

# A GENERATIVE APPROACH TO HULL DESIGN FOR A SMALL WATERCRAFT

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## ABSTRACT

*In the field of ocean engineering, the task of spatial hull modelling is one of the most complicated problems in ship design. This study presents a procedure applied as a generative approach to the design problems for the hull geometry of small vessels using elements of concurrent design with multi-criteria optimisation processes. Based upon widely available commercial software, an algorithm for the mathematical formulation of the boundary conditions, the data flow during processing and formulae for the optimisation processes are developed. As an example of the application of this novel approach, the results for the hull design of a sailing yacht are presented.*

**Keywords:** method; generative design; yacht; optimisation; genetic algorithm

## INTRODUCTION

The design process for watercraft, with a wide spectrum of types and applications, has evolved over many centuries. This activity has always oscillated between art, handcraft and science. Most current knowledge in this field has come from tradition and experience, with creativity representing part of the art and science used for the prediction of design performance. In practice, this process is based on the knowledge and experience of the designer, who first generates or collects various design samples, mostly as two-dimensional sketches. There exist many variants of hull shapes that realise this set of parameters but all of them should be tested in order to choose the best. However, such ideas are not easily shaped and each of them may require a serious amount of time. In addition, the solutions generated by designers are often limited due to the inadequacy of their imagination, preconceptions and personal preferences [1].

With the development of computational methods and computer-based models, many modern designers have rejected their own design intuition and have begun to

rely more on results from digital devices. Most scientific reports in the field of ship design theory deal with very advanced modelling systems and calculations, including computer fluid dynamics, the finite element method and velocity prediction programs, where the results are extremely sensitive to the input data and the method of computing [2], [3], [4]. Nowadays, computer systems for professionals with an initial module for hull creation have reduced in cost and as a result have become common among low-budget designers and non-professionals [5], [6], [7]. In particular, for the design of small ships and recreational vessels, such as yachts and boats, the development of computer-aided drafting techniques and modelling software has enabled designers to circumvent tradition, the artistic aspects of design and personal preferences. Computer software gives an almost ready hull model from the start from a collection of ships, boats, yachts and so on, which can be modified in a parametric or manual way. Knowledge regarding the morphology of hull shape and basic shipbuilding heritage are disappearing. Therefore, most designers design and evaluate using computers [8], [9].

This type of knowledge is often hidden because it is connected either to commercial successful or the inability of many designers to articulate their knowledge in an explicit form [10]. In many cases, a mixture of these factors and sailing myths makes the result incomprehensible and fuzzy [11].

The common use of advanced software, the requirements for high performance in the design process and continuous cost reductions, as well as pressure on designers, have given the perception that hull design is a trivial problem and can be obtained immediately in a single attempt. With such a strong business approach, there is insufficient time for design reflections and possible alternatives never emerge. Since design ideas do not develop, subsequent projects become a repetition of what has already been accomplished. Alternatively, generative systems are innovative approaches that help designers generate many unbiased design samples efficiently. Generative modelling combined with optimisation algorithms is now one of the most widely used computational methodologies for generating and designing geometry in different industries. Such a connection allows designers to script complex generative algorithms for design-space exploration. However, in such a design approach, extensive knowledge is required from a designer who invents and develops the design process. A basic approach for successful design, especially for small ships, must blend scientific knowledge with experiential wisdom and the possibility of a variety of choices, through bringing intuitive and generative processes to the endeavour [12].

This study focuses on the description of a generative approach for the early-stage design of yachts, including, primarily, the search for a hull form. This approach is based on a generative evolutionary framework to configure and optimise designs. Our study contains a description of the method for hull geometry generation. We also indicate the available digital tools for creating such systems and an adopted description of the workflow.

## MOTIVATION FOR RESEARCH

Most design tasks in ship building are connected to the shape of the hull. The main properties of the ship, namely, drag, safety and comfort, are based on the geometry of the hull. In the simplest terms, a new design means the generation of a new hull.

