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THE DISTRIBUTION OF AIR BUBBLE SIZE IN THE PNEUMO-MECHANICAL FLOTATION MACHINE

ROZKŁAD WIELKOŚCI PĘCHERZYKÓW POWIETRZA W PNEUMO-MECHANICZNEJ MASZYNIE FLOTACYJNEJ

The flotation rate constant is the value characterizing the kinetics of cyclic flotation. In the statistical theory of flotation its value is the function of probabilities of collision, adhesion and detachment of particle from the air bubble. The particle – air bubble collision plays a key role since there must be a prior collision before the particle – air bubble adhesion happens. The probability of such an event to occur is proportional to the ratio of the particle diameter to the bubble diameter. When the particle size is given, it is possible to control the value of collision probability by means of the size of air bubble. Consequently, it is significant to find the effect of physical and physicochemical factors upon the diameter of air bubbles in the form of a mathematical dependence.

In the pneumo-mechanical flotation machine the air bubbles are generated by the blades of the rotor. The dispergation rate is affected by, among others, rotational speed of the rotor, the air flow rate and the liquid surface tension, depending on the type and concentration of applied flotation reagents.

In the proposed paper the authors will present the distribution of air bubble diameters on the grounds of the above factors, according to the laws of thermodynamics. The correctness of the derived dependences will be verified empirically.

Keywords: flotation, dispersion of bubbles, bubble size distribution, Rayleigh distribution

Flotacja jest procesem masowym, o którego przebiegu decyduje szereg zdarzeń losowych. Są nimi zderzenia ziarna z pęcherzykiem powietrza oraz trwała adhezja ziarna do powierzchni pęcherzyka. Ze względu na losowy charakter wymienionych wyżej zdarzeń można mówić jedynie o prawdopodobieństwie zajścia zdarzenia. Opisując subprocesy flotacji określonymi prawdopodobieństwami można wyznaczyć wartość stałej prędkości flotacji. Warunkiem koniecznym do zajścia adhezji jest uprzednie zderzenie ziarna z pęcherzykiem. W modelu opracowanym przez zespół Yoon i Luttrell (1989) prawdopodobieństwo

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zderzenia jest wyrażone wzorem (1). W praktyce flotacji wielkość ziarna i pęcherzyka są zmiennymi losowymi o określonych rozkładach. W związku z tym również prawdopodobieństwo zderzenia będzie zmienną losową o określonym rozkładzie. Istnieją modele w których stała prędkości flotacji jest wyrażona przez rozkłady wielkości ziaren i pęcherzyków (wzór 3).

W tym przypadku do wyliczenia średniej wartości stałej prędkości flotacji niezbędna jest znajomości rozkładu wielkości pęcherzyków i wielkości ziarna. Również dla wyliczenia całkowitej powierzchni pęcherzyków przepływających przez jednostkę powierzchni przekroju poprzecznego w jednostce czasu, konieczna jest znajomość rozkładu wielkości pęcherzyków.

W prezentowanej pracy przedstawiono sposób określenia rozkładu wielkości pęcherzyków w pneumo-mechanicznej maszynie flotacyjnej, oparty na rozważaniach heurystycznych.

W procesie dyspergowania powietrza podawanego do komory flotacyjnej rośnie powierzchnia pęcherzyków powietrza, powstających z jednostki objętości gazu, co pociąga za sobą wzrost energii powierzchniowej na granicy faz ciecz-gaz. Założono, że duże bańki powietrza dzielą się na mniejsze pęcherzyki według prawa Boltzmanna rozkładu energii (wzory 10, 12 i 13).

Przy założeniu, że energia pęcherzyków jest zmienną losową o rozkładzie ciągłym, wyliczono funkcję gęstości rozkładu tej zmiennej, wyrażoną wzorem (16). Średnica pęcherzyka powietrza, a zarazem jego energia są zmiennymi losowymi, pomiędzy którymi istnieje zależność funkcyjna wyrażona wzorami (11). Korzystając z twierdzenia odnoszącego się do rozkładu funkcji zmiennych losowych (wzór 17) wyliczono funkcję gęstości rozkładu średnicy pęcherzyków (wzory 18 i 20). Jest to rozkład znany w rachunku prawdopodobieństwa jako rozkład Rayleigha. Na rys.1 podana jest empiryczna funkcja gęstości rozkładu średnicy pęcherzyka.

