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A NEW METHOD FOR AUTOMATIC DEFECTS DETECTION AND DIAGNOSIS IN ROLLING ELEMENT BEARINGS USING WALD TEST

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To detect and to diagnose, the localized defect in rolling bearings, a statistical model based on the sequential Wald test is established to generate a "hypothetical" signal which takes the state zero in absence of the defect, and the state one if a peak of resonance caused by the defect in the bearing is present. The autocorrelation of this signal allows one to reveal the periodicity of the defect and, consequently, one can establish the diagnosis by comparing the frequency of the defect with the characteristic frequencies of the bearing. The originality of this work is the use of the Wald test in the signal processing domain. Secondly, this method permits the detection without considering the level of noise and the number of observations, it can be used as a support for the Fast Fourier Transform. Finally, the simulated and experimental signals are used to show the efficiency of this method based on the Wald test.

Keywords: diagnosis, detection, rolling element bearing, defect, Wald sequential test

1. Introduction

In the industry, a great attention has been given to monitoring conditions and maintenance for the purpose to improve the quality of production. Edwards *et al.* (1998) and Tandon and Choudhury (1999) showed the importance of maintenance as the best way to avoid maintenance problems that are often very expensive. And also how the predictive maintenance techniques have evolved to keep a check of mechanical health by generating information on the machine condition. In rotating machines, the transmission elements: belts, gears and bearings are of major interest in industrial maintenance as the operation of a mechanical system heavily depends on health of these elements. Particularly, the rolling bearing is one of the most critical components that determine machinery health and its remaining life time in modern production machinery (Jayaswal *et al.*, 2008). Robust Predictive Health Monitoring tools are needed to guarantee the healthy state of rolling bearings during the operation. A predictive health monitoring tool indicates upcoming failures which provide sufficient lead time for maintenance planning, as showed by El-Thalji and Jantunen (2015), Mann *et al.* (1995) and Renwick and Babson (1985).

Over the past two decades, several methods have been the subject of studies and developments. Visibly noticed are revolution methods based on mechanical signal processing, which are divided into two main categories, detection and diagnosis, and are based on time-frequency methods and temporal methods or a combination of both. Thus, many methods are born, the scalar indicators such as kurtosis, skew, crest factor (Dron *et al.*, 2004; Pachaud *et al.*, 1997), demodulation and detection of the envelope (Sheen, 2004, 2008), amplitude modulation (Stack *et al.*, 2004), detection of vibration modes (Rizos *et al.*, 1990), de-noising vibratory signals (Bolaers *et al.*, 2004), the spectral density analysis (Krejcar and Frischer, 2011), the Fast Fourier Transform (Lenort, 1995), the statistical model based on hypothesis test as KS-test Kolmogorov and Smirnov (Kar and Mohanty, 2004; Dong *et al.*, 2011; Yang *et al.*, 2005), scalar and vector statistical time series methods (Kopsaftopoulos and Fassois, 2011), neural networks (Samanta and Al-Balushi, 2003), wavelets (Bendjama and Boucherit, 2016), blind source separation (Wang *et al.*, 2014), fuzzy logic (Liu *et al.*, 1996). El-Thalji and Jantunen (2015) and Rai and Upadhyay (2016) reviewed almost all the techniques used in the domain predicting defects.

Typical defects in bearings are localized defects that occur generally in form of tiredness cracking under cyclic pressure of contact (El-Thalji and Jantunen, 2015; Fajdiga and Sraml, 2009; Glaeser and Shaffer, 1996; Ismail *et al.*, 1990; Tauqir *et al.*, 2000). Thus, the detection of cracking is frequently based on detection of the attack. During an abnormal operation, a series of wide band impulses will be generated when the rolling element of the bearing (ball or roller) (Brie, 2000; Ou *et al.*, 2016) goes above the defect at a frequency determined by the shaft speed, geometry of the bearing and the site of the defect (Barkov, 1999; Dyer and Stewart, 1978; Feng *et al.*, 2016; Ma and Li, 1995; Tandon and Choudhury, 1999). The site of the defect depending on the characteristic frequencies gives the possibility of detecting the presence of the defect and performing the diagnosis of the defective part.

The difficulty of detection of localized defects (Niu *et al.*, 2015) is related to the bearing energy which will diffuse through a wide band of frequency and hence it can be easily immersed in the noise (Ma and Li, 1995; Van *et al.*, 2016). Thus, under various operating regimes (varying loads and speeds), many methods remain inefficient for the prediction (El-Thalji and Jantunen, 2015), because it may happen that an excited resonance mode at the beginning of the attack may not be excited later when the defect has developed (Ma and Li, 1995; Mikhlin and Mytrokhin, 2008). In this paper, and to refer on the sequential analysis developed by Wald in the 1940s (Schneeweiss, 2005; Wald, 1943, 1945, 1947, 1949; Wald and Wolfowitz, 1943, 1948), a composite hypothesis test is used for the detection and diagnosis of localized defects in rolling bearings. To this end, it is necessary to be provided with a significant and exact variance without any need to estimate when the resonances modes occur.

