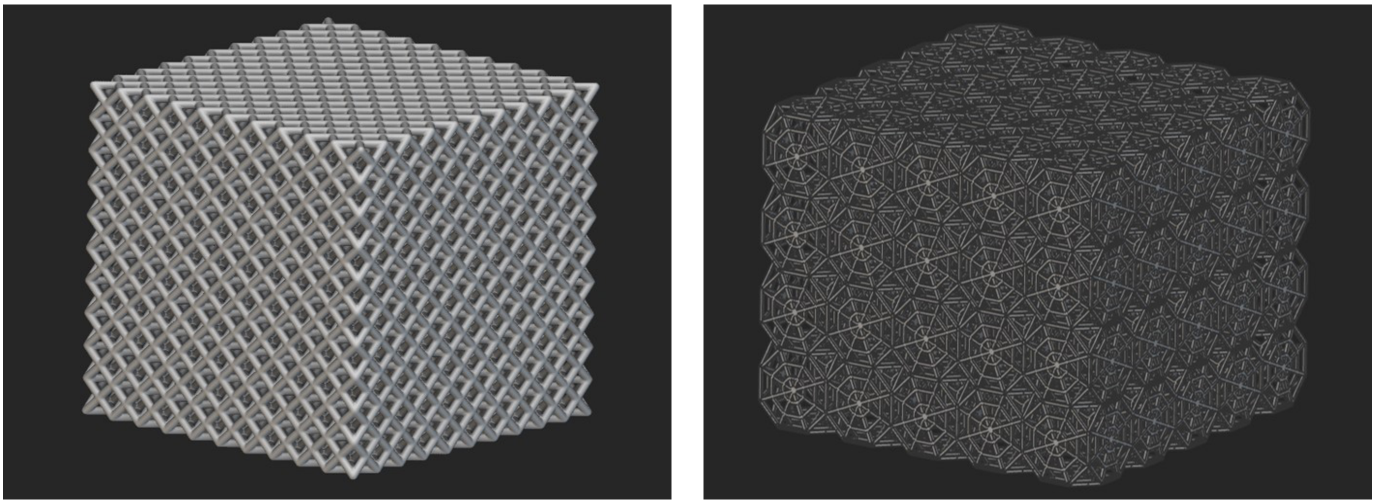
By quantifying the percolation threshold, engineers can design composite materials in an appropriate way to achieve superior mechanical properties such as strength, toughness, and stiffness. For instance, adding carbon nanotubes or graphene to polymers can create a conductive network that significantly enhances the material's mechanical properties once the percolation threshold is reached [20].

Further research on percolation concepts can lead to optimization of the use of materials by ensuring that the manufactured structures have the necessary mechanical properties with minimal resource usage. This is especially relevant for rare and expensive materials.

Percolation theory in spatial structures

While simulating the operating environment of machine and equipment components, engineers have noticed that not all parts of the structure transfer loads to the same extent. Thus, a branch of science related to topological optimization was created. The topological optimization of elements is a computational and mathematical technique used to create optimal material layouts within a defined design space, considering specific constraints and performance requirements. Its main objective is to find the best material distribution to either maximize or minimize particular goals, such as structural strength, stiffness, weight, or other performance criteria [21, 22].

The creation of porous geometry, based on the theory of modeling the physical phenomenon of percolation, also serves as a topological optimization of the element. However, an



a) b)