Wartość średnią wielkości pęcherzyka $\overline{d_b}$ oraz odchylenie standardowe $\sqrt{V(d_b)}$ będące miarą rozrzutu wielkości pęcherzyków wokół wartości średniej zostały wyliczone na podstawie związku pomiędzy parametrem rozkładu Rayleigha a wartością średnią i wariancją (wzory 21 i 22). Dla uzyskania zależności parametru rozkładu oraz średniej wielkości pęcherzyka od warunków fizycznych i fizykochemicznych w komorze flotacyjnej zastosowano zasadę zachowania energii dla średniej wielkości pęcherzyków (wzory 23-27). Uzyskane wyrażenia na średnią wartość średnicy pęcherzyka oraz parametr rozkładu średnicy pęcherzyka (odpowiednio wzory 31 i 32) w sposób jawny podają zależność tych wielkości od napięcia powierzchniowego roztworu flotacyjnego, wydatku powietrza oraz mocy przekazywanej do układu flotacyjnego.

Słowa kluczowe: flotacja, dyspersja pęcherzyków, rozkład wielkości pęcherzyków, rozkład Rayleigha

1. Introduction

Flotation is a mass process and its course is determined by numerous random events. These are collisions of a particle with an air bubble and the permanent adhesion of a particle to an air bubble surface. Due to the random character of the above events we can only speak about a probability of occurrence. Thus, describing the subprocesses of flotation by the defined probabilities, which affect the rate of the process we can determine the value of flotation rate constant , indicate its relationship with the process parameters and properties of mineral (Brozek et al., 2003b). This value have macroscopic character and should include information on the factors affecting the flotation process. The prior particle-bubble collision is the necessary condition for the adhesion to occur. In the last decades numerous models of flotation rate constant were developed (Schuhmann, 1942; Sutherland, 1948; Bełogłazov, 1947; Pogorełyj, 1962; Varbanov et al., 1993; Geidel, 1985; Schubert & Bischofberger, 1979; Flint & Howarth, 1971; Yoon & Mao, 1996; Yoon & Luttrell, 1989; Schimmoller et al., 1993; Jiang, 1991; Brozek et al., 2003a). In these models the value of this constant depends upon the particle and bubble sizes or upon probability of the collision which depends upon the ratio of particle and bubble sizes. In the model developed by Yoon's and Luttrell's team (1989) the event probability is :

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$$P_c = A\left(\frac{d_p}{d_b}\right)^2 \tag{1}$$

where:

A — constant depending upon the value of Reynolds' number for the bubble,
d_p — particle diameter,
d_b — buble diameter.

In the flotation practice the particle and air bubble sizes are random variables of fixed distributions. Accordingly, also the collision probability will be a random variable of the fixed distribution. The shape of this distribution will affect the distribution of the flotation rate constant. Vorbanov, Forssberg and Hallin (1993), applying Radoev's et al. (1990) research results, presented the expression for the flotation rate constant, taking into account the particle hydrophobic properties, characterized by the θ wetting angle:

$$k = \frac{3V_g\, d_p\,(1-\cos\theta)}{2\pi\, S\, d_b^2} \tag{2}$$

where:

θ — particle equilibrium wetting angle,
V_g — air flow rate [m^3/s],
S — area of the flotation chamber cross-section,
d_b — bubble diameter,
d_p — particle diameter.

If the particle and air bubble sizes have fixed size distributions, described by the probability density functions as $f(d_p)$ and $f(d_b)$, respectively, the authors suggest replacing them in formula (2) by their mean values. Then the flotation rate constant is expressed by the formula (Vorbanov et al., 1993):

$$k = \frac{3V_g\,(1-\cos\theta)\displaystyle\int_0^{d_p} d_p\, f(d_p)\, dd_p}{2\pi\, S\displaystyle\int_0^{d_b} d_b^2\, f(d_b)\, dd_b} \tag{3}$$

In this case, it is necessary to know the size distribution of air bubble in order to calculate the mean value of the flotation rate constant. Also the calculation of the bubble surface area flux is enabled by knowing the air bubble size distribution. As it can be seen from the above examples, information about the distribution of bubble sizes in the flotation chamber will facilitate the analysis of the flotation process course of a given raw material in the concrete physicochemical conditions. This paper presents the method of determining the distribution of bubble sizes in the pneumatic mechanical flotation machines, based upon heuristic considerations.