Generally, each design task begins by making the main assumptions and a preliminary architectural sketch of the general arrangement. Such a design task does not have only one solution. As a result, the design process is a choice between different goals and it is always an attempt to find a compromise. The traditional approach to solving this problem is a synthesis of theory and practice. The main parameters and dimensions and volume and weight demands are predicted using parametrical formulae. These equations are the result of collecting data on ships already built (theoretical part). In this phase, the expert's wealth of knowledge and experience is essential (practical part) because there are infinitely many

variants of shapes for one collection of its main parameters. The naval architect should proceed to bring all data together in a creative way. The best result from a set of possibilities is then chosen. This short moment in a long timeline of the design process often decides the final results of the work.

The removal of this choice is practically impossible in the next phases of the project. This is the traditional approach. Generally, in this approach, there is a lack of tests and research on different concepts to reach a synergy of knowledge. This is a basic disadvantage of this approach because the diversity of design alternatives is crucial for the conceptual phase of the design process. Using the generative approach allows the knowledge of the designer to be supplemented with computational decision support that provides real-time spatial feedback during conceptual design. Simultaneously, utilising a large repertoire of parametrical formulae and design knowledge together with the precise criteria of spatial evaluation presents design challenges.

For ensuring the realisation of the design assumptions and creating functional and efficient spatial formations, both theory and an application of a parametric design process are necessary. In generative design, a basic layout of an input computer-aided design model must be first created. Such an efficient geometric system is a challenge in itself. Design specifications and constraints are then defined. Various computational simulations are later executed to obtain a set of optimised solutions. However, the possibility of assessing various design options based on a similar knowledge base as in the traditional approach is the main advantage of the proposed generative approach (Fig. 1).

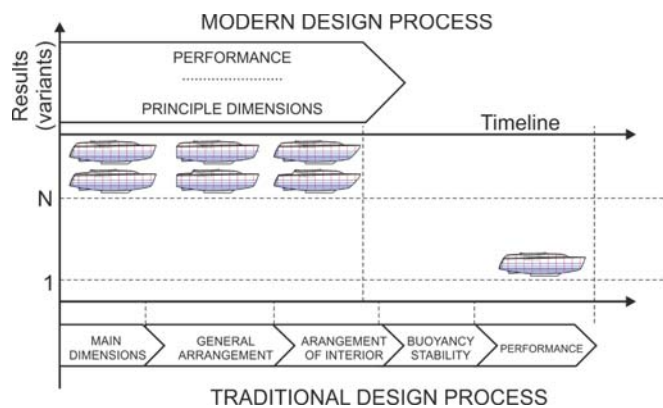


Fig. 1. Traditional versus modern approach for design process

## PROPOSED NEW APPROACH

The concept of the proposed method of designing small watercraft is based on the strategy of generative design, consisting of the simultaneous use of elements related to the theory of ship design and modern computer modelling techniques of free geometry. The very process of generating solutions is considered a “black box” that the designer can influence by changing the input elements or by using a generative tool for creating shapes. Most of the generative

decisions are made automatically by the software and only some – crucial ones – must be made by the designer. Due to the subjectivity of the target function (the results of the design are also influenced by aesthetic impressions), the final selection from a population of obtained results also must be made by the designer. The elements supporting the selection of the final solution are the extensive visualisation of results in the form of a three-dimensional (3D) graph, enabling the classification and grouping of related solutions. This approach can be defined as semi-automated.

The problem of multi-objective optimisation can be solved by methods such as the weighted criteria method, hierarchical optimisation and the minimax method. However, due to the huge number of expected solutions during the design of watercraft, the evolutionary method seems most suitable. Such an approach is known as evolutionary multi-objective optimisation. The most popular technique for engineering is a genetic algorithm [13]. Key parameters for such a process are decisive variables (mathematical formulae controlling the object configuration), design targets (formulae that create space for searching for minima or maxima), parameters (variables that influence the design target) and limits (definition of the acceptable space of solutions).

