NON-EUCLIDEAN GEOMETRY IN THE MODELING OF CONTEMPORARY ARCHITECTURAL FORMS

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Summary. When seeking inspiration for development of spatial architectural structures, it is important to analyze the interplay of individual structural elements in space. A dynamic development of digital tools supporting the application of non-Euclidean geometry enables architects to develop organic but at the same time structurally sound forms. In the era of generative design and highly advanced software, spatial structures can be modeled in the hyperbolic, elliptic or fractal geometry. This paper focuses on selected non-Euclidean geometric models which are analyzed in generative processes of structural design of structural forms in architecture.

Keywords: elliptic geometry, hyperbolic geometry, fractal geometry

1 Introduction

The design of forms in contemporary architecture is an increasingly biomimetic process in which architects seek unconventional but also geometrically logical functional and spatial solutions. Mimicking the technology of nature creates beautifully modeled, fluid and ephemeral projects and is currently one of the most interesting trends in architecture. The key for understanding the structure of organic forms is abandoning the Euclidean geometry, which does not describe the elements of the natural world: it simplifies it using "ideal" forms such as: line, circle, square, cube [1]. The development of digital technologies opens new possibilities in the search for the optimum shapes of structural forms, through a description of biological structures and development of dynamic models in a non-Euclidean geometry.

"Geometry is a discipline, and I'm very disciplined" [2] – these are the words of a prominent American architect of Chinese descent, Ieoh Ming Pei, who designed, among others, the Bank of China Tower in Hong Kong (1982-1989), the Grand Louvre in Paris (1983-1993) or the Miho Museum in Shigaraki, Japan (1992-1997). Executions of I. M. Pei's designs confirm his words by showing beautiful forms built out of elements of the Euclidean geometry. Skillful and conscious use of geometry is even more important when the development of architectonic forms is a process combining several different geometries and the resulting surfaces are manifolds seamlessly combining objects with different curvatures. In such cases, the discipline in geometry, invoked by I. M. Pei, becomes an indispensable tool for developing free architectural forms.

2 Application of non-Euclidean geometry in modeling of architectural forms based on selected examples

2.1 Elliptic geometry

One of the non-Euclidean geometries is the elliptic geometry, also known as spherical geometry (the geometry of a spherical surface), which is a special case of the Riemann geometry for a constant and positive curvature. Elliptic geometry is a two-dimensional metric geometry in which, given a point not placed on a line, there is not even one disjoint line passing through that point and the sum of internal angles of any triangle is greater than 180°. Moreover, in elliptic geometry, all the straight lines are closed lines with a finite length and two different points may be connected by two segments.

In the history of architecture, elliptic geometry elements were used in domes, of which some of the most notable are: the *Pantheon* dome in Rome (diameter of 43.3 m), *Hagia Sophia* in Istanbul (one of the most magnificent Byzantine churches, built in 532-537) or the *St. Peter's Basilica* in Rome (dome with an internal diameter of 42.0 m and 52.0 m high, designed by Michelangelo Buonarroti). As the building technology developed and digital tools were implemented, the capacity to use elliptic geometry for originating structural forms and developing new tendencies in architecture has increased.

The search for unconventional architectural forms using non-Euclidean geometry has led to interesting and often more effective engineering solutions. An example of such a structural form design is the *30 St Mary Axe* in London (located in the City of London, i.e. London's main financial district), designed by Norman Foster and built in 2001-2004. The office building is 180 meters tall and has 40 floors (floors 39-40 are home to some of the highest bars and restaurants in London). The unique shape of the object, similar to a cone, has made it possible to reduce the side wind pressure and eliminate drafts at the street level, which often occur near high risers. Moreover, the tower uses cutting-edge technology to reduce energy consumption, including a double glass façade that protects the inside of the building against heat in the summer and provides effective thermal insulation in winter. The use of elliptic geometry in the development of the building's structural form has allowed its designers to obtain the unique shape and the achieve optimized engineering solutions. The streamlined tower is a distinctive element of London's skyline, at the same time setting new trends in the design of high-risers.