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2. Size of air bubbles

If air bubbles are generated by a capillary of inner diameter d_i, then (analogically as for an appearing drop of liquid flowing out of the capillary), total buoyancy force of the bubble is counterbalanced by surface tension force (Cho and Laskowski, 2002, Krzan and Małysa, 2002):

$$\frac{\pi d_b^3}{6}(\rho_c - \rho_g)\, g = \pi\, d_i \sigma \tag{4}$$

from which:

$$d_b = \sqrt[3]{\frac{6 d_i \sigma}{(\rho_c - \rho_g)\, g}} \tag{5}$$

In the above formulas d_b denotes the air bubble diameter, ρ_g air density while σ surface tension of the flotation solution. As it can be seen from formula (5), the lower the surface tension of the solution, the smaller the air bubbles. This observation concerns the conditions in which covering the air bubble (which is breaking off from the capillary) with the surface-active compound reaches the state of equilibrium.

The surface tension of the solution decreases with the growth of concentration of surface-active compounds according to the following dependence (Scheludko, 1969):

$$\sigma = \sigma_o - kT\Gamma_\infty \ln(Kc + 1) \tag{6}$$

where:

- σ_o — water surface tension,
- k — Boltzmann's constant,
- T — temperature,
- c — concentration of the surface-active compound,
- Γ_∞ — maximum adsorption on the liquid-gas interface,
- K — constant.

Then:

$$d_b = \sqrt[3]{\frac{6 d_i \left[\sigma_o - kT\Gamma_\infty \ln(Kc+1)\right]}{(\rho_c - \rho_g)\, g}} \tag{7}$$

Butanol and hexanol are applied as a collector at coal flotation. The increase of collector concentration results in decreasing the size of air bubbles (in the conditions of adsorption equilibrium) which, consequently, results in the increase of probability of collision between a particle and a bubble in an elementary act of flotation and, therefore, in the increase of flotation rate constant.

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3. Minimum size of air bubbles

If an air bubble is mineralized completely, then, with its too small dimensions, it may appear that the gravity of the aggregate will be larger than its buoyancy force. Such an aggregate will not be elevated to the froth layer. For the simplified conditions of equilibrium of both forces the bubble size will be a minimum value below which the aggregate will not flow up to the froth layer (Szatkowski & Freyberger, 1985b):

$$\frac{N_p \pi d_p^3 (\rho_s - \rho_c)}{6} = \frac{\pi d_{b\min}^3 \rho_c}{6} \tag{8}$$

where

$N_p = \dfrac{\pi d_{b\min}^2}{d_p^3}$ — number of spherical particles on the surface of the completely mineralised bubble,

ρ_c — liquid density,

ρ_s — density of particles under flotation.

After substituting to formula (8) for N_p, an expression for the minimum bubble size is obtained:

$$d_{b\min} = \frac{\pi(\rho_s - \rho_c) d_p}{\rho_c} \tag{9}$$

Therefore the minimum size of bubbles depends upon density and size of particles under flotation. For example, for the coal of particle density ρ_s = 1.5 Mg/m^3 and ρ_c = 1.0 Mg/m^3, $d_{b\min} = 1.57 d_p$. In real flotation conditions the aggregate is in motion and is affected by the medium drag force. That is why the drag force should be added to the left side of equation (8). Because of it, the minimum bubble size should be slightly larger than the calculated one from formula (9).

4. Distribution of bubble diameter in the chamber of a flotation machine

In mechanical flotation machines the distribution of bubble diameter is affected by the following factors: rotational speed of the rotor, air flow rate, method of delivery of air to the flotation chamber which depends upon the machine type as well as surface tension of the flotation solution. A certain contribution to the final form of distribution is also to be made by the phenomena which is reverse to the process of dispergation of air bubbles, i.e. the phenomena of coalescence (Prince and Blanch, 1990).

