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# COMPARATIVE STUDY OF MODELS AND A NEW MODEL OF FERROFLUID VISCOSITY UNDER MAGNETIC FIELDS AND VARIOUS TEMPERATURES

# BADANIE PORÓWNAWCZE MODELI I NOWY MODEL LEPKOŚCI FERROFLUIDU W POLU MAGNETYCZNYM I RÓŻNYCH TEMPERATURACH

#### Key words:

magnetic fluid, ferrofluid, rotational viscosity, magnetoviscous effect, rheology.

Abstract:

Ferrofluid is a substance with a controllable viscosity that is used in various systems for dispersing mechanical energy, such as brakes or vibration dampers. It is also used in seals or loudspeakers. An increase in the magnetic field affects the formation of particle structures inside the carrier fluid, which increases internal friction. Existing mathematical models that describe the increase in viscosity do not provide satisfactory results for commercial and undiluted ferrofluids. In this study, we measured viscosity, which refers to the increased resistance a magnetic fluid creates when it flows under shear. Various synthetic oil-based ferrofluids with known saturation magnetization values and different particle distributions were selected for the study. The temperature range of  $25-80^{\circ}$ C and the value of the shear rate of  $100 \text{ s}^{-1}$  were taken into account. The aim of the study is to compare existing mathematical models with experimental results and to propose a model that best describes the effect of the magnetic field on the increase in viscosity of the fluids studied. The proposed model is based on dividing the particle distribution into two fractions and applying a correction factor. The results showed that the difference in theoretical and experimental values does not exceed 6.5%. Research and results have potential applications in the design and development of synthetic oil-based ferrofluid applications where significant temperature changes occur.

Słowa kluczowe: ciecz magnetyczna, ferrociecz, lepkość rotacyjna, efekt magnetolepkości, reologia.

# Streszczenie:

Ferrociecz to substancja o sterowalnej lepkości, która znajduje zastosowanie w różnych systemach dyssypacji energii mechanicznej, takich jak hamulce czy tłumiki drgań. Stosowana jest również w uszczelnieniach czy głośnikach. Wzrost pola magnetycznego wpływa na powstawanie struktur cząsteczkowych wewnątrz cieczy nośnej, co zwiększa tarcie wewnętrzne. Istniejące modele matematyczne opisujące wzrost lepkości nie dostarczają zadawalających wyników dla komercyjnych i nierozcieńczonych ferrocieczy. W pracy dokonano pomiarów lepkości, która odnosi się do wzrostu oporu, jaki stawia ciecz magnetyczna podczas przepływu pod wpływem siły ścinającej. Do badań wybrano różne ferrociecze na bazie oleju syntetycznego o znanej wartości magnetyzacji nasycenia oraz różnym rozkładzie cząstek. Wzięto pod uwagę zakres temperatury 25–80°C oraz wartość szybkości ścinania 100 s<sup>-1</sup>. Celem pracy jest porównanie istniejących modeli matematycznych z wynikami eksperymentalnymi i zaproponowanie modelu, który najlepiej opisuje wpływ pola magnetycznego na wzrost lepkości badanych płynów. Proponowany model bazuje na podziale rozkładu cząstek na dwie frakcje oraz zastosowaniu współczynnika korekcyjnego. Wyniki wykazały, że różnica w wartościach teoretycznych i eksperymentalnych nie przekracza 6,5%. Przeprowadzone badania i uzyskane wyniki mają potencjalne zastosowanie w projektowaniu i rozwoju aplikacji ferrocieczy na bazie oleju syntetycznego, gdzie występują znaczące zmiany temperatury.

#### **INTRODUCTION**

Magnetic fluids are suspensions of particles with magnetic properties in a carrier liquid. They belong

to the group of materials with controlled rheological properties. Depending on the particle size, two types of magnetic fluid can be distinguished: ferrofluids (FF), which are produced based on magnetic

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particles with diameters in the order of nanometres (approximately 5–20 nm), and magnetorheological fluids (MRF) with diameters approximately 5– $-15 \mu$ m.

