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A SIMPLIFIED WAY OF ESTIMATING A DYNAMIC LOAD RATING FOR THRUST ROLLER BEARINGS

UPROSZCZONY SPOSÓB SZACOWANIA NOŚNOŚCI DYNAMICZNEJ ŁOŻYSK WZDŁUŻNYCH WAŁECZKOWYCH

Key words: ISO 281, rolling bearings, dynamic load rating.

Summary: The method of calculating the bearing capacity of rolling bearings is described in the ISO 281 standard. The calculation procedure for roller thrust bearings presented there, depending on the value of the nominal bearing angle, requires the selection of one of two formulas. Then, using the table, one reads the value of the factor depending on the geometry of the bearing components. To facilitate and speed up calculations (and perhaps also increase their accuracy), this article proposes a formula that is adapted to numerical applications, replaces linear interpolation with a proper non-linear function and allows calculations to be made for a specific value of the nominal bearing angle, but not within the range of 15°. The difference between the values calculated according to the proposed formula and the value calculated according to ISO 281 is, on average, around 3%.

Słowa kluczowe: ISO 281, łożyska toczne, nośność dynamiczna.

Streszczenie: Sposób obliczania nośności łożysk tocznych jest opisany w normie ISO 281. Przedstawiona tam procedura obliczeniowa dla łożysk wzdłużnych wałeczkowych, w zależności od wartości nominalnego kąta działania łożyska, wymaga wyboru jednego z dwóch wzorów. Następnie trzeba, korzystając z tabeli, odczytać wartość współczynnika zależnego od geometrii części składowych łożyska. By ułatwić i przyspieszyć obliczenia (a być może także zwiększyć ich dokładność), w niniejszym artykule zaproponowano wzór, który jest przystosowany do aplikacji numerycznych, zastępuje interpolację liniową właściwą funkcją nieliniową i umożliwia wykonanie obliczeń dla konkretnej wartości nominalnego kąta działania łożyska, a nie dla zakresu o rozpiętości 15°. Różnica między wartościami obliczonymi według proponowanego wzoru a wartością obliczoną według normy ISO 281 średnio wynosi ok. 3%.

INTRODUCTION

Most rolling bearings lose their usability due to surface fatigue. Surface fatigue is a type of wear in which local loss of consistency and associated material losses are caused by material fatigue as a result of cyclical contact stress (within Hertz stress limits) in the surface layer of mating elements (rolling or sliding with slipping), with lubricated contact (pitting) or dry (spalling) **[L. 1, 2]**. The element's service life until reaching the limit state due to fatigue wear was adopted as the term "rolling contact fatigue" (RCF).

The history of calculating bearing life began over 60 years ago when Gustaf Lundberg from the Chalmers Institute of Technology and Arvid Palmgren from the SKF AB bearing company applied Weibull's theory of probability **[L. 3, 4]** regarding fatigue of material to determine the life of rolling bearings. Their fundamental work of 1947 **[L. 5, 6]** and 1952 **[L. 6, 7]** regarding internal stress distribution, equivalent loads, and statistical bearing life distribution shaped the foundations of ANSI/ABMA and ISO standards describing bearing life, giving rise to catalogues of bearing manufacturers. First, the simplest method for calculating bearing life

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adopted by ISO in 1962 was presented in ISO 281, and then it was modified, and the current version dates from 2007 **[L. 8, 9, 10]**. Research carried out over the last several years, e.g., **[L. 11–24]**, was not reflected in the standard.

THE PURPOSE OF THE ARTICLE

In all versions of the standard, to calculate the dynamic load capacity, it is necessary to select the appropriate formula depending on the bearing geometry and read the corresponding data from the table. This procedure facilitates difficult numerical calculations and quick estimation of bearing capacity for various bearing construction variants.

In addition, the tables are discrete (provide selected values), and intermediate values, according to the comment in ISO 281, should be calculated using linear interpolation. However, the parameters given in the tables are not linearly dependent, which makes this interpolation inaccurate.

The third disadvantage of calculations according to ISO 281 is that, for example, for an nominal contact angle between 45° and 60°, the column "α = 50°" should be used – similarly for other values, in ISO 281 standard, the angles are approximate and apply to angle ranges (15°).