The air pumped to the flotation chamber is dispersed by turbulence of the flotation medium, generated by rotational speed of the rotor. Large air bubbles which flow from the tubes are crushed into lots of smaller bubbles. In this process the surface of air bubbles, originating from the volume

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unit of gas, increases which results in the growth of surface energy on the gas-liquid interface. This energy is produced by liquid vortexes in the zone of intense vortex motion between the rotor and rotor stator. It was assumed that a large air bubble is divided into smaller bubbles according to Boltzmann's law of bubble energy distribution (Pigon & Ruziewicz, 1980):

$$n(w) = n_p \exp\left(-\frac{w - w_p}{E_c V_b}\right) \tag{10}$$

where:

- w — bubble surface energy,
- w_p — threshold level of bubble surface energy below which there are no bubbles (minimum level of bubble energy determining the minimum bubble size),
- E_c — constant depending upon intensity of coalescence,
- $n(w)$ — number of bubbles of w energy,
- n_p — number of bubbles of threshold energy w_p, originating from a gas volume unit,
- V_b — volume of a large air bubble leaving the tube transporting air to the flotation chamber.

Volume of a large air bubble is determined from the formula analogical to formula (5) whereas d_i denotes the inner diameter of the tube while the place of d_b is taken by a diameter of a large air bubble, marked further in the text as D.

Energies w_p and w are expressed by the following formulae:

$$w_p = \pi d_m^2 \sigma \tag{11a}$$

$$w = \pi d_b^2 \sigma \tag{11b}$$

where:

- σ — liquid surface tension,
- d_b — bubble diameter,
- d_m — diameter of the smallest bubbles.

If the air flow rate is equal to V_g [m^3/s], then the product $V_g\, n(w) = N(w)$ will present the number of air bubbles of energy w originating in the unit of time:

$$N(w) = N_p \exp\left(-\frac{w - w_p}{E_c V_b}\right) \tag{12}$$

while $N_p = V_g\, n_p$ denotes the number of bubbles of energy w_p, originating in the unit of time.

The relation

$$\frac{N(w)}{N_o} = f_1(w) = \frac{N_p}{N_o} \exp\left(-\frac{w - w_p}{E_c V_b}\right) \tag{13}$$

represents the fractional part of bubbles of energy w, originating in the unit of time while N_o denotes the total number of bubbles originating in the unit of time.

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Under the assumption that the energy of bubbles is a random variable, function $f_1(w)$ should be the probability density function of this random variable. If the function $f_1(w)$ is to be the probability density function, it has to fulfill a standardizing condition, i.e.:

$$\int_{w_p}^{\infty} f_1(w)\,dw = 1 \tag{14a}$$

$$A\int_{w_p}^{\infty} \exp\left(-\frac{w-w_p}{E_c V_b}\right) dw = 1 \tag{14b}$$

where A — constant.

After integration we obtain:

$$A = \frac{1}{E_c V_b} \tag{15}$$

Finally, the probability density function of bubbles energy is expressed by the formula:

$$f_1(w) = \frac{1}{E_c V_b}\exp\left(-\frac{w-w_p}{E_c V_b}\right) \tag{16}$$

The diameter of air bubble and also its energy are random variables between which there is a function dependence, expressed by formula (11b). If the distribution of bubble diameter is to be found, the theorem concerning the distribution of the function of random variables should be used (Gerstenkorn and Śródka, 1972):

$$f(d_b) = f_1\left[w(d_b)\right]\left|w'(d_b)\right| \tag{17}$$

where: $w'(d_b)$ is the derivative of function (11b) to d_b.

According to formulas (17) and (16), the probability density function of bubble diameter will be as follows:

$$f(d_b) = \frac{2\pi\sigma}{E_c V_b} d_b \exp\left[-\frac{\pi\sigma\left(d_b^2 - d_m^2\right)}{E_c V_b}\right] \tag{18}$$

Therefore the random variable, which is the bubble diameter, is subject to distribution known in probability calculus as Rayleigh's distribution. It belongs, similarly as Maxwell's and Weibull's distribution, to the family of gamma distributions and constitutes a particular case of the so-called generalised gamma distribution. The empirical distributions of bubble diameter, obtained by many authors, have the form of Raileigh's distribution (Szatkowski & Freyberger,1988, Grainger, 1988, Prince & Blanch, 1990, Małysa et a. 1999a,b, Imhof et al., 2003, Fallenius, 1987, Grau & Heiskanen, 2002, Hupka et al., 1994, Rodrigues & Rubio, 2003). Fig. 1 presents the probability density function of bubble diameter obtained by the team of Imhof et al. (2003).