2. Problem position

2.1. Probability Density Function (PDF) of vibrations of rolling bearings

To characterize vibration of rolling bearings, which is supposed to be a stationary stochastic process, and the PDF can describe the percentage in time when the signal reaches a given amplitude x. For the given amplitude, the PDF is estimated by

$$P(x) = \lim_{\Delta x \to 0} \frac{P_r\{x \leqslant x(t) \leqslant x + \Delta x\}}{\Delta x} = \lim_{\Delta x \to 0} \frac{1}{\Delta x} \sum_{i=1}^{j} \frac{\Delta t_i}{T} \tag{2.1}$$

where T is the total time of observation and Δt_i is the i-th duration while $x(t)$ is inside the interval $[x, x + \Delta x]$. For vibration without a defect, which represents healthy functioning, the distribution of the amplitude can be considered as a Gaussian process. This vibratory signature will have a well-defined variance σ_0^2 which is different from the variance σ_1^2 of a signal with a localized fault (Fig. 1) and, consequently, the overall vibration of the bearing will be constituted by two alternately periodic parts with different variances (Ma and Li, 1995).

2.2. Sequential Probability Test (SPRT)

Introduce now the sequential probability test (SPRT) of a simple null hypothesis H_0 which indicates the good operating condition and a simple alternative hypothesis H_1 which indicates the presence of a defect, based on N independent observations $x_1, x_2, \ldots, x_N$ having a common probability density function developed by Wald (1945, 1949) and Weiss (1956).

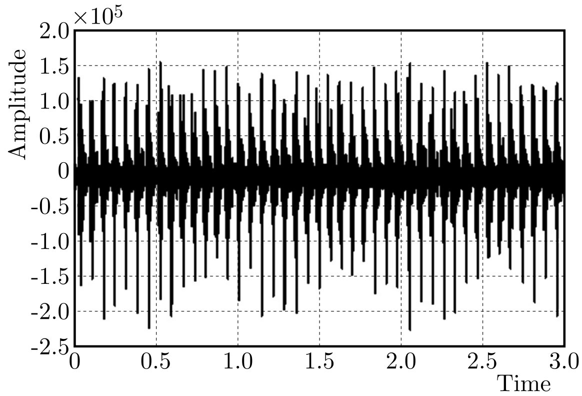

Fig. 1. Real signal of the rolling bearing with a defect

The hypotheses are

$$\begin{aligned} H_0: \qquad & P(\mathbf{x}/H_0) = \frac{1}{\sqrt{(2\pi)^N\sigma_0^N}}\exp\Big(-\sum_{i=1}^N \frac{x_i^2}{2\sigma_0^2}\Big) \\ H_1: \qquad & P(\mathbf{x}/H_1) = \frac{1}{\sqrt{(2\pi)^N\sigma_1^N}}\exp\Big(-\sum_{i=1}^N \frac{x_i^2}{2\sigma_1^2}\Big) \end{aligned} \tag{2.2}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_N]$ and σ_i^2 are the variances with $\sigma_0^2 < \sigma_1^2$.

For the analysis of any vibratory signal, certainly one of the two variances will be retained outside the test hypothesis H_1 and one will have information whether or not it occurs with one of the characteristic frequencies of the rolling bearings (inner race, outer race, ball and cage). In the case of healthy rolling bearings, during a time $\varDelta t$ for the signal $x(t)$, all measurements of M observations will have a Gaussian distribution given by relation $(2.2)_1$, In the case of a defective bearing given by $(2.2)_2$ and by varying the number M of observations in the time $\varDelta t$, and as soon as M is sufficiently large, and it is always possible to calculate the estimated variance σ_1^2 of the acquired vibratory signal with the defect in the rolling bearing. The variances σ_0^2 for healthy rolling bearings could be calculated by

$$\sigma_0^2 = \frac{1}{M}\sum_1^M x_i^2 \tag{2.3}$$

and $\sigma_1^2 = M^{-1}\sum_1^M x_i^2$ is considered as an estimated variance of the defect signal. Such an estimate will lead to a test for probability of both detection or false alarm (Ma and Li, 1995).