Such a process can be performed by the following steps:

- (1) Definition of a task:
  - Preliminary formulation of input data: particulars of the designed object; a conceptual sketch; basic assumptions; form topology of hull; criteria for evaluation.
- (2) Parametrisation of the geometrical model:
  - Selection of software set;
  - Formulation of the geometry generator for an applied method of parametrisation, the definition of variables and their range of changes.
- (3) Establishment of criteria:
  - Formulation of goals for the optimisation process by the selection of measured data;
  - Establishment of the constraints for a model.
- (4) Generation of a population of possible solutions:
  - Selection of control parameters for optimisation;
  - Generation of a population of solutions and its preliminary assessment.
- (5) Post-processing:
  - Creation of a design space of solutions;
  - Selection of a method of exploring the design space of solutions. For a small population of results obtained, such a process can be performed by the designer using the Pareto front. For a large population of solutions, automation of the process must be applied;
  - Selection of a solution for further design steps.

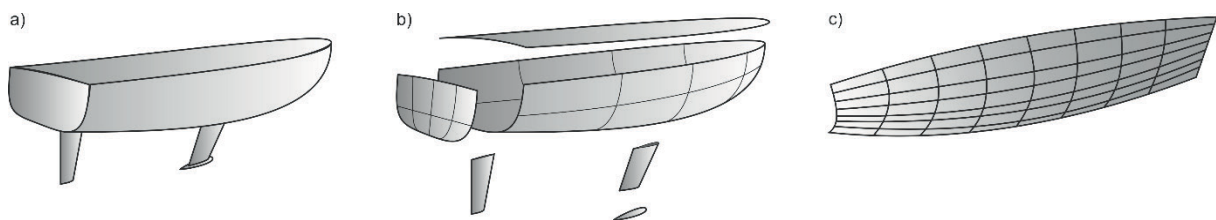


Fig. 2. Decomposition of hull form into a single-surface set: a) complex form; b) regions; c) single surface

## DESCRIPTION OF PARAMETRIC MODEL OF A HULL

The traditional source of description of the form of a ship's hull is a drawing of its body lines, which represents the data of the presented form as a set of 2D sections made in three perpendicular planes. Contemporary approaches to the description of 3D shapes are based upon the use of 3D analytically described surfaces. There are some different methods in the literature [14] that are proposed to describe the parametric modelling of the hull shapes of small watercraft.

These techniques were developed for various kinds of ship and for various types of hull, such as single/multi-chine planning hulls [15], [16] and rounded hulls for yachts and boats [17], [18]. The level of advance and complication of the geometrical modelling technique depend on the aim considered by the researchers. Some of these can be used only for a yacht or a boat. There are also universal approaches that help to create an extensive range of hull forms [14]. The accuracy and correctness of the gained model are different. However, the larger the range of possible applications, the more complex the parametric model and the more variables to control. The general models usually consist of many regions and patches. This level of division can be an impediment in the conceptual stage of the design process. In this phase, the designers are looking for alternatives, not the final solution. The most simplified case is a description by one patch (one continuous surface). Such a case is adequate for a smooth and simple geometry, like a sailing boat (Fig. 2).

The proposed scheme of design for a sailing yacht hull surface is divided in two stages, namely, pre-defining a set of control curves and final surface modelling.

The most popular way of describing the curves and surfaces is by using the object-type non-uniform rational B-spline (NURBS). The properties of such an object are influenced by the:

- localisation of the control points;
- definition of node vectors;
- weight coefficients for control points;
- degree of controlling polynomial [19], [20].

The NURBS elements allow the use of interpolation techniques to model the surface patches of a hull, such as by generating a grid of points – a tensor-product patch or a network of curves – a blended patch, a sweeping composite patch or a lofted patch.

## TOOLS FOR PARAMETRIC MODELLING

The process of modelling watercraft is complicated due to the complexity of the object, as well as the dispersion of design works, so tools for semiautomatic design space exploration should be implemented. This approach delivers a wide population of acceptable variants of solutions relatively quickly, but the selection of the best set is completed by the designer.