This study is concerned with ferrofluids, and viscosity change is a feature that describes the resistance to the rotation of particles, or particle structures, in a carrier fluid under forced flow in magnetic field conditions. The ability to change the viscosity of ferrofluids is commonly used in various industrial applications [L. 1] and biological processes [L. 2], so an understanding of the mechanism is important.

The volume concentration of magnetic particles in commercial ferrofluids usually does not exceed 15%. Without a magnetic field, the particles increase the viscosity relative to the carrier fluid **[L. 3]**. In addition to hydrodynamic interaction, particle-to-particle interaction can also occur due to their magnetic properties. The importance in this case is the size and shape. The thermal energy (kT) also has an influence. Significant interparticle interaction can lead to the formation of chains or clusters of particles. Also, increasing the shear rate breaks down structures and reduces the viscosity of ferrofluids **[L. 4]**.

The fluid's viscosity depends on the magnetic field, even with weak dipole interparticle interaction. This is due to the magnetic field's interaction with the magnetic moment [L. 5]. On the one hand, shear flow causes magnetic particles to rotate because of the difference in viscous friction forces acting in opposite directions. When the direction of the magnetic field is perpendicular to the vorticity of the flow, i.e. to the axis of rotation of the particles, the magnetic moment of the particle tries to align itself with its direction. This causes the magnetic moment to shift and interact with the hydrodynamic torque of the particle when it rotates in the viscous carrier fluid. The existence of two opposing torques interferes with the free rotation, which macroscopically increases the viscosity. If the magnetic field and vorticity are parallel, the effect disappears because the alignment of the particle's magnetic moment with the magnetic field direction is not disturbed by the particle's rotation [L. 6]. The above model assumes that the magnetic moment of the particle is fixed to the crystal structure so that the rotation of the magnetic moment is related to the rotation of the particle itself. This mechanism is called Brownian relaxation. The second mechanism is Neel relaxation, which is related to the fact that it

is possible to rotate the magnetic moment relative to the crystal structure of the particle. The process that dominates depends primarily on the size of the particles. For magnetite particles, Brownian relaxation dominates for diameters greater than 13 nm.

Shliomis proposed a theoretical model that describes the dependence of viscosity on the magnetic field [L. 7]. He introduces the concept of rotational viscosity  $\eta_{,,}$  which adds to the viscosity of the carrier fluid. This concept is based on a single particle model without magnetic dipoledipole interactions. The Shliomis model agrees well with the experiment only in the case of dilute ferrofluid [L. 8]. In commercial ferrofluids, there is always an intermolecular interaction in the case of magnetic field interaction [L. 9]. Therefore, there are mechanisms different from those only based on rotational viscosity. The viscosity change in these fluid types is called a 'magnetoviscous effect'. Chain formation is assumed to occur primarily for particles with diameters greater than 15 nm. This leads to the conclusion that a ferrofluid should be described as a suspension containing a large proportion of small particles that affect rotational viscosity and a small number of large particles that can form structures and dominate the magnetoviscous behaviour. Ferrofluids can also exhibit phenomena such as negative viscosity, which occurs in the presence of an alternating magnetic field [L. 10].

Changing the viscosity of ferrofluids is a complex problem, as various phenomena must be considered. This has been the subject of much research, but to date, there has not been a universal model to determine viscosity for commercial oilbased ferrofluids. This paper compares existing mathematical models with experimental results for typical ferrofluids with 15–40 kA/m saturation magnetizations and a temperature range of 25– -80°C. On the basis of this, a model that best describes the effect of the magnetic field on the viscosity increase of ferrofluids and considers the distribution of magnetic particles is proposed.

#### MATHEMATICAL MODELS

Experimental and theoretical research on the viscosity change in magnetic fluids under external magnetic fields has been carried out over the past 40–50 years, and different mathematical models have been created. As above-mentioned, the

Neel and Brownian mechanisms influence the equilibrium of a particle in an external magnetic field during fluid flow. Each of these relaxation mechanisms is described by a corresponding relaxation time.