Figure 1: *30 St Mary Axe Tower*, design by Foster & Partners, London; a**)** London City panorama, b) interior of the restaurant on the top floor of the skyscraper source: a), b) Foster + Partners Ltd., [accessed on 7 May 2013] http://www.fosterandpartners.com/projects/swiss-re-headquarters-30-st-mary-axe

An interesting example of the application of elliptic geometry in smaller buildings is the house design inspired by the shape of a shell. *Shell House* is a luxury villa located in the Karuizawa province (near Nagano, Japan), designed by Japanese architect Kotaro Ide

(ARTechnic) in 2005 and built in 2006-2008. The shape of the building is based on two tubes with elliptical cross-section, which intertwine creating a smooth organic form centralized around a large fir tree. The ellipses form unique space both inside and outside of the building, making it difficult to separate the elements such as walls, roof or parts of the garden.

Figure 2: *Shell House*, design by Kotaro Ide (ARTechnic), Karuizawa; a) view from the south-west, b), c) view from the terrace on the fir, d) interior - view of the entrance to the terrace source: a , b, c) „ArchDaily", *Shell/ARTechnic architects*, 17.01.2009 [accessed on 7 May 2013] http://www.archdaily.com/11602/shell-artechnic-architects/

The house is in tune with its surrounding, forming a consonant system with a sculpture-like, organic structure. The large glass surfaces characteristic of the *Shell House* (looking towards the garden) enhance the impression that the villa has fused harmoniously with the surrounding landscape. The elliptical shell has been made of reinforced concrete and landscaping objects and some interior decoration elements are made out of the Japanese ulin tree. By using material technologies offering resistance to the changing weather conditions in the region (humid summers and cold winters) the leisure villa is fully prepared for year-round use.

2.2 Hyperbolic geometry

Hyperbolic geometry may be obtained from the Euclidean geometry when the parallel line axiom is replaced by a hyperbolic postulate, according to which, given a line and a point which is not on the line, there are least two different lines passing through the point that have no common points with that line. In hyperbolic geometry, a plane is the surface of a saddle, geodesic lines are hyperboles and the sum of internal angles of any triangle is less than 180°. One of the hyperbolic geometry models is the inside of a circle, i.e. the Klein's model, or another model with similar characteristics, i.e. the Poincaré Model. In the Poincaré hyperbolic geometry model, points of a plane are points of a borderless disk, lines are sections of lines and circular curves perpendicular to the disk's border [3]. The hyperbolic plane geometry model discovered by Henri Poincaré in 1882 was a frequent subject of work of Maurits Cornelis Escher, the eminent Dutch painter and graphic artist, whose works to this day remain the source of inspiration for architects. The Poincaré model was the foundation of M.C. Escher's work in the “Circle Limit” series. The use of hyperbolic geometry in architecture can be traced through the work of prominent engineers, designers and architects of the twentieth century, including P.L. Nervi, M. Nowicki, E. Saarinen, O. Niemeyer, F. Candela, E. Torroja. The work of engineers is particularly interesting, as it shows their search for the optimum structural forms using hyperbolic geometry.