In the context of the listed imperfections of the ISO 281 standard, the purposes of the work are the following:

- Simplifying the calculation procedure and adapting it to numerical applications,
- The replacement of linear interpolation with an appropriate non-linear function, and
- Enabling calculations to be made for the exact angle α, not for a range of 15°.

ESTIMATION OF DYNAMIC LOAD RATING OF CYLINDRICAL ROLLER THRUST BEARINGS ACCORDING TO ISO 281

The ISO 281 standard provides methods for calculating the basic dynamic rating of rolling bearings and basic rating life. The basic dynamic rating is defined as a constant stationary load which a rolling bearing can theoretically endure for a basic rating life of one million revolutions. The basic rating life is associated with 90% reliability for bearing operating under conventional operating conditions **[L. 8, 9]**.

For the purposes this paper, the symbols given in ISO **[L. 8, 9, 25]** and the following apply:

C_a – basic dynamic axial load rating, in Newtons;
D_{we} – roller diameter applicable in the calculation of load ratings, in millimetres;
D_{pw} – pitch diameter of bearing, in millimetres;
L_{we} – effective roller length applicable in the calculation of load ratings, in millimetres;
Z – the number of rolling elements;
b_m – rating factor for contemporary, commonly used, high quality hardened bearing steel in accordance with good manufacturing practices the value of which varies with bearing type and design;
f_c – factor which depends on geometry of the bearing components, the accuracy to which the various components are made, and the material;
α – nominal contact angle, in degrees.

Estimating dynamic load rating according to ISO 281 should start with calculating the quotient values:

– for α = 90°
$$\frac{D_{we}}{D_{pw}} \tag{1}$$

– for $\alpha \neq 90°$
$$\frac{D_{we}\cos\alpha}{D_{pw}} \tag{2}$$

Then one can read the value of the factor fc from the table contained in the standard. The first two columns of the table relate to bearings in which the angle α = 90°. The first column presents the values of the quotient (1) from 0.01 to 0.3 every 0.01 – and in the second, the corresponding values of the coefficient f_c. The third column presents the values of the quotient (2), and in the next three represent the corresponding values of the coefficient f_c for the angles a of 50°, 65°, and 80°. The table is supplemented by a comment which shows the following:

- The values of the f_c coefficient for the values of the quotients (1) or (2) other than those given in the table should be calculated using linear interpolation;
- Column "α = 50°" is used for angles from 45° to 60°, column "α = 65°" for angles from 60° to 75°, and column "α = 80°" for angles 60°–75° and 75°–90°.

Depending on the angle α, the axial dynamic load capacity, C_a, of single row thrust roller bearings is expressed by the following formulas **[L. 8, 9]**:

– For α = 90°
$$C_a = b_m f_c L_{we}^{7/9} Z^{3/4} D_{we}^{29/27} \tag{3}$$

– For $\alpha \neq 90°$
$$C_a = b_m f_c \left(L_{we}\cos\alpha\right)^{7/9} tg\alpha\ Z^{3/4} D_{we}^{29/27} \tag{4}$$

PROPOSED CHANGES IN THE METHOD OF CALCULATING DYNAMIC LOAD RATING

To simplify the calculation procedure and adapt it to numerical applications, one can replace linear interpolation with a proper non-linear function and enable calculations for the exact angle α (and not for the range) in accordance with the purpose of the work as follows:

- Develop a formula that would not require the use of tables,
- Develop one general formula for all values of the angle α (also for α = 90°).

To achieve this, replace the expression

$$f_c\left(\cos\alpha\right)^{7/9} tg\alpha \tag{5}$$

from Formula (4) such that

– It will allow the calculation of f_c (without tables).
– For angle $\alpha = 90°$, it will reduce to f_c.

For this purpose, the parameter M_α has been introduced, which, for the angle $\alpha = 90°$, will be 1, and for the other values of the angle α, it will be calculated from the following trigonometric expression:

$$M_\alpha = \begin{cases} \text{for } \alpha = 90° & 1 \\ \text{for } \alpha \neq 90° & \left(\cos\alpha\right)^{7/9} tg\alpha \end{cases} \tag{6}$$

So Formulas (3) and (4) can be reduced to one equation of the following form:

$$C_a = b_m L_{we}^{7/9} Z^{3/4} D_{we}^{29/27} f_c M_\alpha \tag{7}$$

Then the task of simplifying the method of estimating the dynamic load rating of thrust roller bearings comes down to developing formulas that allow calculating the product $f_c M_\alpha$ for various angles α and diameter ratios, without the need to use tables.