The prevalence of each relaxation mechanism is determined by the crystallographic anisotropy energy of the particle, which is proportional to its volume:

$$E = K_U V$$

where:  $K_U$  is the anisotropy constant; V is the volume of the particle.

For small particle sizes, the Neel relaxation mechanism plays a decisive role:

$$\tau_{N} = \tau_{0} exp \left(\frac{K_{U}V}{kT}\right) \cdot \left(\frac{K_{U}V}{kT}\right)^{-1}$$
(1)

where:  $\tau_0$  is the Larmor precession damping time of the magnetic moment; *k* is Boltzmann's constant; and *T* is the absolute temperature of the fluid.

It should be noted that the value of the constant  $K_U$  depends on the temperature and shape. The value of  $\tau_0$  can change in the range of  $10^{-8}$ – $10^{-13}$ .

According to (1),  $\tau_{N}$  increases exponentially with particle volume  $V = \frac{\pi d^{3}}{6}$ , and at a certain particle diameter value, the magnetic moment rotates with the particle rotation.

For large-volume particles (with relatively large diameters), the anisotropy energy becomes much larger than the thermal energy. The moment relaxation time in this case is equal to the rotational diffusion time of the particle in the viscous fluid:

$$\tau_B = \frac{3\eta_c V_m}{kT} \tag{2}$$

where:  $\eta_c$  – dynamic viscosity of the carried fluid;  $V_m = \frac{\pi (d+2s)^3}{6}$  – particle volume with surfactant layer thickness (s).

From the condition  $\tau_N = \tau_B$ , it is possible to estimate the critical value of particle diameter  $d_{cr}$  under which one relaxation mechanism starts to dominate. **Fig. 1** shows the variation of the characteristic relaxation times of the Neel and Brownian mechanism as a function of the diameter of the magnetite nanoparticles for three different values of the anisotropy constant  $K_U$ :10,20,40 $\frac{kJ}{m^3}$  and different values of the surfactant thickness s = 1, 2, and 3 nm.

According to the calculations, the critical diameters are, respectively: 12.75 nm ( $K_U = 40$  kJ/m<sup>3</sup>), 16.35 nm ( $K_U = 20$  kJ/m<sup>3</sup>), and 21 nm ( $K_U = 10$  kJ/m<sup>3</sup>). According to [**L. 12**], the critical diameter value for magnetite particles is 13 nm. Based on the analyses in this study,  $K_U = 40$  kJ/m<sup>3</sup> is assumed. The characteristic length of the oleic acid chain itself is about 2.77 nm. The effective length of the attached one to the magnetite particle may be shorter. Therefore, the thickness of the surfactant layer is considered about 2 nm.



Fig. 1. Dependence of the relaxation time of magnetite particles on their diameter

Rys. 1. Zależność czasu relaksacji cząstek magnetytu od ich średnicy

In the absence of a magnetic field, the rotational velocity  $\eta_f$  of the particles is equal to the local rotational velocity of the ferrofluid. In this case, the viscosity of the ferrofluid can be calculated using Einstein's formula:

$$\eta_{f0} = \eta_c \left( 1 + 2.5\varphi_h \right) \tag{3}$$

where:  $\varphi_h$  – particle volume fraction.

Equation (3) describes well liquids with particle concentrations below 3–5%, for which interparticle interactions can be neglected. For ferrofluids with low and moderate volume fractions, equation (4) can also be used. It was independently developed by Roscoe **[L. 13]** and Brinkman **[L. 14]**:

$$\eta_{f0} = \frac{\eta_c}{\left(1 - \varphi_h\right)^{2.5}}$$
(4)

Equation (4) cannot be used to determine viscosity in high particle volume fractions because it does not consider the so-called particle packing limit. As the concentration increases, Equation (4) gives underestimated viscosity values.