3. Sequential test

3.1. The likelihood ratio test with simple choice

The likelihood ratio test (PRT) of the σ_1^2 measurement could then be expressed as follows (Ma and Li, 1995; Paulson, 1947)

$$\begin{cases} \text{if} \qquad \xi(\mathbf{x}) > \mu \qquad \text{choose } H_1 \\ \text{if} \qquad \xi(\mathbf{x}) < \mu \qquad \text{choose } H_0 \end{cases} \tag{3.1}$$

where $\xi(\mathbf{x})$ is the likelihood ratio, which is defined by

$$\xi(\mathbf{x}) = \frac{P(\mathbf{x}/H_1)}{P(\mathbf{x}/H_0)} = \frac{\sigma_1^N}{\sigma_0^N}\exp\Big(\frac{\sigma_1^2-\sigma_0^2}{2\sigma_0^2\sigma_1^2}\sum_{i=1}^N x_i^2\Big) \tag{3.2}$$

By taking the natural logarithm of the two parts, the test can be simplified into

$$\begin{cases} \text{if} \quad f(\mathbf{x}) > \gamma \quad \text{choose} \ H_1 \\ \text{if} \quad f(\mathbf{x}) < \gamma \quad \text{choose} \ H_0 \end{cases} \tag{3.3}$$

where

$$f(\mathbf{x}) = \sum_{i=1}^{N} x_i^2 \qquad \gamma = \frac{2\sigma_0^2\sigma_1^2}{\sigma_1^2 - \sigma_0^2} \ln\Big(\frac{\sigma_1^N}{\sigma_0^N}\mu\Big) \tag{3.4}$$

Then probability P_f of the false alarm and the detection probability P_d of the PRT are

$$\begin{aligned} P_f &= P(f(\mathbf{x}) > \gamma/H_0) = P\Big(\sum x_i^2 > \gamma/H_0\Big) = \int\limits_{\sum x_i^2 > \gamma} P(\mathbf{x}/H_0)\, d\mathbf{x} \\ P_d &= P(f(\mathbf{x}) > \gamma/H_1) = P\Big(\sum x_i^2 > \gamma/H_1\Big) = \int\limits_{\sum x_i^2 > \gamma} P(\mathbf{x}/H_1)\, d\mathbf{x} \end{aligned} \tag{3.5}$$

From equations (2.2) and $(3.5)_1$, the P_f is a decreasing monotone function of the parameter γ.
Integration of equations (3.5) leads to

$$P_f = \exp\Big(\frac{-\gamma}{2\sigma_0^2}\Big) \qquad P_d = \exp\Big(\frac{-\gamma}{2\sigma_1^2}\Big) \tag{3.6}$$

where σ_0^2 is the variance measured during healthy operation, σ_1^2 –variance measured during unspecified operation and γ – the threshold of the test determined by

$$\gamma = -2\sigma_0^2 \ln P_f \tag{3.7}$$

while combining P_d with P_f we will have

$$P_d = P_f^{\frac{\sigma_0^2}{\sigma_1^2}} \tag{3.8}$$

Using equation $(3.5)_1$ to determine the probability of the false alarm P_f which corresponds to threshold equation (3.7) on the one hand and, on the other, using this same threshold will give the maximum probability of detection P_d defined by equation $(3.5)_2$ related to the variance σ_1^2 which is an unknown parameter estimated in one duration of the previously signal fixed. It can be deduced that the uniformly most powerful test (UMP) exists in the sense of Neyman-Pearson criterion which maximizes P_d (3.8) for a given P_f because the optimal probability rate test (PRT) (3.3) for each $\sigma_1^2 > \sigma_0^2$ could be completely defined apart from the knowledge of the true variance σ_1^2 of the signal defect. Finally, the UMP test is defined by system (3.3) and is constructed by equations (3.1), (3.2) with a determined γ by the pre-established false alarm probability α, where α is the threshold of significance

$$P_f(\gamma) = \alpha \tag{3.9}$$

3.2. Wald sequential test

Contrarily to the classical test (test with a simple choice), one is not obliged to make a choice between the two hypotheses H_0 and H_1, consequently, one deals with another type of test. If the size of observations is fixed, the construction of the test leads to the sharing of possible values of the statistical domain in three regions (Wald, 1945; Berger and Wald, 1949; Wolfowitz, 1949; Sobel and Wald, 1949)

$$\Psi^{(n)} = \Psi(x_1, x_2, \dots, x_n) \tag{3.10}$$

that is the region of probable values and the region of improbable values (knowing that the basic hypothesis H_0 is true). If a given value of $\Psi(x_1, x_2, \dots, x_n)$ falls into the region of improbable values, the basic hypothesis is rejected. The sequential test, that is, the test based on a sequential procedure of observation, is built up as follows. For each value of