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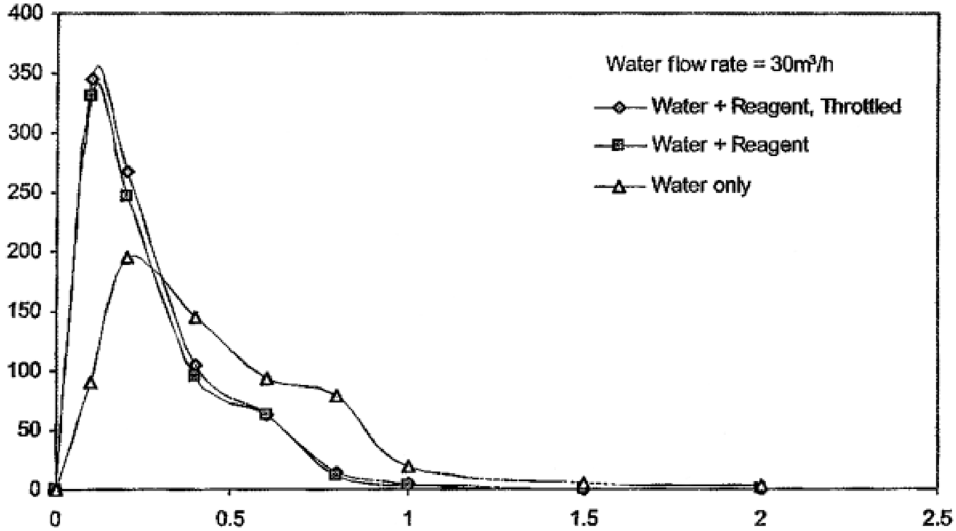

Fig. 1. Probability density function of bubble diameter (Imhof et al., 2003)

Denoting:

$$\frac{\pi\sigma}{E_c V_b} = \lambda_b \tag{19}$$

Formula (18) takes the form:

$$f(d_b) = 2\lambda_b d_b \exp\left[-\lambda_b\left(d_b^2 - d_m^2\right)\right] \tag{20}$$

The values λ_b is called the parameter of Rayleigh's distribution.

The mean value of bubble diameter $\overline{d_b}$ and standard deviation $\sqrt{V(d_b)}$ being the measure of spread of bubble diameter round the mean value are equal to, respectively:

$$\overline{d_b} = \int_{d_m}^{\infty} d_b f(d_b) dd_b = 2\lambda_b \int_{d_m}^{\infty} d_b^3 \exp\left[-\lambda_b\left(d_b^2 - d_m^2\right)\right] dd_b = \frac{\Gamma(1{,}5)}{\sqrt{\lambda_b}} \tag{21}$$

$$\sqrt{V(d_b)} = \left[\int_{d_m}^{\infty} \left(d_b - \overline{d_b}\right)^2 f(d_b) dd_b\right]^{\frac{1}{2}} = \left[\frac{\sqrt{\lambda_b \Gamma(2) - \Gamma^2(1{,}5)}}{\lambda_b}\right]^{\frac{1}{2}} \tag{22}$$

where:

$V(d_b)$ — variance of random variable,

Γ — gamma function.

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The detailed course of the curve of probability density function depends on the value of parameter λ_b. As it results from formulae (21) and (22), both the mean value of bubble diameter and standard deviation decrease with the growth of λ_b parameter. It means that the concentration of bubble diameters round the mean value increases.

5. Dependence of distribution parameter upon physical and physicochemical conditions in the flotation chamber

As mentioned before, rotational speed of the rotor, air flow rate and also the method of delivery of air into the flotation chamber as well as the surface tension of flotation solution affect the distribution of air bubble diameter. The distribution parameter should depend openly upon these factors. In order to formulate this dependence, the energy conservation law was applied to the mean diameter of bubble.

This law is expressed by the following formula:

$$E_1 - E_2 = E_3 \tag{23}$$

where:

E_1 — surface energy of bubbles of $\overline{d_b}$ [J/s], diameter, dispersed in a unit of time,
E_2 — surface energy of large air bubbles entering the flotation chamber in a unit of time [J/s],
E_3 — energy delivered to the chamber in a unit of time by rotor blades generating turbulence of the flotation medium [J/s].