Assuming that at a specific concentration value, the ferrofluid results in a change of properties, and a liquid behaves like a solid, Rosensweig **[L. 15]** developed the equation for the determination of the viscosity of liquids with volumetric concentration values up to 30%:

$$\eta_{f0} = \eta_c \left[ \left( 1 - 2.5\varphi_h + \left( 2.5\varphi_m - 1 \right) \right) \left( \frac{\varphi_h}{\varphi_m} \right)^2 \right]^{-1} \quad (5)$$

where:  $\varphi_m$  – maximum volumetric concentration (usually assumed to be 0.74).

Particle sizes are varied, and this influences ferrofluid viscosity. Another model describing changes in the viscosity of ferrofluids with a high-volume concentration of particles is the Chong model [L. 16], which later was modified to [L. 17]:

$$\eta_{f0} = \eta_c \left( 1 + 2.25 \frac{\varphi_m \varphi_h}{\varphi_m - \varphi_h} \right)^2 \tag{6}$$

When an external magnetic field is applied, only particles with diameters larger than  $d_{cr}$  have their own angular velocity  $\overline{\omega}_p$ , which may be different from the angular velocity of the fluid  $\overline{\Omega}(\omega_p \neq \Omega)$ , resulting in an additional magnetic moment [L. 18]:  $6\eta_f \varphi(\overline{\omega_p} - \overline{\Omega}) = \overline{M} \times \overline{H}$ , where:  $\overline{M}$  – magnetization, which can be described by a Langevin function for the case of a stationary magnetic field in equilibrium; H – magnetic field strength. In this case, frictional forces can cause a change (increasing) in viscosity. Since these frictional forces are the result of the external magnetic field, the viscosity turns out to be a function of the dimensionless field strength  $\xi$ [L. 18]:

$$\eta_f = \eta_c \left( 1 + \frac{5}{2} \varphi_h + \frac{3}{2} \varphi_h \frac{\xi - tanh(\xi)}{\xi + tanh(\xi)} \right)$$
(7)

$$\eta_f = \eta_c \left( 1 + \frac{5}{2} \varphi_h + \frac{3}{2} \varphi_h \frac{\xi L^2(\xi)}{\xi - L(\xi)} \right)$$
(8)

where:

 $\xi = \mu_0 \frac{mH}{kT} - \text{dimensionless field strength};$   $m = M_s \frac{\pi \langle d^3 \rangle}{6} - \text{magnetic moment of the particle};$   $L(\xi) = \operatorname{coth}(\xi) - \xi^{-1} - \text{dimensionless Langevin's function};$   $M_s - \text{saturation magnetization};$ d - diameter of the particle.

Equations (7) and (8) are obtained under the assumption that the direction of the magnetic field is perpendicular to the velocity vector, which is characteristic of a plane Couette flow. The last term in (7) and (8) corresponds to the so-called rotational viscosity  $\eta_r$ , i.e. the additional resistance that arises due to the hydrodynamic resistance to rotation of the particle in the carried fluid when the external magnetic field is applied. The first equation (7) describes the properties of ferrofluids with low concentrations of magnetic particles under the condition of small values of ferrofluid vorticity. Equation (8), on the other hand, is used to determine the viscosity of ferrofluids with higher values of ferrofluid vorticity.

#### FERROFLUIDS AND THEIR PROPERTIES

**Fig. 2** shows the particle distributions of the studied ferrofluids obtained by the DLS method **[L. 19]**. In addition, a theoretical log-normal distribution curve is shown. The corresponding figures also indicate the critical diameter ( $d_{cr} = 12.7$  nm) value. The mean diameter values for ferrofluids are: 17.5 nm for FF1, 19.7 nm for FF2, and 14.9 nm for FF3.

The volume concentration of magnetite particles (Fe<sub>3</sub>O<sub>4</sub>) in selected fluids can be determined from the formula:

$$\varphi = \frac{M_s}{M_m} \tag{9}$$

where:  $M_s$  – saturation magnetization of ferrofluid, calculated from the particle distribution obtained by the DLS method,  $M_m = 412$  kA/m – saturation magnetization of magnetite [L. 20].