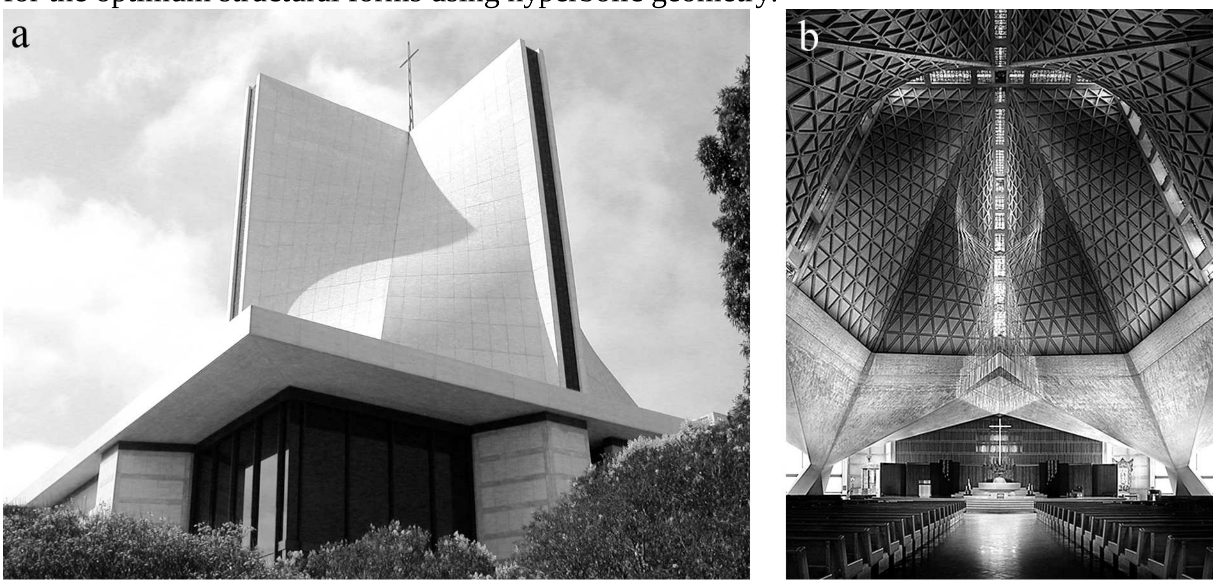

Figure 3: *St. Maria Cathedral* in San Francisco, California, design by P.L.Nervi and P. Belluschi; view of the object from the outside, b**)** interior view of the hyperbolic vault source: a) „Wikipedia – The Free Encyclopedia”, *St. Mary’s Cathedral in San Francisco,* 2008 [accessed on 8 May 2013] http://pl.wikipedia.org/wiki/Plik:St_Mary%27s_Cathedral_-_San_Francisco.jpg fot. Claire Sullivan, „Design Folio”, *St Mary’s Cathedral – San Francisco,* 12.08.2010, [accessed on 8 May 2013], <http://designfolio.co.nz/_blog/Design_Folio_NZ/page/91/#_>

One of the most interesting examples of such search is the *Cathedral of Saint Mary* in San Francisco, California, by P.L. Nervi (in cooperation with Pietro Belluschi) built in 1971. The building's unusual form has its origins in the geometry of a hyperbolic parabola, where the individual planes skew up, closing the structure and forming a cross where they intersect. Thanks to the geometry used, P.L. Nervi has created a unique form with a particularly attractive play of light, offering interesting visual effects outside and inside of the church. Additionally, due to the rigidity of hyperbolic parabola surfaces, the unique roof has been supported by just four pillars. The structural form obtained by P.L. Nervi is a result of the search for effective architectural and structural solutions through the use of hyperbolic geometry.

Another interesting example is the *Philips Pavilion* designed for the first International Trade Fair in Brussels after World War II. The project authors: Le Corbusier and Iannis Xenakis, have designed not only the architecture of the pavilion, but also created a 10-minute light and sound show. The form of the pavilion, based on the shape of a hyperbolic paraboloid, was made out of a thin concrete in the form of prefabricated panels suspended on a steel wire structure. Through the modern technologies and the experimental combination of architecture and sound, the *Philips Pavilion* reflected the renewal and reconstruction of Europe after the war. Also, the novel approach that the architects took to develop the pavilion's form, by employing hyperbolic geometry, introduced new trends in world architecture.