Deriving the formula replacing the tables with coefficient values f_c

In order not to need a table to determine the value of the f_c coefficient, it must be enclosed in the form of a formula which, as described in the previous section, depends on the angle and the ratio of the roller diameter to the pitch diameter (1), (2). The following chart (**Fig. 1**) shows the fc values read from the ISO 281 standard for individual angles α (201 cases).

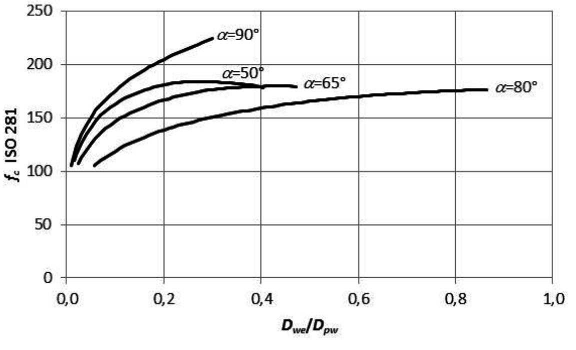

Fig. 1. The values of the f_c factor according to the table in the ISO 281 standard

Rys. 1. Wykres wartości współczynnika f_c opracowany na podstawie tabeli w normie ISO 281

For each of the angles α separately, the relationship between the D_{we}/D_{pw} quotient and the f_c coefficient can be described by a logarithmic function, obtaining high fit values (f_c' means approximation f_c):

– For $\alpha = 50°$ $$f_c' = 22.338\ln\left(\frac{D_{we}}{D_{pw}}\right) + 210.82 \quad R^2 = 0.9158 \tag{8}$$

– For $\alpha = 65°$ $$f_c' = 25.335\ln\left(\frac{D_{we}}{D_{pw}}\right) + 204.65 \quad R^2 = 0.9791 \tag{9}$$

– For $\alpha = 80°$ $$f_c' = 27.347\ln\left(\frac{D_{we}}{D_{pw}}\right) + 183.11 \quad R^2 = 0.9968 \tag{10}$$

– For $\alpha = 90°$ $$f_c' = 36.839\ln\left(\frac{D_{we}}{D_{pw}}\right) + 264.09 \quad R^2 = 0.9884 \tag{11}$$

As measures of the quality of the approximation function (f_c') fit to the data from ISO 281 (f_c), the following were adopted:

– The coefficient of determination **[L. 26]** $$R^2 = \left(\frac{\operatorname{cov}(X,Y)}{\sigma_x \sigma_y}\right)^2 \tag{12}$$

– The maximum relative difference $\max\left|\frac{f_{ci}-f_{ci}^{'}}{f_{ci}}\right|$ (13)

– The average relative difference $\frac{\sum_{i=1}^{201}\left|\frac{f_{ci}-f_{ci}^{'}}{f_{ci}}\right|}{201}$ (14)

The maximum relative difference between the value of the f_c coefficient selected from the ISO 281 standard and calculated from Formulas (8) to (11) is 10%, and the average relative difference is 1.4% (201 cases were analysed, i.e. all resulting from the data in the table, which is in ISO 281; f_{ci} stands for the ith case).

Derivation of dependence replacing trigonometric functions

If, in Formula (4), instead of the cosine function, there was a sine function, then for the angle α = 90°, the expression with the exponent 7/9 would simplify to L_{we} (sin 90° = 1). Then, in this part, Formula (3) would be a special case of formula (4) for α = 90°. However, Formula (4) has the cosine function (a cos90° = 0); therefore, a separate formula for α = 90° is given in the ISO 281.

In addition, in Formula (4), there is tg α, which is not in Formula (3). From the properties of trigonometric functions, the following is known:

$$\cos\alpha \cdot tg\alpha = \sin\alpha \quad (15)$$

Therefore, for α = 90°, this product would be 1 and Formulas (3) and (4) would not be different, but one cannot forget the exponent, and, in fact, in Formula (4), the expression related to the angle α has the following form:

$$\left(\cos\alpha\right)^{7/9} tg\alpha \quad (16)$$

which is not equal to sin α.