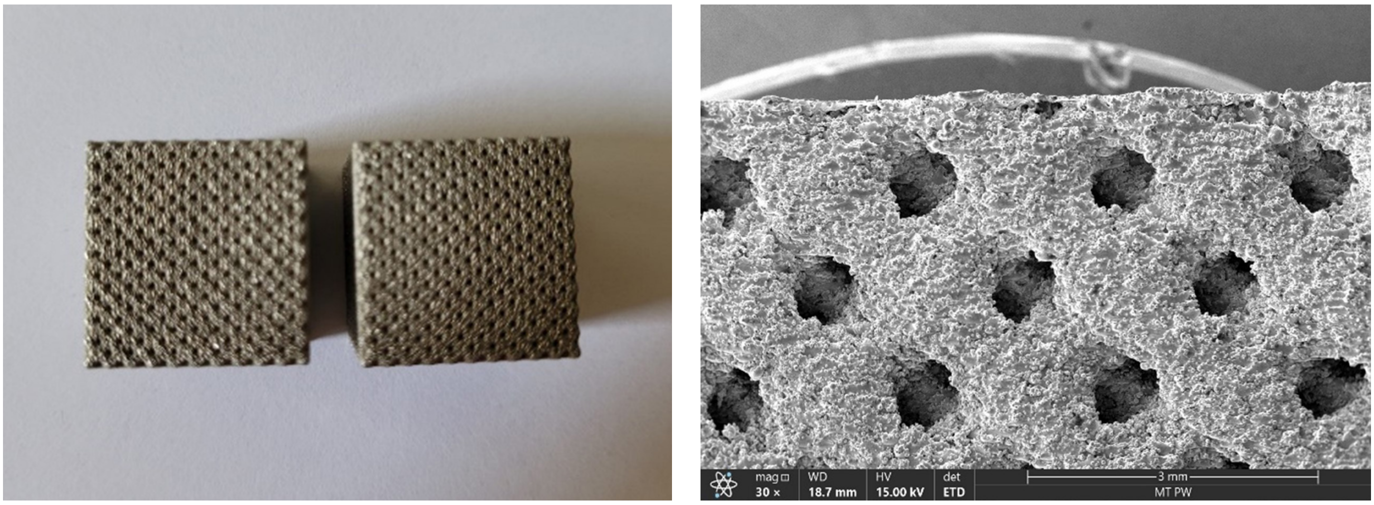
Fig. 2. Example of 3D CAD model of spatial porous geometries a) face-centered cubic structure b) cubic structure imitating a spider's web

element designed in this way would optimally take the shape of a porous spatial geometry, not continuous. Thus, this structure would be able to carry similar loads, with a significant reduction in weight. Most often, the geometry of such structures comes from nature, e.g. spider's web, honeycomb, fractals, but there are also models designed by mathematicians [23]. Triply Periodic Minimal Surfaces (TPMS) structures are particularly interesting from the point of view of adaptation to materials engineering [24].

The starting point for creating a mathematical model determining the degree of percolation and the percolation threshold for individual spatial structures is to create a database of 3D models (Fig. 2) that can be a starting point for FEM analyses and, after production, for microscopic, strength, fatigue and other tests.

The key to applying the percolation theory in spatial structures is to understand and appropriately parameterize the network of connections between the structure's nodes. A properly designed structure network, according to the percolation theory, should strengthen each other after exceeding the percolation threshold, multiplying the mechanical properties.

Due to the complex network of connections occurring in percolation structures, the manufacturing technology that allows for the most accurate reproduction of the designed geometries is 3D printing (Figure 3). Previously, spatial structures with complex geometry were difficult to adapt to industrial solutions, but the possibility of producing them from various materials (plastics, metals, ceramics) using 3D printing will probably change this state of affairs.



a) b)

Fig. 3. Cubes made using L-PBF 3D printing technology with FCC spatial geometry a) upper plane b) image at x30 magnification

Percolation phenomenon application

The use of porous structures otherwise percolation structures in spatial geometries does not have to be limited only to theoretical models aimed at developing a methodology for assessing the parameters of the percolation network in a given spatial system. Elements that are equally durable and at the same time much lighter are desirable in every area of industry. Examples of applications of the described phenomenon include all types of energy absorbers, supporting structures (building, technical, vehicles), industrial filters, implants (Fig. 4 and Fig. 5) and many others.