The equation below is used to determine the hydrodynamic volume concentration, considering the thickness of the surfactant:



Fig. 2. Particle size distributions of ferrofluids (volume weighted)

Rys. 2. Rozkłady wielkości cząstek badanych ferrocieczy (ważone objętościowo)

$$\varphi_h = \varphi \frac{\left(\left\langle d \right\rangle + 2s\right)^3}{\left\langle d^3 \right\rangle} \tag{10}$$

where:  $\langle d \rangle$ - mean diameter value.

### Table 1. Physical properties of examined ferrofluids

Table 1. Właściwości fizyczne badanych ferrocieczy

Label	Saturation magnetization <i>M<sub>s</sub></i> kA/m	Density ρ <sub>F</sub> g/ml	Viscosity $\eta_{f0}$ (B = 0 T, t = 25°C) Pa·s	Viscosity of the base fluid $\eta_c$ (T = 25°C) Pa·s	Volume Concentration Ø %	Volume Concentration $\varphi_h$ %
FF1	35.1	1.33	0.33	0.221	8.5	16.4
FF2	47.7	1.415	0.2	0.096	11.6	21
FF3	46.1	1.426	0.64	0.355	10.9	17.6

shown in Table 1.

Since the volume concentration of the dispersed phase in the fluids is 8-12%, the viscosity of the

synthetic oil was determined based on equation (4) and the viscosity curves  $\eta_f = \eta_f(B)$  obtained experimentally. The parameters of the fluids are

### FERROFLUIDS VISCOSITY MEASUREMENT RESULTS

Rheological properties of the ferrofluids studied were measured on an MCR 301 rheometer using a 20 mm diameter plate-plate geometry. The rheometer was equipped with an additional module (MRD 180), which allows measurements to be carried out in a magnetic field and with variable temperature values.

**Fig. 3** shows the viscosity dependence of the studied ferrofluids under 100 s<sup>-1</sup> shear rate and different temperature values T = 25, 40, 60, and

80°C. The same figure shows the relative values of the change in the viscosity of the fluids at different temperatures. According to the tests, the maximum increase in the viscosity of fluid FF1 is approximately 62%, for fluid FF2 82% and for fluid FF3 approximately 90% at T = 25°C. It should be noted that the maximum change in the viscosity of FF3 is almost independent of the temperature change. It depends mainly on the change in the viscosity of the carried fluid.

When comparing the particle distribution of fluids (**Fig. 2**), one can observe that for fluids FF2



Fig. 3. Dependence of ferrofluid viscosity on temperature

Rys. 3. Zależność lepkości ferrocieczy od temperatury

and FF3, the volume proportion of particles with diameters larger than the critical value of 12.7 nm is higher. Approximately 81% of the particle volume for fluid FF3 and 97% for fluid FF2 will change the direction of the magnetic moment by rotating in the carrier fluid. For fluid FF1, this percentage is only 63%. According to the theory, the higher the volume proportion of particles whose relaxation follows a Brownian mechanism, the greater the viscosity changes that can be expected. Furthermore, this is confirmed by the graphs of the relative viscosity change (**Fig. 3**).

In **Fig. 4**, the dependence of viscosity on magnetic field H is shown for minimum and maximum value of temperature. Continuous black line 1 presents the experimental values. The continuous red line 2 is the result of equation (8). The mean values of diameter  $\langle d_{\rm FF1} \rangle = 20.21$  nm and  $\langle d_{\rm FF3} \rangle = 23.51$  nm are calculated on the basis of the by DLS method distributions obtained. The curves presented confirm the conclusion that the model (8) cannot be used to determine the viscosity of ferrofluids with an average concentration of magnetic particles. However, better results can be obtained, if the correction factor *a* is used:

$$\eta_{f} = \eta_{f0} + \eta_{c} a \frac{3}{2} \varphi_{h} \frac{\xi L^{2}(\xi)}{\xi - L(\xi)}$$
(11)

In **Fig. 4 a)** the change in the viscosity of ferrofluid FF1 at 25°C is shown and curves 3 and 4 are calculated for the 3 and 6.6 values of the correction factor respectively. **Fig. 4 b)** represents the change in the viscosity of ferrofluid FF1 at 80°C and curves 5 and 6 are obtained for a = 1.5 and a = 2.35. It should be noted that ferrofluid FF1 has the smallest particle volume concentration and higher volume fraction of smaller particles than ferrofluids FF2 and FF3.