As the digital environment for the 3D modelling, as well as visualisation of results in the presented approach, RhinoCeros® [21] software has been selected, with Grasshopper® as an additional module [22]. Such generative tools make it possible to solve multi-object-oriented algorithms and to generate the hull form directly in a digital environment, creating the possibility of simultaneous modification of geometry and analysis of intermediate results, as illustrated in Fig. 3. As the optimisation tool, the “Octopus” plugin of Grasshopper® software is used [23] which gives the possibility of evolutionary multi-objective optimisation with the application of the strength Pareto evolutionary algorithm [24].

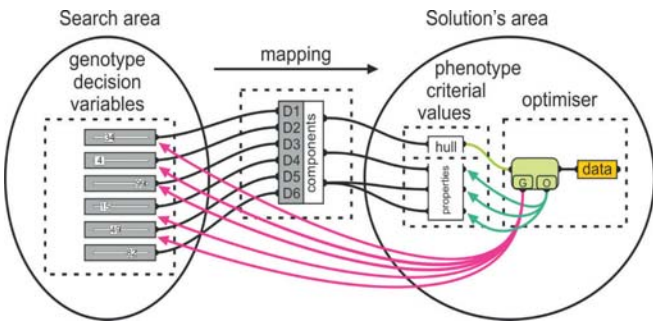


Fig. 3. Schematic of optimisation process with parameters and components (on basis of [23])

## CASE STUDY

The proposed design method has been used for the example of a sailing yacht with the following given requirements: simple topology; speed  $v = 7$  knots; preliminary main dimensions of overall length  $L = 12.00$  m, maximum breadth of hull  $B_{max} = 6.00$  m and height of hull  $H = 2.00$  m.

## Parameterisation of Geometrical Model

Since the surplus of control points leads to the generation of many analysed cases, rational limitations on the number of control points must be applied. For the presented approach, a nine-point parametric model was applied. Points are used for the generation of a network of five control curves, which are the basis for a NURBS surface as a one-patch hull. Modification of the control points (Fig. 4a) leads to a transformation of the resulting surface (morphing) (Fig. 4b).

The control points of the model are marked by  $P_{ij}$  for  $i=0,1,2$  and  $j=0,1,2$ . The control curves are described by the following shortcuts: the keel profile *KLP*; the side of deck *SOD*; the transom edge *TRA*; the basic station *STA*; the stem profile *SMP*.

The boundary dataset and possibilities of transformation for the performed task are limited by the main dimensions of the designed hull, as well as the symmetry of the object formed. The range of variability of the decision variables is presented in Table 1.

Tab. 1. Collection of decision variables

No.	Coordinates	Range	No.	Coordinates	Range
1	$z_{00}$	$\langle 0; H \rangle$	6	$y_{21}$	$\langle 0; B/2 \rangle$
2	$y_{20}$	$\langle 0; y_{21} \rangle$	7	$x_{02}$	$\langle 0; L \rangle$
3	$x_{01}$	$\langle 0; x_{02} \rangle$	8	$z_{01}$	$\langle 0; z_{00} \rangle$
4	$x_{11}$	$\langle 0; x_{02} \rangle$	9	$w_{10}$	$\langle 0.01, 0.02, \dots, 2.00 \rangle$
5	$x_{21}$	$\langle 0; L \rangle$	10	$w_{11}$	$\langle 0.01, 0.02, \dots, 2.00 \rangle$

## Set of Criteria

The design of any engineering object is a multi-goal process. A definition of such a set of results of the process is required (values as well as their limit, i.e., minimum or maximum) and is the basis for the assessment of the quality of solutions obtained and the finalisation of the process. Due to this, the set of optimisation criteria must be established. The objective function should have the form of a parametric formula, which can be adopted in the digital design space. The goals should allow for the assessment of different aspects of the design. For the analysed case, the following set of criteria, which represent the specific properties of a sailing yacht, have been selected and represented as a vector containing all applied expressions:

$$f(x) = [f_1, f_2, f_3, f_4]^T \quad (1)$$

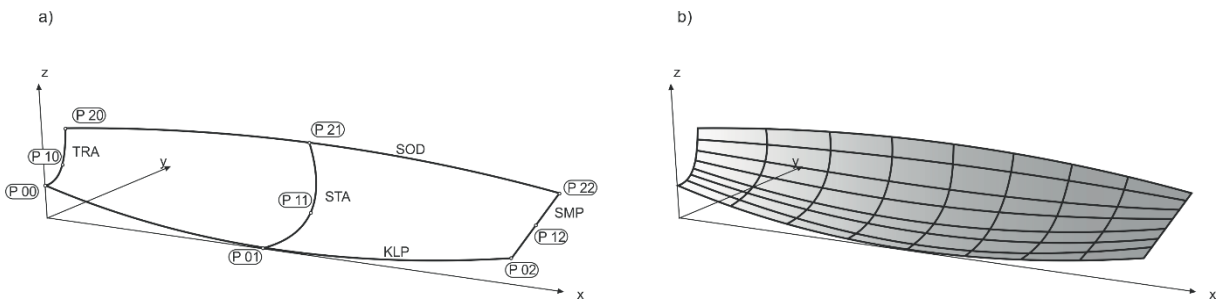


Fig. 4. a) Control curves with location of decisive variables. b) Final surface modelling

where  $f_1$  is the mass criterion [t],  $f_2$  is the resistance criterion [N],  $f_3$  is the safety (stability) criterion [m] and  $f_4$  is the space criterion [-]. The objective functions had a structure enabling their implementation in the developed computer module and therefore the calculations of criterion quantities can be carried out directly in the modelling environment.

– Mass criterion:

$$f_1(x) = M_S = q_K \cdot S_K + M_W \quad (2)$$

where  $M_S$  is the mass of the light ship [t],  $q_K$  is the unitary mass of the shell surface and is equal to 0.017 [t/m<sup>2</sup>],  $S_K$  is the area of the shell surface [m<sup>2</sup>] and  $M_W$  is the mass of equipment and is equal to 5.55 [t]. Data for the masses of the shell and equipment are taken from a yacht of similar size [25].

– Resistance criterion:

$$f_2(x) = R_T = R_R + R_V \quad (3)$$

where  $R_V$  is the viscosity resistance according to the ITTC 78 method [26] and  $R_R$  is the residual resistance [27].

To estimate the residual resistance value  $R_R$ , the formula developed at the end of the twentieth century was adopted as part of the Delft Systematic Yacht Hull Series study ([25]). It is currently believed to be the most reliable and comprehensive approximation regarding the prediction of sailboat hull resistance [28], [29]. It is expressed by the following relationship:

$$R_R = \left\{ a_0 + a_1 \cdot C_p + a_2 \cdot LCB + a_3 \cdot \frac{B_{WL}}{T} + a_4 \cdot \frac{L_{WL}}{\nabla^{1/3}} + a_5 \cdot C_p^2 + a_6 \cdot C_p \cdot \frac{L_{WL}}{\nabla^{1/3}} + a_7 \cdot LCB^2 + a_8 \cdot \left( \frac{L_{WL}}{\nabla^{1/3}} \right)^2 + a_9 \cdot \left( \frac{L_{WL}}{\nabla^{1/3}} \right)^3 \right\} \cdot \nabla \cdot \rho \cdot g \quad (4)$$

where  $\alpha_{0-9}$  are the regression coefficients,  $\nabla$  is the volume of displacement [m<sup>3</sup>],  $\rho$  is the water density [kg/m<sup>3</sup>],  $g$  is gravity and is equal to 9.81 [m/s<sup>2</sup>],  $L_{WL}$  is the length of the waterline [m],  $B_{WL}$  is the breadth of the waterline [m],  $T$  is the hull draft [m],  $LCB$  is the longitudinal centre of buoyancy [%],  $LCF$  is the longitudinal centre of float [m] and  $C_p$  is the prismatic coefficient [-].