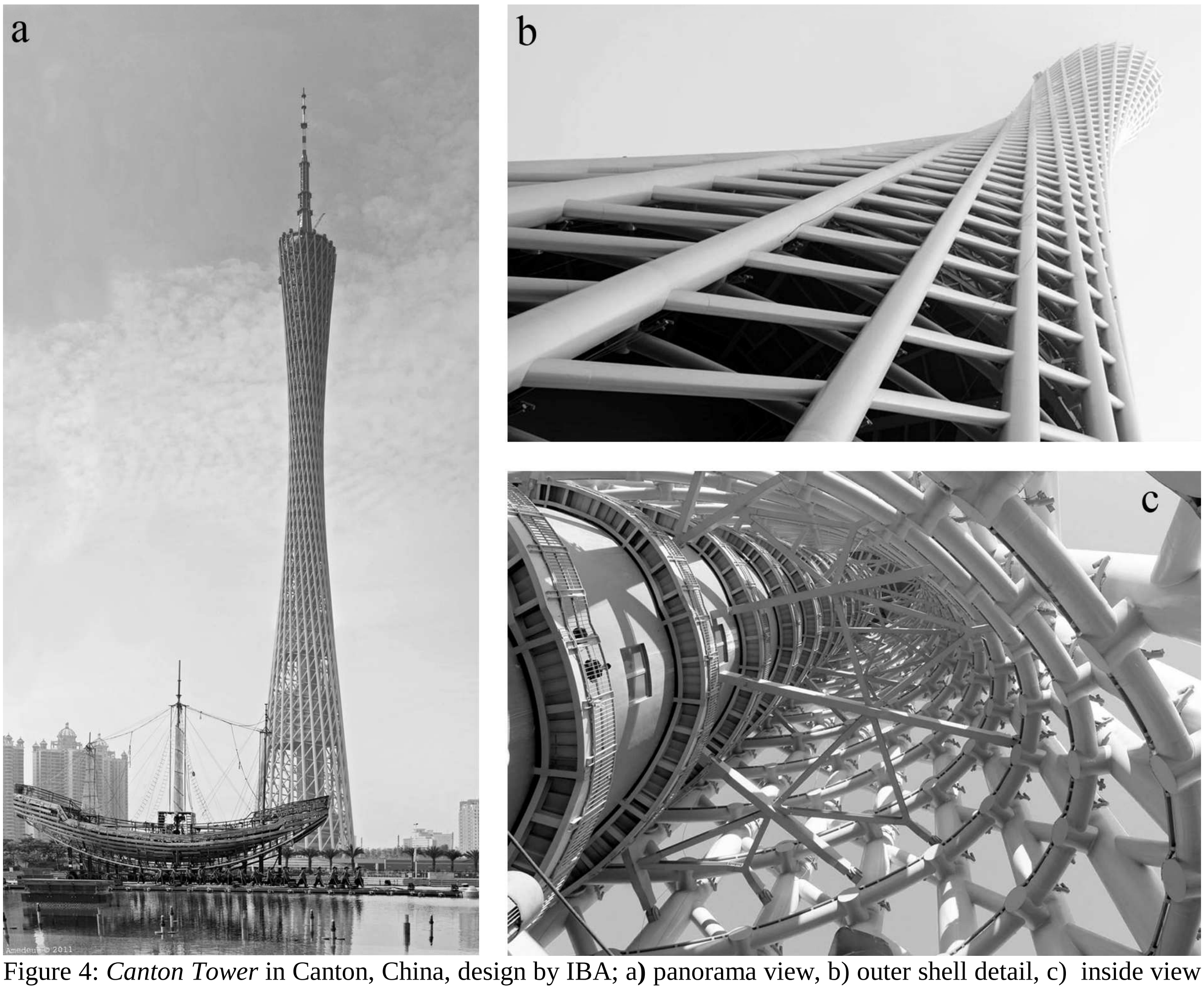

Figure 4: *Canton Tower* in Canton, China, design by IBA; a**)** panorama view, b) outer shell detail, c) inside view of the tower structure source: a) fot. Rudi Amadeus Blondia, [accessed on 8 May 2013] http://www.hasselblad.com/hoc/photographers/rudi-amedeus-blondia.aspx b) Information Based Architecture (IBA), [accessed on 8 May 2013], http://www.iba-bv.com/ c) [accessed on 8 May 2013] <http://www.cantontower.com/en/Raiders.aspx?code=0201>

As the digital tools developed, the capacity to use hyperbolic geometry in the quest for better engineering solutions has increased. As a result of the application of advanced digital modeling and digital simulation software, hyperbolic geometry is used to design some of the most technologically advanced buildings in the world. One of the examples is the *Canton Tower*, a television tower in China, designed by the IBA – Information Based Architecture office (Mark Hemel and Barbara Kuit), which was opened in 2010. The *Canton Tower* is

600m tall, with 106 levels above ground and two underground levels (the usable area of the building is 114,054m²). The extraordinary hyperbolic shape of the tower was generated by two ellipses, of which one is at the level of the foundations, while the other has been pivoted and raised to level 450m (the roof level). The unique torsion of the bars connecting the ellipses has introduced rigidity into the whole system and has made its structural form light and sculpture-like. *Canton Tower* serves as an observation tower, with observation platforms of different character located at various heights; some of them take the form of open gardens, others – protruding observation decks with transparent floors. Restaurants occupy the top floors.

2.3 Fractal geometry

"Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth nor does light travel in a straight line." – this is a quote from *The Fractal Geometry of Nature* by Benoit Mandelbrot, in which the author developed a new branch of mathematics used to describe and analyze the irregularities found in natural structures. Fractal geometry describes fractals, self-similar objects forming small structures in a wide range of magnification levels. The characteristic features of fractal geometry elements include: self-similarity (every fractal part, no matter how low the structural level, bears similarity to the entire structure), fractal dimension (describing the complexity of a fractal) and iterative structure (the same procedure being repeated for an infinite number of times).