$$\nu = 1, 2, \dots, n, n+1 \tag{3.11}$$

the domain Γ_ν of possible values of the critical statistics $\Psi(x_1, x_2, \dots, x_n)$ is divided into three disjoined regions: $\Gamma_\nu^{H_0}$ – region of probable values, $\Gamma_\nu^{H_1}$ – region of improbable values and Γ_ν^* – region of doubtful values (knowing that H_0 is true)

$$\Gamma_\nu = \Gamma_\nu^{H_0} \cup \Gamma_\nu^{H_1} \cup \Gamma_\nu^* \tag{3.12}$$

where $\nu = 1, 2, \dots$ with each step ν of the sequential procedure of observation. After having recorded the observations $x_1, \dots, x_\nu$, $\nu = 1, 2, \dots$ one makes a decision relying on the following rule which defines the Wald test: if $\Psi(x_1, x_2, \dots, x_n) \in \Gamma_\nu^{H_0}$ one accepts H_0; if $\Psi(x_1, x_2, \dots, x_n) \in \Gamma_\nu^{H_1}$ one accepts H_1 and if $\Psi(x_1, x_2, \dots, x_n) \in \Gamma_\nu^*$ the problem remains open until the ν-th observation. For this reason, the region Γ_ν^* is called the region of indetermination or the region of the observation pursuit.

For the establishment of the Wald test of probability, one considers two simple hypotheses of the following form, see Wald (1945, 1947) and Wald and Wolfowitz (1948)

$$\begin{array}{ll} H_0 & \text{The observation is extracted from a density population } f(x,\theta_0) \\ H_1 & \text{The observation is extracted from a density population } f(x,\theta_1) \end{array} \tag{3.13}$$

The critical statistics of this test is defined by the relation (Wald, 1945, 1947; Paulson, 1947)

$$\Psi^{(\nu)} = \ln \frac{f(x_1,\theta_1) \cdots f(x_i,\theta_1)}{f(x_1,\theta_0) \cdots f(x_i,\theta_0)} = \sum_{i=1}^{\nu} \ln \frac{f(x_i,\theta_1)}{f(x_i,\theta_0)} \tag{3.14}$$

where: $f(x_i,\theta_0) = P(\mathbf{x}/H_0)$ with $\theta_0 = \sigma_0^2$ and $f(x_i,\theta_1) = P(\mathbf{x}/H_1)$ with $\theta_1 = \sigma_1^2$.

$P(\mathbf{x}/H_0)$ and $P(\mathbf{x}/H_1)$ could be drawn from equation (2.2). So, one establishing the likelihood ratio, the critical statistics would be expressed as follows

$$\Psi^{(\nu)} = \ln \left\{ \left[\frac{\exp\Big(-\frac{1}{2\sigma_1^2}\sum_{i=1}^{\nu} x_i^2\Big)}{\sqrt{(2\pi)^\nu}\sigma_1^\nu} \right] \Big/ \left[\frac{\exp\Big(-\frac{1}{2\sigma_0^2}\sum_{i=1}^{\nu} x_i^2\Big)}{\sqrt{(2\pi)^\nu}\sigma_0^\nu} \right] \right\} \tag{3.15}$$

After simplification of equation (3.15) and arrangement of the logarithmic term, one gets

$$\Psi^{(\nu)} = \frac{\sigma_1^2 - \sigma_0^2}{2\sigma_0^2\sigma_1^2} \sum_{i=1}^{\nu} x_i^2 + \frac{\nu}{2} \ln \frac{\sigma_1^2}{\sigma_0^2} \tag{3.16}$$

The three regions are defined roughly by relations (3.11), (3.12), (3.13) and (3.16) that define the completely Wald test (WT) (Aïvazian, 1986; Wald, 1947; Weiss, 1956)

$$\begin{aligned} &\Gamma_\nu^{H_0} = \Big\{\Psi :\ \Psi^{(\nu)} \leqslant \ln \frac{\beta}{1-\alpha}\Big\} \qquad \Gamma_\nu^{H_1} = \Big\{\Psi :\ \Psi^{(\nu)} \leqslant \ln \frac{1-\beta}{\alpha}\Big\} \\ &\Gamma_\nu^* = \Big\{\Psi :\ \ln \frac{\beta}{1-\alpha} \leqslant \Psi^{(\nu)} \leqslant \ln \frac{1-\beta}{\alpha}\Big\} \end{aligned} \tag{3.17}$$

Wald test (3.17) is more optimal than all tests between hypotheses (3.13) with risks of the first and second species lower than the respective given values α and β.

Values of α and β (Aïvazian, 1986) are: 0.1, 0.05, 0.025, 0.01, 0.005, 0.001, 0.002.

4. Rolling element bearings defects detection

4.1. Detection procedure

The detection procedure is divided into many steps which can be stated as follows:

1 – Take the discrete vibration for M samples, which is larger than the amount of the characteristic period of the defect.