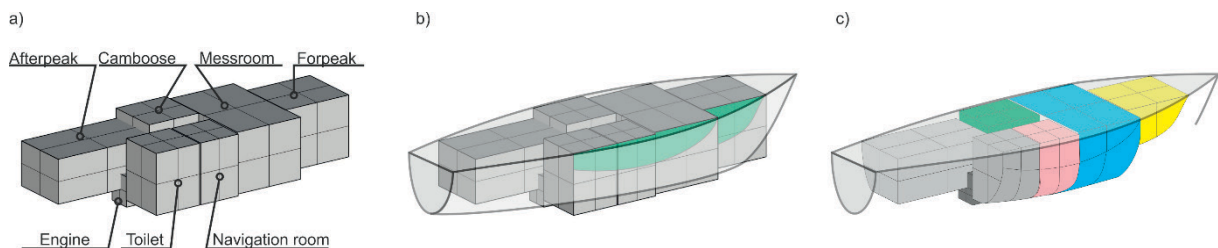


Fig. 5. Definition of internal volume criterion: a) solid model of volume required; b) model of hull and interior; c) model of internal space

– Safety criterion

It is difficult to determine a stability parameter for the purposes of optimisation. However, in conjunction with other objective functions, this will allow us to build a wide spectrum of secure solutions. In this study, it was assumed that the universal measure of stability is the initial metacentric height. This value is the relation between the hydrostatic features of the hull and the mass properties of the entire object, with both depending on the geometry of hull. For simplicity, it is assumed that the vertical centre of gravity for the generated hull equals the draft. It should be assumed that at this design stage, qualitative results are more important than accurate quantitative results. Due to this, the formula for  $h_0$  is as follows:

$$f_3(x) = h_0 = VCB + \frac{I_{xx}}{\nabla} - T \quad (5)$$

where  $h_0$  is the initial metacentre height [m],  $VCB$  is the vertical coordinate of the centre of buoyancy [m] and  $I_{xx}$  is the initial transverse moment of inertia of the waterline [m<sup>4</sup>].

– Space criterion

Based on the preliminary concept of functionality with a set of initial linear dimensions from the design assumptions, a 3D model of internal space has been built using cuboid blocks as functional components (Fig. 5a). The key point for such a requirement is a block model for internal space requirements. To define such a criterion, two limited functions are introduced. The first one is for required internal volume and the second one is for the disposed volume. For the first requirement, the relationship between the total volume of the 3D model,  $V_{BC}$  (Fig. 5a), and the volume that is limited to the design hull shape,  $V_{BI}$  (Fig. 5c), is as follows:

$$f_4(x) = V = 1 - \frac{V_{BI}}{V_{BC}} \quad (6)$$

where  $V_{BI}$  is the volume of solid model limited to the designed hull shape [m<sup>3</sup>],  $V_K$  is the total volume of the designed hull shape [m<sup>3</sup>] and  $V_{BC}$  is the total volume of the solid model of the required internal space [m<sup>3</sup>].

## CONSTRAINTS

Due to the nature of evolutionary multi-objective optimisation software, technically rational constraints of the evolution process must be introduced. Depending on the analysed object, such functions are defined based upon geometrical, exploitation, strength, technological or economic requirements. In this research, the constraints are of geometrical, topological, functional and quantitative character.

The geometrical constraints relate to positions between the control points. The range within which any point can shift is strictly defined and relates to the technique of surface generation. Due to this, one can expect that each prepared geometry is like the hull of the specific type of ship.

The topological constraints depend on the technique used for modelling of the surface patches. One must check which system of building a surface is most suitable for our design. This technique can be changed at any time during the design process or it can deal with it as decision variables.

The topological constraint is a binary variable having two possible values called “true” and “false.” This function checks whether the block, which represents a functional system, is inside or outside the hull.