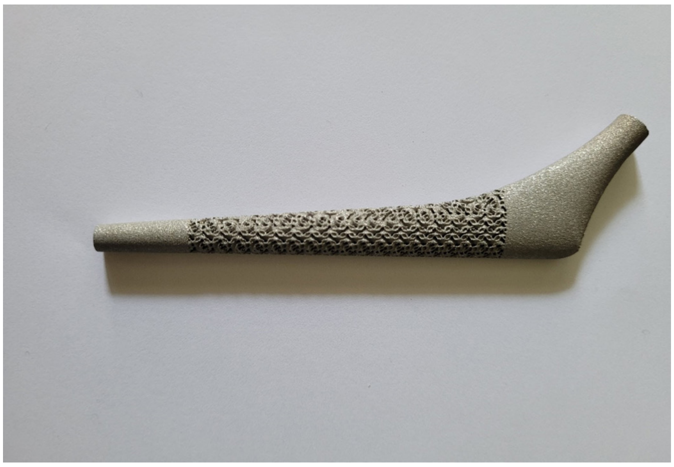


Fig. 4. Hip joint endoprosthesis stem – percolation structure

In the case of the structure observed in Figure 5, it can be noticed that there are discontinuities in the structure, i.e. gaps or pores, which fulfill a very important function in the case of a medical implant, because the bone structure after surgery has the ability to integrate with the endoprosthesis by growing into it. This reduces the risk of complications and increases strength and durability of the implant. Therefore, due to the growth of bone tissue deep into the implant, the whole structure becomes a composite and a double percolating structure.

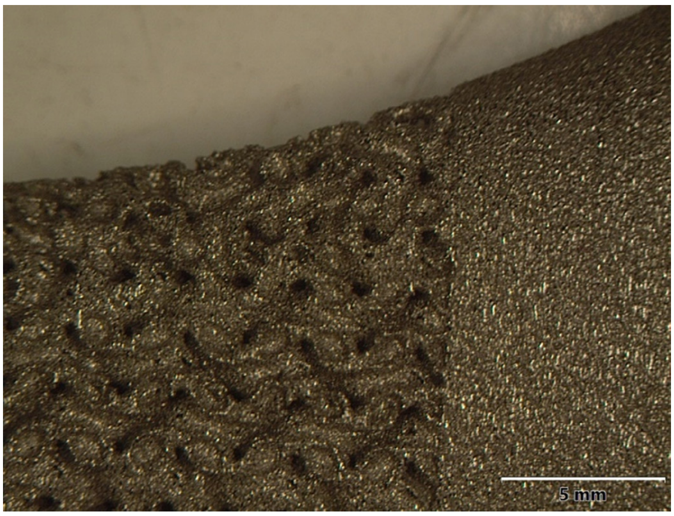


Fig. 5. Porous structure in the central part of the hip joint endoprosthesis stem – 8x magnification

The indicated medical implant is a good demonstrator to illustrate the need for research on the percolation phenomenon at many levels - designing the structure on a macro scale allows for obtaining several dozen percent lighter, equally durable (confirmed by FEM simulations) element, the geometry of which fulfills additional application functions. Additionally, the work should be performed on the microstructure of the material from which the print is made, by appropriate modeling of the microstructure and its subsequent production using a combination of carefully prepared 3D printing and heat treatment processes.

Conclusions

By understanding percolation relationships in spatial structures and the mechanisms of their formation, it is possible to design percolation patterns, which will then be implemented in a controlled manufacturing process, contributing to the creation of materials with the desired mechanical properties.

The microstructures of materials should be considered in three dimensions, taking into account the arrangement of phases and the potential occurrence of percolation patterns between them.

Porous materials are becoming increasingly significant in all fields of the industry. The use of percolation theory can create scope for manufacturing materials with the unique properties, expanding their applications beyond traditional uses.

The percolation theory may contribute to the explanation of phenomena in the field of crack mechanics as a result of the analysis of the paths of progressive micro-damage to the spatial structure.

Exceeding the percolation threshold in the considered system causes a sudden increase in the strength properties of the entire structure.

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