2 – Select a window of size N for the test.

3 – Estimate the variance σ_0^2 by using equation (2.3).

4 – Suggest a choice of α and β.

5 – Position the window at the beginning of recording of the vibration.

6 – Compute $\Psi^{(N)}$ by using equation (3.16).

7 – Define the intervals of the three regions by the terminals $a = \ln[\beta/(1-\alpha)]$ and $b = \ln[(1-\beta)/\alpha]$.

8 – Make the test by using equation (3.17).

9 – Generate a hypothetical signal defined by

$$h(i) = \begin{cases} 0 & \text{if} \quad H_0 \quad \text{is true} \quad (\Psi \leqslant a) \\ 1 & \text{if} \quad H_1 \quad \text{is true} \quad (\Psi \geqslant b) \end{cases}$$

If $a \leqslant \Psi \leqslant b$, carry on with pursue for data opening another window (here, one does not make a decision but only increases the size of the window).

10 – After generation of the hypothetical signal, if a defect is present, there will be a data vector composed of two values 0 and 1. If 1, then appears periodically with a period of the characteristic frequency of the bearing and is considered defective.

11 – To compare the detected frequency with the main characteristic frequencies of the rolling bearings, it would be very easy to locate the defect so the diagnosis could be established by comparing the multiple of this frequency detected with that of the most well-known defects.

4.2. Test plan

Based on the detection procedure described in Section 4.1, a test plan can be established which is shown by the procedure diagram shown in Fig. 2. So that the experiment is valid, one chooses N as a small fraction of the characteristic period of the defect, that is to say one fifth (Ma and Li, 1995). By examining step 10 in the detection procedure in Section 4.1 (to show if there is periodicity), one uses autocorrelation of the signal, a peak in the autocorrelation function reveals the periodicity of the signal, and the value of the time of this peak will give the period of the defect T_d. Consequently, one can determine the frequency of the defect f_d, and comparing it with the characteristic frequency f_c, one can establish the diagnosis.

5. Validation of the model by simulated and experimental signals

5.1. Validation of the model by simulated signals

5.1.1. Generation of the simulated signals

To simulate the defect, a bearing of the type NJ2204ECP has been used. The shaft speed is $n = 1500$ rpm, the characteristic frequencies are determined by the relations from Appendix A1,

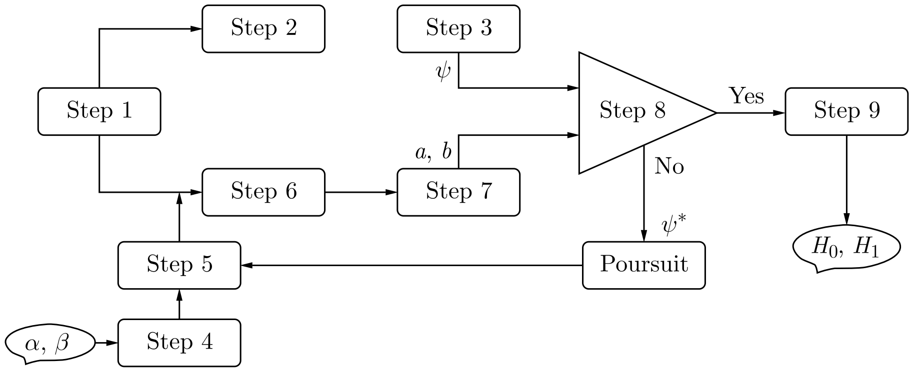

Fig. 2. Test plan

where the frequency of the cage is: $f_{cage} = 0.39fr$ (9.74 Hz), the frequency of the outer race: $f_{or} = 0.39Zfr$ (87.68 Hz), the frequency of the inner race $f_{ir} = 0.61Zfr$ (137.32 Hz), and the frequency of the ball: $f_{re} = 4.754fr$ (118.85 Hz); where: $fr = 25$ Hz, Z is the number of balls. For NJ2204ECP: $Z = 9$, $d = 7.5$ mm, $D = 34$ mm, $\alpha = 0$. The reference signal (Fig. 3a) is taken as a sinusoid of frequency 25 Hz, amplitude equal to unit and a null phase. The simulated defect signal (Fig. 3b) is considered as the sum of a sinusoid of frequency 25 Hz, amplitude equal to unit and the null phase, a sinusoid of frequency 87 Hz of amplitude 10 times the unit (representing a defect of frequency 87 Hz, which corresponds to the frequency of the outer race, as one can use the function pulstran available in Matlab which generates a series of impulses), and a Gaussian white noise centered with variance equal to 1 generated by the function "randn" available in Matlab with a signal noise ratio SNR = 20 dB. The thresholds of significances are fixed at $\alpha = 0.05$ and $\beta = 0.002$.