The quantitative constraints refer to the function of objectives described by parametric methods, such as the total mass of the object and the bare hull resistance and stability, for which there are restrictions related to the allowable range of variability of parameters of the formula used. The following constraints have been identified in this study as numerical values:

- For the resistance criterion: the scope of applicability of the approximate method [27].
- For the stability criterion: the minimal metacentre height should be greater than 0.5 m according to the rules of classification societies [30].
- For the space criterion: the generated shapes have an evenly distributed volume along the length of the hull and the trim angle cannot be greater than 3°.

## GENERATION OF SOLUTIONS

The generation process of a set of potential solutions consists of the following steps (Fig. 6):

- 1) A set of decisive variables (coordinates of control points and weights for control points) with a limited range defined;
- 2) The shape of the body of the hull is generated, the mass balance is obtained, the buoyancy equation is solved and the draft is estimated. The set of hydrostatic parameters for the calculated draft is generated;
- 3) The data generated are evaluated according to the fulfilment of constraints, as well as the objective functions;
- 4) For a correctly established dataset, the geometrical form of the hull, as well as the values of the criteria functions, are transferred to be evaluated by a genetic algorithm;
- 5) A set of acceptable solutions is generated, forming a Pareto front.

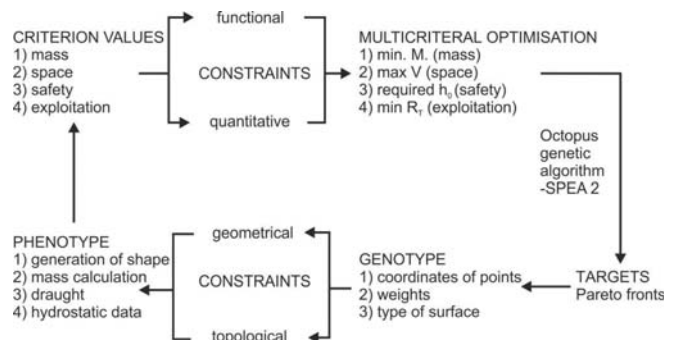


Fig. 6. Optimisation loop based on [31]

## POST-PROCESSING

Post-processing consists of the following activities:

- Visualisation of the results obtained in the form of graphs and diagrams;
- A review of the obtained solutions in the working space of RhinoCeros® software;
- Classification of the solution set;
- Grouping of the solution set.

An example visualisation of the results obtained in the form of graphs and diagrams originating from the optimisation software is presented in Fig. 7, where only Pareto optimal solutions are plotted. Presentation in these forms of graphs allows the designer to consider “soft” criteria like the waterline curvature, the visual form of the hull transom angle and so on.

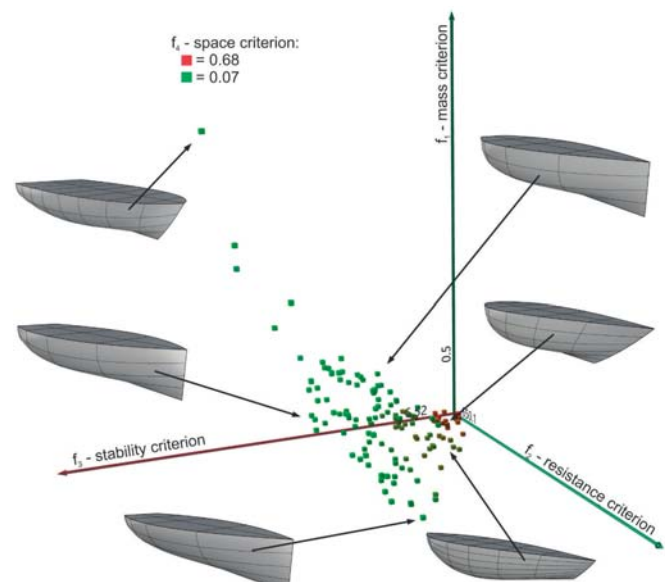








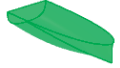



























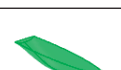



Fig. 7. Graphical visualisation selected from the results in relationship to the Pareto optimal solutions

The solutions obtained were compared in terms of the value of criterion functions and the hull shape delivered with them. The range of results obtained and the individual values of the objective function for selected cases are presented in Table 3.