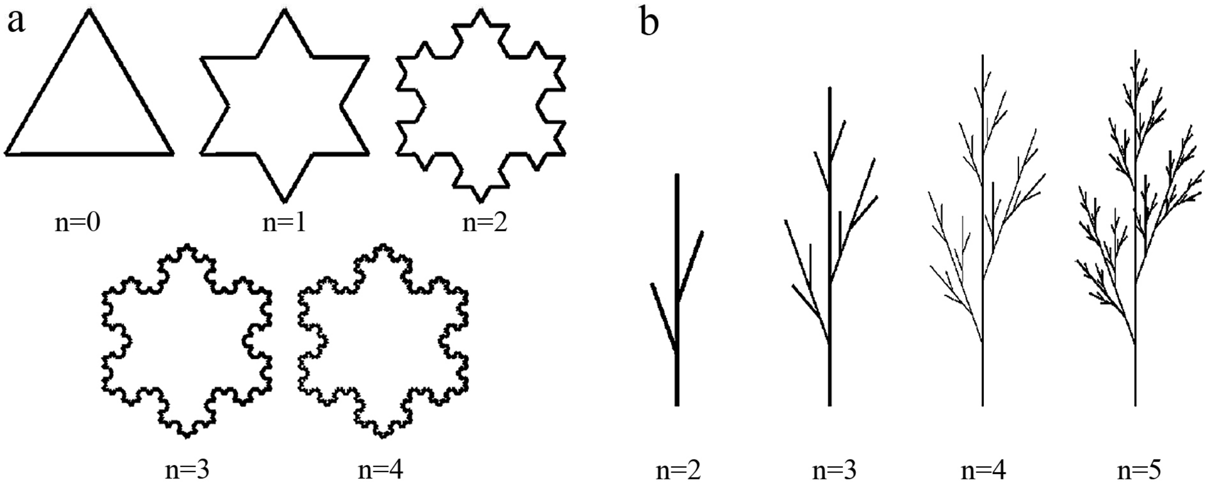

Figure 5: Fractal structures; a**)** construction of the Koch curve, b**)** iterative development of the 'plant-fractal' using the L-system source: a) , b) "Tree Draw Honours Project", *L-systems,* [accessed on 9 May 2013] <http://people.cs.uct.ac.za/~mdanoher/TreeDrawWebsite/lsystems.html>

With the unified fractal theory created by Mandelbrot in 1975, it has become possible to describe and examine unusual geometric structures that are irregular and disjointed, which until then have been considered in mathematics as a special, indescribable case. Mandelbrot created a language that has made it possible to integrate all of the earlier fractal objects (including the Cantor set, Koch curve, Peano curve, Gilbert curve, Sierpinski triangle) and noted that what seemed to be an exception in mathematics is a rule in the natural world around us.

The fractal equivalents observed in nature, such as e.g. clouds, plants or plate tectonics, confirm the assumption that the fractal theory is a natural path of development and changes that occur in the universe. At the same time however it is a mathematical language which may be used to describe for example a natural phenomenon that is seemingly chaotic and random. The developing computer technology has made it possible to conduct ever more accurate research on the fractal theory and the ability to use the fractal language allows has allowed us to create a geometry which mirrors the natural route of development. One of the interesting applications is the use of chaotic structures in architecture and demonstration of complexity based on specific order and similarity.

Scientists have been investigating the history of architecture in search for elements of the fractal philosophy. This influence is quite clearly visible in the architecture of Indian temples, in Gothic and Art Nouveau architecture. Gothic is mentioned as a particularly fitting example, since both the decoration of a column base and of the head are a reduced version of the entire building. Also Art Noveau, with its stylized repetition of details in different sizes, has some features of the fractal geometry. Mandelbrot himself pointed at the example of the opera house by Charles Garnier, where the significant disseverance and adequate details and ornaments referred to fractal forms. The work of Antonio Gaudi, Victor Horta or Charles Rannie Mackintosh also invokes the expressive rhythms and patterns present in nature. All those works however were created through intuition, before the fractal theory was noticed and described.

A completed project that has fully consciously applied fractal architecture is the *Federation Square* in Melbourne designed by Lab Architecture Studio (Peter Davidson and Don Bates) in 1997-2001. The building and the adjacent square creates a unique urban and architectural structure designed in the spirit of fractal geometry. The square, intended for public use (with cultural and commercial functions) is the key element of the entire concept. On one hand, it organizes the lives of the individual buildings located around it, while on the other hand it is an impressive, large area with a precisely designed surface. It consists of sandstone triangles of different colors and sizes, arranged according to the Penrose tiling. The non-periodic divisions that the architects used in the *Federation Square* project make up a chaotic pattern that repeats periodically after shift [4].