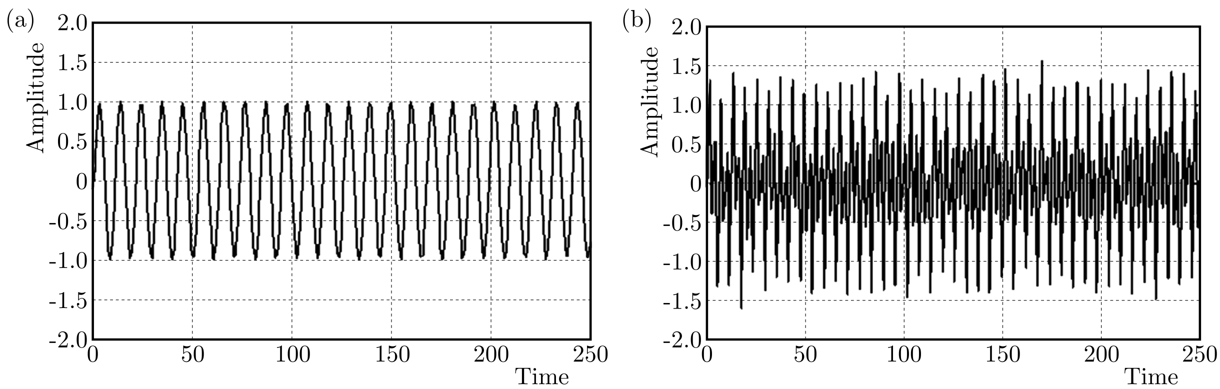

Fig. 3. (a) Reference signal, (b) defect signal

5.1.2. Interpretation

One can say that periodicity of a hypothetical signal (h-signal, Fig. 4a) appeared in the function of autocorrelation (Fig. 4b) reveals the existence of a defect. To determine its frequency, one carries out Fourier fast transform (FFT) of the hypothetical signal, which reveals visually the frequency of the defect (87 Hz) which corresponds indeed to the characteristic frequency of the outer race (Fig. 4c). Consequently, one can affirm that the plan suggested for detection and diagnosis of the defect in the bearing has succeeded and to made diagnosis of the defective part. During healthy running, the hypothetical signal will be zero, the autocorrelation of the h-signal will not reveal any periodicity, and the FFT will confirm the absence of the defect.

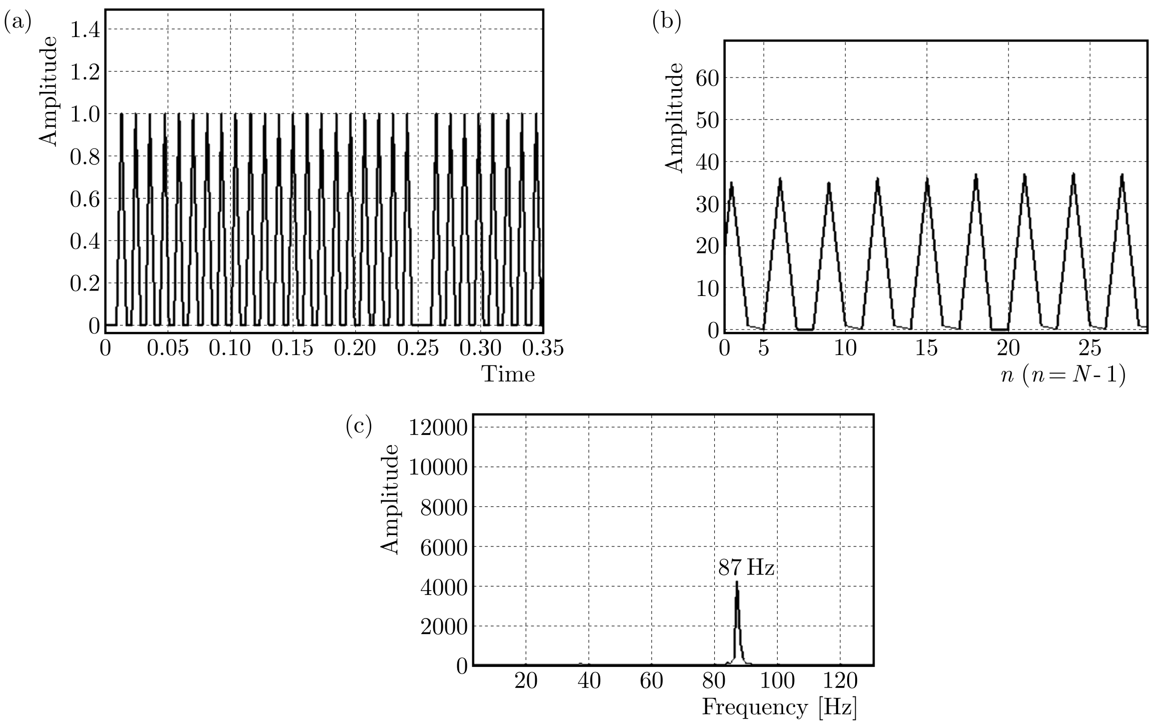

Fig. 4. (a) Hypothetical signal, (b) autocorrelation of hypothetical signal, (c) FFT of hypothetical signal