In the tables of ISO 281, four angle values α are considered: 50°, 65°, 80°, and 90°. Expression (16) assumes for these angles the following values: 0.85, 1.10, 1.45, and 1 (for 90° angle).

Expression (5) can be replaced by products of Formulas (8) to (10) and Expression (16):

– For α = 50° $f_c^{'}\left(\cos\alpha\right)^{7/9} tg\alpha = 18.878\ln\left(\frac{D_{we}}{D_{pw}}\right)+178.16$ $R^2 = 0.9158$ (17)

– For α = 65° $f_c^{'}\left(\cos\alpha\right)^{7/9} tg\alpha = 27.805\ln\left(\frac{D_{we}}{D_{pw}}\right)+224.6$ $R^2 = 0.9791$ (18)

– For α = 80° $f_c^{'}\left(\cos\alpha\right)^{7/9} tg\alpha = 39.74\ln\left(\frac{D_{we}}{D_{pw}}\right)+266.9$ $R^2 = 0.9968$ (19)

– For α = 90° Formula (11) in its unchanged form.

The use of these formulas frees one from the need to use a table, but it introduces another four formulas that should be used depending on the angle, which is difficult to call a simplification. However, these formulas can be saved in general form as

$$f_c^{'}M_\alpha = a\ln\left(\frac{D_{we}}{D_{pw}}\right)+b \quad (20)$$

where M_α (6) for the angle α = 90° is 1, and for the other values of the angle, α is calculated from Expression (16), while a and b are coefficients dependent only on the angle α, which can be approximated by the following formulas:

$$a = 0.5045\alpha - 5.1308 \quad R^2 = 0.8742 \quad (21)$$

$$b = 2.2752\alpha + 71.128 \quad R^2 = 0.9254 \quad (22)$$

Inserting (21) and (22) into (20) produces:

$$f_c^{'}M_\alpha = \left(0.5045\alpha - 5.1308\right)\ln\left(\frac{D_{we}}{D_{pw}}\right)+2.2752\alpha+71.128 \quad (23)$$

The directional coefficient of the relationship between the expression $f_c M_\alpha$ calculated according to ISO 281 and according to Formula (23) is 0.9794 (with the free word equal to 0); therefore, Formula (23) was corrected by dividing it by this value and obtained the following:

$$f_c^{'} M_\alpha = \left(0.5151\alpha - 5.2387\right)\ln\left(\frac{D_{we}}{D_{pw}}\right) + 2.3229\alpha + 72.6241 \qquad (24)$$

For all 201 cases that can be calculated on the basis of data from the ISO 281, a calculation was made according to the proposed Formula (24), and the effects of the comparison are presented in **Fig. 2**.

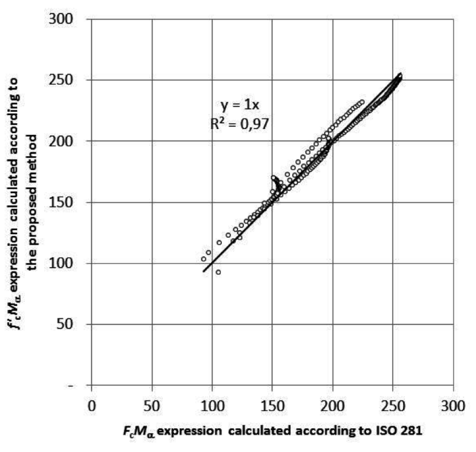

Fig. 2. Relationship between the $f_c M_\alpha$ calculated according to ISO 281 and the proposed formula
Rys. 2. Zależność między wyrażeniem $f_c M_\alpha$ obliczonym wg normy ISO 281 i wg proponowanego wzoru

Using the measures of fit quality described by Formulas (12), (13), and (14), it can be concluded that Formula (24) gives a high approximation accuracy. The coefficient of determination is 0.97, the maximum difference is 13%, but the average difference is about 3%. The points in the graph are arranged in several series, which is the effect of the angle ranges and two formulas adopted by the ISO 281 standard. However, in reality, the phenomenon is continuous (which further justifies the use of the proposed formula).