In **Figs. 4 c)** and **d)**, the change in the viscosity of ferrofluid FF3 at 25°C and 80°C, respectively, are shown. Curves 3, 4, and 5 are calculated for the 3, 5 and 6.6 values of the correction factor. As follows from **Fig. 4 c)** and **Fig. 4 d)**, changing the value of the correction factor cannot lead in this case to a good agreement between the theoretical and experimental data. For a ferrofluid with higher particle volume concentration, the experimental curves have a different initial inclination in comparison with FF1. In ferrofluid FF1, for H greater than 240 kA/m, the viscosity is independent



Rys. 4. Lepkość ferrocieczy FF1 oraz FF3

of the field value  $\left(\frac{\partial \eta}{\partial H}\Big|_{H>240kA/m} \rightarrow 0\right)$ . The analysis of **Figs. 4 a)** and **b**) has shown, that the angle of initial inclination of the curve  $\eta_f = \eta_f(H)$ under small values of the field depends on the temperature:  $\left.\frac{\partial \eta}{\partial H}\right|_{T_1} > \frac{\partial \eta}{\partial H}\Big|_{T_2}$  at  $T_2 > T_1$ . Which cannot be said about the curves for ferrofluid FF3, for which:  $\left.\frac{\partial \eta}{\partial H}\right|_{T_1} \approx \frac{\partial \eta}{\partial H}\Big|_{T_2}$ . In equations (8) and (11), only the last

In equations (8) and (11), only the last component depends on the particle size and magnetic field. This is the so-called rotational viscosity, as named by M. Shliomis but for diluted fluids without particle interaction. **Fig. 5** shows the dependence of this component on different values of the mean diameter and temperature. The larger the average value of the particle diameter, the larger the initial inclination angel of the viscosity curve in a weak field. A change in temperature has an effect on this component of viscosity but not as strong as the mean diameter of the particles.



Fig. 5. Dependence of ferrofluid viscosity on mean particle diameter and temperature in the presence of a magnetic field

Rys. 5. Zależność lepkości ferrocieczy od wartości średniej średnicy cząstek oraz temperatury w polu magnetycznym

In accordance with the analysis, to describe the change in the viscosity of the studied ferrofluids it is proposed to divide the particles, whose relaxation follows the Brownian mechanism, into two fractions. The first fraction consists of larger mean diameter values and affects viscosity values at low field, while the second fraction contains smaller diameters and affects viscosity at strong magnetic fields:

$$\eta_{f} = \eta_{b} \left( 1 + \frac{5}{2} \varphi_{h} \right) + a \eta_{b} \left( \varphi_{h1} \frac{\xi(\langle d_{1} \rangle, H) L^{2} \left( \xi(\langle d_{1} \rangle, H) \right)}{\xi(\langle d_{1} \rangle, H) - L \left( \xi(\langle d_{1} \rangle, H) \right)} + \varphi_{h2} \frac{\xi(\langle d_{2} \rangle, H) L^{2} \left( \xi(\langle d_{2} \rangle, H) \right)}{\xi(\langle d_{2} \rangle, H) - L \left( \xi(\langle d_{2} \rangle, H) \right)} \right)$$
(12)

where:

 $\varphi_{h1}$ ,  $\langle d_1 \rangle$  – volume concentration and mean value of diameter for the first fraction of particles,  $\varphi_{h2}$ ,  $\langle d_2 \rangle$  – volume concentration and mean value of diameter for the second fraction of particles, A – correction factor.