The difference $E_1 - E_2$ is a change (increase) of surface energy caused by the increase of bubble surface in a unit of time. This energy is delivered from outside by a rotating rotor (E_3).

The above energies are expressed by the following formulas:

$$E_1 = \left(\pi \overline{d_b}^2 \sigma\right)\left(\frac{V_b}{\frac{\pi \overline{d_b}^3}{6}}\right)\left(\frac{V_g}{V_b}\right) = \frac{6\sigma}{\overline{d_b}} V_g \tag{24}$$

$$E_2 = \left(\pi D^2 \sigma\right)\left(\frac{V_b}{\frac{\pi D^3}{6}}\right)\left(\frac{V_g}{V_b}\right) = \frac{6\sigma}{D} V_g \tag{25}$$

$$E_3 = \eta P \tag{26}$$

where:

D — diameter of a large (dispersed) bubble, entering the flotation chamber,
η — efficiency of the motor mechanical system,
P — power of rotor depending on its rotational speed,
V_g — gas flow-rate.

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The first factor in formula (24) represents surface energy of the bubble of diameter $\overline{d_b}$, the second one is the number of bubbles of diameter $\overline{d_b}$, originating from a large bubble of volume V_b, while the third factor is the number of large bubbles flowing into the chamber in a unit of time. Analogical factors occur in formula (25). The product $(1 - \eta)P$ represents energy losses occurring in the mechanical system.

After substituting expressions (24)-(26) to formula (23) we obtain:

$$\frac{6\sigma}{\overline{d_b}}V_g - \frac{6\sigma}{D}V_g = \eta P \tag{27}$$

The mean value of bubble diameter, calculated from formula (27), is :

$$\overline{d_b} = \frac{6DV_g\sigma}{6V_g\sigma + \eta PD} \tag{28}$$

The following boundary:

$$\lim_{V_g \to \infty} \overline{d_b} = D \tag{29}$$

means that with the increase of air flow rate without a change of rotor power the diameter of bubbles grows and the rate of air dispergation decreases. It was confirmed experimentally (Gorain et al., 1995).

The size of a large air bubble D, calculated from formula (5), after taking into account that $\rho_c >> \rho_g$, is equal to:

$$D = \sqrt[3]{\frac{6d_i\sigma}{\rho_c g}} \tag{30}$$

After substituting for D into formula (28) the following expression is obtained for a mean bubble size:

$$\overline{d_b} = \frac{6V_g\sigma\sqrt[3]{d_i}}{\left(36\sigma^2V_g^3\rho_c g\right)^{\frac{1}{3}} + \eta P\sqrt[3]{d_i}} \tag{31}$$

It results from the analysis of this formula that the mean bubble diameter decreases with the growth of rotor power (increase of the rotational speed of rotor) and with decreasing the surface tension of the flotation solution (at the unchanged air flow rate).

The mean bubble diameter is connected with the distribution parameter by dependence (21). Taking into account expression (31), the distribution parameter of the bubble diameter in the mechanical flotation chamber is expressed by the formula:

$$\lambda_b = \left[\frac{\left(36\sigma^2V_g^3\rho_c g\right)^{\frac{1}{3}} + \eta P\sqrt[3]{d_i}}{6{,}77\,V_g\sigma\sqrt[3]{d_i}}\right]^2 \tag{32}$$

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The rotor power is proportional to its rotational speed. Therefore in formulas (31) and (32) the physical conditions in the flotation chamber are characterized by the rotational speed of rotor, the energy dissipation depending upon the properties of flotation pulp and air flow-rate while the physicochemical conditions are characterized by surface tension depending on concentration and type of reagent (O'Connor et al., 1990). Moreover, the distribution parameter depends on the method of air delivery, considered by diameter d_i of the pipe (capillary) which delivers the air.

6. Conclusions

1. The assumption of Boltzmann's distribution of energy of air bubbles leads to Rayleigh's distribution of bubble diameter. This type of distribution is confirmed by numerous literature items concerning bubble size.
2. The application of energy conservation law in relation to the bubbles generated in the flotation chamber enabled the authors to combine Rayleigh's distribution parameter and the mean bubble diameter with physical and physicochemical parameters of the flotation medium.

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