Modified formula for dynamic load rating

After substitution (24) to (7), a simplified formula was obtained for the dynamic load capacity of thrust roller bearings:

$$C_a = b_m L_{we}^{7/9} \cdot Z^{3/4} \cdot D_{we}^{29/27} \cdot \left[\left(0.515\alpha - 5.24\right)\ln\frac{D_{we}}{D_{pw}} + 2.32\alpha + 72.6\right] \qquad (25)$$

The only source of differences between the dynamic load rating values calculated according to ISO 281 and according to the proposed Formula (25) is the expression $f_c M_\alpha$. Therefore, these differences amount to an average of about 3%, which is described in the commentary to Formula (24) and **Fig. 2**. **Table 1** presents examples of calculations made on the basis of ISO 281 and according to the proposed formula (25).

Table 1. Dynamic load rating of the example bearings calculated on the basis of ISO 281 and the proposed formula (25)
Table 1. Nośność dynamiczna przykładowych łożysk obliczona na podstawie ISO 281 oraz proponowanego wzoru (25)

No.	b_m	D_{we}	D_{pw}	L_{we}	α	Z	f_c^{ISO281}	C_a^{ISO281}	$C_a^{(25)}$	Relative
	–	[mm]	[mm]	[mm]	[°]	–	–	[kN]	[kN]	difference
1	1	3	30	5	90	15	175.7	15.2	16.2	6.3%
2	1	20	500	40	90	13	143.4	432.0	449.1	4.0%
3	1	10	40	30	90	11	215.4	217.4	226.5	4.2%
4	1.1	3	30	5	50	15	160.9	13.0	13.5	4.0%
5	1.1	20	500	40	50	13	139.5	390.6	406.2	4.0%
6	1.1	10	40	30	50	11	183.7	172.3	177.8	3.2%
7	1.1	3	30	5	65	15	144.7	15.2	15.1	0.3%
8	1.1	20	500	40	65	13	124.7	453.5	439.1	3.2%
9	1.1	10	40	30	65	11	173.6	211.5	204.6	3.3%
10	1.1	3	30	5	80	15	123.0	17.1	16.7	1.9%
11	1.1	20	500	40	80	13	105.6	508.5	472.0	7.2%
12	1.1	10	40	30	80	11	142.8	230.4	231.3	0.4%
13	1.15	30	300	40	50	15	160.9	810.6	842.8	4.0%
14	1.15	30	300	40	65	13	144.7	850.4	848.1	0.3%
15	1.15	30	300	40	80	11	123.0	844.4	828.6	1.9%

SUMMARY

The dynamic bearing capacity provided by manufacturers is experimentally verified in durability tests **[L. 27, 28]**, due to the large spread, covering at least 20 bearings, which makes them costly and time consuming. In unit applications for which special rolling nodes are designed, e.g., large-size coronary bearings, it is not possible to carry out such tests, so it is worth improving the tools for calculating the load capacity. One of such tools is the ISO 281 standard, which, however, due to its specificity, described in chapter two, makes it impossible to quickly assess the load capacity for many variants of the rolling node structure.

Thus, in accordance with the purpose of the work, a method of calculating the dynamic load rating of cylindrical roller thrust bearings (Formula 25) was proposed, which accomplishes the following:

– Simplifies and speeds up calculations (by deriving one general formula for all values of the angle α, also for $\alpha = 90°$);
– Facilitates the use of numerical methods (the developed formula does not require the use of tables); and,
– Makes the calculation result more dependent on bearing parameters than ISO 281 (by replacing the linear interpolation of data from the table with a proper non-linear function and enabling calculations for a specific angle α, not for a range of 15°).

The average difference between the dynamic load capacity calculated according to ISO 281 and calculated according to the proposed formula (25) is about 3%, and in no case exceeds 13%.

Therefore, the differences are small, and they do not necessarily mean a worse estimation of the dynamic load rating based on the proposed formula (25). In calculations according to ISO 281, the values of the f_c coefficient for the quotient values other than those given in the table are obtained by linear interpolation, which is less precise than the proposed method (and requires additional calculations). An even greater source of imprecise estimation of dynamic bearing capacity based on ISO 281 is the fact that, for angles α from 45° to 60°, the values of f_c from the column "$\alpha = 50°$" are used, while the next two columns of the table from the standard use, respectively, within the ranges of 60°–75° and 75°–90°. Therefore, in the ISO 281 standard, the angles are approximate and relate to the ranges of angles (every 15°), similarly to the ranges of diameters, and the developed Formula (25) allow calculations for any values of these parameters.

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