For ferrofluids selected, the parameters are given in **Table 2**.

In **Fig. 6**, continuous lines present the experimental values of the FF3 ferrofluid. Dotted lines denote the best possible fit which can be achieved by the correction parameter *a* and equation (11). The dashed lines denote the viscosity change according to equation (12) and selected parameters (**Table 2**). The difference in theoretical and experimental values for FF1 fluid does not exceed 6.5%; FF2 fluid – 5,3%; FF3 fluid does not exceed 3.5%.

The FF1 ferrofluid has the smallest particle concentration and saturation magnetization compared to the other two studied ferrofluids, and the largest volume of particles, the magnetic moment of which changes its direction without the rotation of the particle itself. For this fluid, only 63% of the particles can affect the viscosity according to the known theoretical model (8). But the known viscosity model does not allow us to describe the viscosity of such ferrofluids well. Curves 2 for all studied ferrofluids are much lower than the experimental values (Fig. 4). It can be concluded that in these fluids the particle interaction exists and cannot be neglected. This interaction results in the formation of aggregates, which probably mostly consist of the particles with larger diameters. This in turn leads to a change in the concentration of magnetic particles of the ferrofluids. In this study, correction factor

Table 2. Selected parameters of ferrofluids

 Tabela 2.
 Dobrane parametry ferrocieczy

Label	φ <sub>h1</sub> ,%	<d<sub>1&gt;, nm</d<sub>	φ <sub>h2</sub> ,%	<d2>, nm</d2>	а			
					T = 25°C	$T = 40^{\circ}C$	$T = 60^{\circ}C$	T = 80°C
FF1	37	16.5	63	25.24	6.6	4.45	4.15	2.5
FF2	72	15.44	28	28.6	7.3	4.3	2.4	1.34
FF3	77	16.1	23	29.26	6.65	6.6	6.55	6.45



**Fig. 6.** Comparison of viscosity models for ferrofluid FF3 Rys. 6. Porównanie modeli lepkości dla ferrocieczy FF3

*a* describes in some way such changes. The lack of viscosity-temperature dependence for FF3 fluid may mean that the agglomeration structures do not change significantly with increasing temperature, and the change in the viscosity of the ferrofluids is mainly related to the change in the viscosity of the carrier liquid. In this case, the thermal energy is lower than the energy of the particle interaction that resulted in the formation of structures.

#### CONCLUSIONS

Ferrofluid properties, namely its viscosity, depend on a number of parameters. In particular, on the volume fraction of magnetic particles, the type and viscosity of the carrier fluid, the magnetic properties of the particle material, the particle size and shape, the particle size distribution, the magnetic field parameters, and the temperature.

There is still a lack of theoretical models to describe the change in the viscosity of commercial ferrofluids with an average concentration of particles based on synthetic oils in the presence of an external magnetic field and at different operating temperatures. Most of the known models are for



dilute fluids or fluids with a high concentration of magnetic particles. These models are developed for carrier fluid with viscosity lower than typical synthetic oils.

The results obtained from the rheological studies indicate large differences between the actual viscosity values of the ferrofluids and known theoretical models. Most researchers point out the impossibility of using the mean diameter value to determine viscosity and possible presence of agglomerates of different shapes.

In this paper, a new method for determining the viscosity of commercial ferrofluids based on the known particle size distribution, the magnetic properties of the particle material, and the saturation magnetization of the ferrofluids is presented. The method uses the M. The Shliomis equation for rotation viscosity but modelling the ferrofluid as a system of two fractions and takes into account the possibility of increasing the volume fraction of particles due to agglomeration structures.

The results obtained on the basis of the proposed equation describe the viscosity of the studied ferrofluids well, not only qualitatively but quantitatively (max. difference does not exceed 6.5%). However, as a result of the influence of

the temperature, this method can only be used for a specific type of ferrofluid. In further analysis, studies should be carried out to develop a method for dividing particle fractions and selecting a correction factor to determine a more universal model – also taking into account the influence of shear rate.

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