5.2. Validation of the model by experimental signals

5.2.1. Generation of the experimental signals

The test stand consists of a reinforced concrete frame, isolated from the ground by shock-absorbing studs. Two rows of shafts each having diameter of 60 mm and length of 680 mm are mounted in an open loop and fixed to the chassis by four rolling bearings with an average stiffness of $3 \cdot 10^7$ daN/m as shown in Fig. 5.

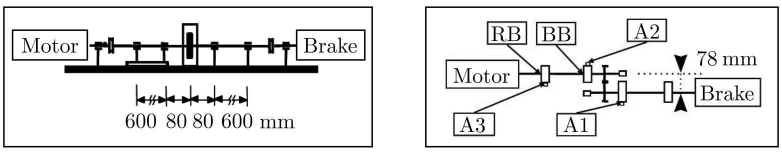

Fig. 5. Architecture of the teststand (RB – roller bearing, BB – ball bearing)

The bearings in the vicinity of the test gear pair have ball bearings of the type 6012, while the outer bearings are roller bearings of the type NU1013. The shaft lines are connected in rotation by test gears. The applied speed and torque are measured by an electronic device composed of a motor and a brake.

The dynamic behavior of the system can be studied using measurements of the acceleration, transmission error and noise. The accelerations are measured using piezoelectric accelerometers ENDEVCO 224C whose resonance frequency is 32 kHz. The accelerometers are mounted by gluing small duralumin pellets onto the accelerometers which are screwed. The tests are carried out on a spur gear with helical teeth. The gear ratio is 36/38 with modulus $m = 2$. The geometric characteristics of the ball and roller bearing are given in Table 1.

Type of defect: To simulate the scaling on the bearings, a notch of 1.7 mm and depth of 0.088 mm is made using a fine grinder as shown in Fig. 6. The roller bearing is removable without "NU type" destruction or specialized tooling.

Table 1. Geometric characteristics of the ball and roller bearing

Geometric characteristics	Ball bearing	Roller bearing
Middle diameter to center of balls D [mm]	77.7	80.55
Diameter of ball d [mm]	9	7
Number of balls Z	14	21
Angle of contact α	0	0

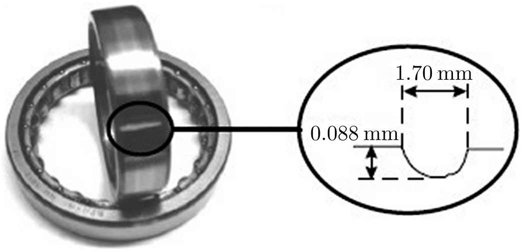

Fig. 6. Defective inner race geometry of the roller bearing

Monitoring conditions: The applied load is equal to 12 daN m and 4300 rpm speed test. The characteristic frequencies of the ball bearing and the roller bearing are calculated by the geometrical formulas given in Appendix A1.

Table 2. Characteristic frequencies of the ball and roller bearings

Bearing type	F_r [Hz]	F_{cage} [Hz]	F_{or} [Hz]	F_{ir} [Hz]	F_{er} [Hz]
6012	71.67	31.68	443.56	559.77	627.02
NU1013	71.67	32.72	687.11	817.89	830.91

F_r – rotating frequency, F_{cage} – frequency of the cage,
F_{or} – frequency of the outer race, F_{ir} – frequency of the inner race,
F_{er} – frequency of the ball or roller

Experimental signals: The acquired reference signal and the acquired signal defect are shown in Fig. 7a and 7b.

5.2.2. *Interpretation*

The detection and diagnostic plan applied to the experimental signals shown in Fig. 7a and Fig. 7b is able to detect the fault frequency applied to the bearing inner ring shown in Fig. 8a. It shows the presence of state "1" of the hypothetical signal and Fig. 8b shows a frequency of 814 Hz very close to the fault frequency which is equal to 817.89 Hz. It indicates that the plan has reacted well in establishing a correct diagnosis.