Tab. 3. Comparison of results

No.	Range of results		1	2	3	4	5	6
Crit. \ Vis.								
1 [t]	6.52–8.19		8.19	7.98	7.75	7.56	7.36	7.53
2 [N]	650–1438		1438	1271	1238	1374	1367	1305
3 [m]	0.52–6.55		6.55	4.55	3.96	2.36	1.08	1.90
4 [-]	0.067–0.694		0.069	0.067	0.069	0.070	0.14	0.091
No.	7	8	9	10	11	12	13	14
Crit. \ Vis.								
1 [t]	7.39	7.52	7.70	7.55	7.50	7.33	7.28	7.22
2 [N]	1196	1177	1080	1024	1055	1090	1057	1003
3 [m]	1.28	2.78	3.31	2.70	2.07	1.13	2.14	2.02
4 [-]	0.122	0.072	0.071	0.073	0.086	0.144	0.092	0.11
No.	15	16	17	18	19	20	21	22
Crit. \ Vis.								
1 [t]	7.11	7.00	7.08	6.83	6.89	6.89	6.99	7.12
2 [N]	1155	1059	985	1021	956	888	913	832
3 [m]	0.63	1.49	1.53	0.86	0.51	0.55	1.09	0.96
4 [-]	0.258	0.191	0.179	0.321	0.378	0.365	0.214	0.225
No.	23	24	25	26	27	28	29	30
Crit. \ Vis.								
1 [t]	6.66	6.87	6.86	6.69	6.82	6.78	6.75	6.73
2 [N]	833	788	713	779	697	681	667	668
3 [m]	0.64	0.99	0.61	0.70	0.80	0.72	0.62	0.58
4 [-]	0.462	0.299	0.367	0.471	0.344	0.367	0.392	0.401
No.	31	32	33	34	35	36	37	38
Crit. \ Vis.								
1 [t]	6.60	6.63	6.64	6.52	6.52	6.59	6.62	6.59
2 [N]	754	723	693	675	666	662	650	661
3 [m]	0.69	0.60	0.52	0.55	0.53	0.53	0.56	0.53
4 [-]	0.546	0.511	0.505	0.694	0.695	0.561	0.600	0.561

## CONCLUSIONS

This study presents the concept of a novel approach to the design problems of small vessels with the use of a parallel design with multi-criteria optimisation processes. Based upon widely available commercial software, an algorithm for the mathematical formulation of the boundary conditions, the data flow structure during processing and the formulae for the optimisation processes have been developed.

Some important advantages can be noted:

- Efficiency of the design process:
  - The wide space of possible analysed projects allowing different but acceptable geometries offers more possible solutions and gives the possibility of comparison as well as delivering new and original ideas;
  - The use of computer programs significantly accelerates the process of generation of the expected data;
  - Joining separate computer-aided design tools in one process, like evolutionary multi-criteria optimisation and iterative formulae for decision making, gives a concise tool for efficient design process performance.
- Automation of process:
  - The genetic algorithm and evolutionary methods can create some unexpected positive solutions, offering the possibility of breaking traditional design schemes.
- Analysis of the results:
  - Graphical representation of the results accelerates and makes the analysis and decision process easier.
- Time and labour consumption of the process (Table 4):
  - The processing time for one design case amounts to seconds. This radically accelerates the design process in comparison to the classical approach;
  - Consequently, the number of cases it is possible to analyse is large and cannot be calculated in a reasonable time using the classical approach;
  - Digital results provide the possibility of carrying out some simulation tests during the design phase, which leads to reduced time and costs;
  - The proposed algorithm can work based upon low-cost, widely-available software and an average-class computer.

Tab. 4. Process performance data

Computer	Processor	Intel® Core™ I7-4500U CPU @ 1.80 GHz 2.40 GHz
	RAM	12.0 GB
	Operating system	64 bites, processor x64
Algorithm	Decision variables	9
	Criteria	4
		7
Parameters	22	

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