6. Diagnosis plan

To establish a good diagnosis of defects, it is necessary to know a significant number of defects. Thus, by comparing the frequency detected by the Wald test presented before with the characteristic frequencies we can locate the defect. By comparing the defect frequency with the main defects of the rolling bearings (Barkov, 1999), we can establish the diagnosis by using the frequency of modulation presented in the work of Barkov (1999). For the plan suggested by Fig. 2 it is possible to establish the diagnosis of the bearing defective part and its nature.

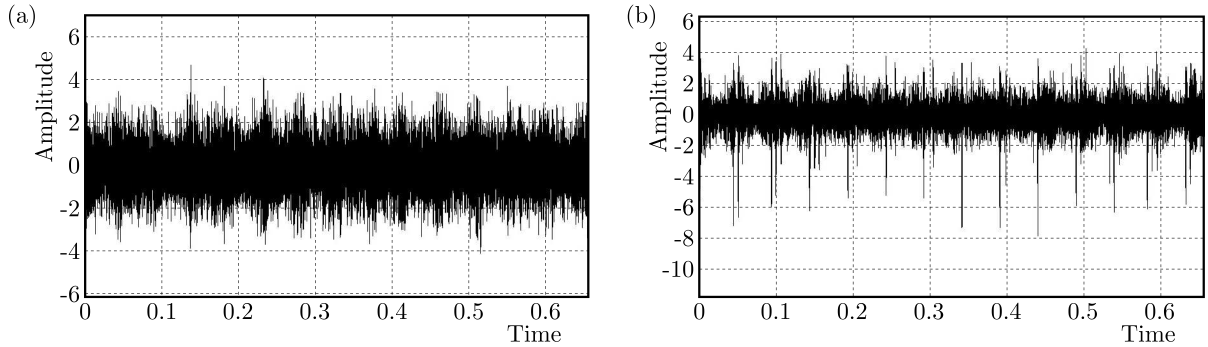

Fig. 7. (a) Experimental reference signal, (b) experimental defect signal

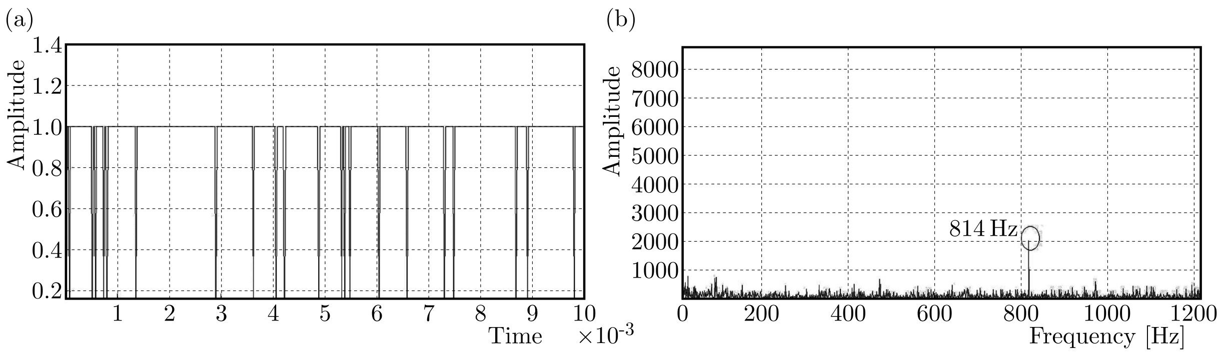

Fig. 8. (a) Experimental hypothetical signal, (b) FFT of the experimental hypothetical signal

7. Conclusion

The detection and diagnosis plan based on the Wald test is described. This plan can be applied to measurements of the bearings vibration signals with and without defects under various loads and speeds. The effectiveness of the suggested detection plan is illustrated in Fig. 4 for the simulated signal and in Fig. 8 for the experimental signal. The plan works very well with vibratory signals of wide bands. Finally, the plan is very promising for automatic detection and diagnostic applications.

Acknowledgment

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Appendix 1

Characteristic frequencies of the bearing (Barkov, 1999):

— frequency of the cage

$$f_{cage} = \frac{f_r}{2}\Big(1 - \frac{d}{D}\cos\alpha\Big)$$

— frequency of the outer race

$$f_{or} = Z\frac{f_r}{2}\Big(1 - \frac{d}{D}\cos\alpha\Big)$$

— frequency of the inner race

$$f_{ir} = Z\frac{f_r}{2}\Big(1+\frac{d}{D}\cos\alpha\Big)$$

— frequency of the ball

$$f_{re} = f_r\frac{d}{D}\Big[1+\Big(\frac{D}{d}\Big)^2\cos\alpha\Big]$$

where α is the angle of contact, d [mm] – diameter of the ball, D [mm] – middle distance to the center of balls, Z – number of balls, f_r [Hz] – rotating frequency ($f_r = n/60$), n [rpm] – shaft speed.

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