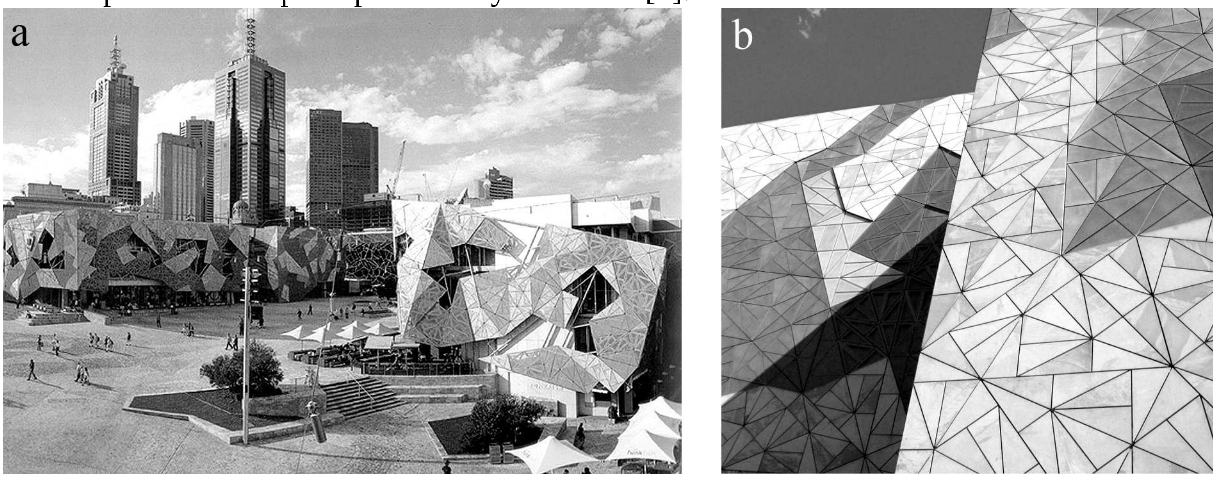

Figure 6: *Federation Square*, Melbourne, design by Lab Architecture Studio (Peter Davidson, Don Bates); a**)** birds eye view, b) detail of the Penrose tiling facade, Source: a) fot. Andrew Hoobs and Adrian Lander, “Elsewhere”, *Spectacular Buildings*, [accessed on 9 May 2013], http://www.thecityreview.com/spectacular.html b) [accessed on 9 May 2013], <http://www.flickr.com/photos/deanmelbourne/2727865589/>

Many of the classic fractal structures, known and described in mathematics, mimics the shapes observed in nature. For example, the Koch curve, which resembles a snowflake, or the Barnsley fern fractal, so named because of its striking resemblance to an actual fern leaf. In 1968, a Hungarian biologist Aristid Lindenmayer used the “Pythagorean tree” structure to introduce the concept of L-systems. L-system is a digital generator of fractal structures, faithfully reflecting the dynamics of plant growth. Fractal structures observed in nature can be generate using generative design tools based on the iterations of an algorithmic code [5].

Additionally, L-systems allow you to create a virtual microclimate by combining gravity, tropism and interactions between various elements of the model's structure, i.e. 'plants' and contact with obstacles [1]. This approach to the modeling of forms creates opportunities for including other interactive systems in the growth process, such as collection of rainwater and solar energy.

An interesting example of using the L-system for the development of architectural form is *The Tote* restaurant in Mumbai, with a bar and a banquet hall, designed by Serie Architects. The building was developed in 2009 in cooperation with the Facet Construction Engineering Pvt. Ltd. design office as a conversion of historic buildings comprising the Mumbai Race Course complex. Given a set of requirements from the historic preservation office, the designers kept the outer shape of the building, but proposed a novel structure of roof supports over the floor space of 2,500 m². As an analogy to the green areas surrounding the building, the architects proposed a continuation of tree-shaped structures inside of the restaurant. The arboral, asymmetric structures support the roof while intersecting with each other. The tree structure is designed as a steel truss built out of I-beam and pipe profiles. The building is an example of how digital tools may be used for modeling fractal geometry forms, with special emphasis on the morphogenesis process. The clear inspiration with the environment is also an attempt to harmoniously place the new 'object-system' in its environment.

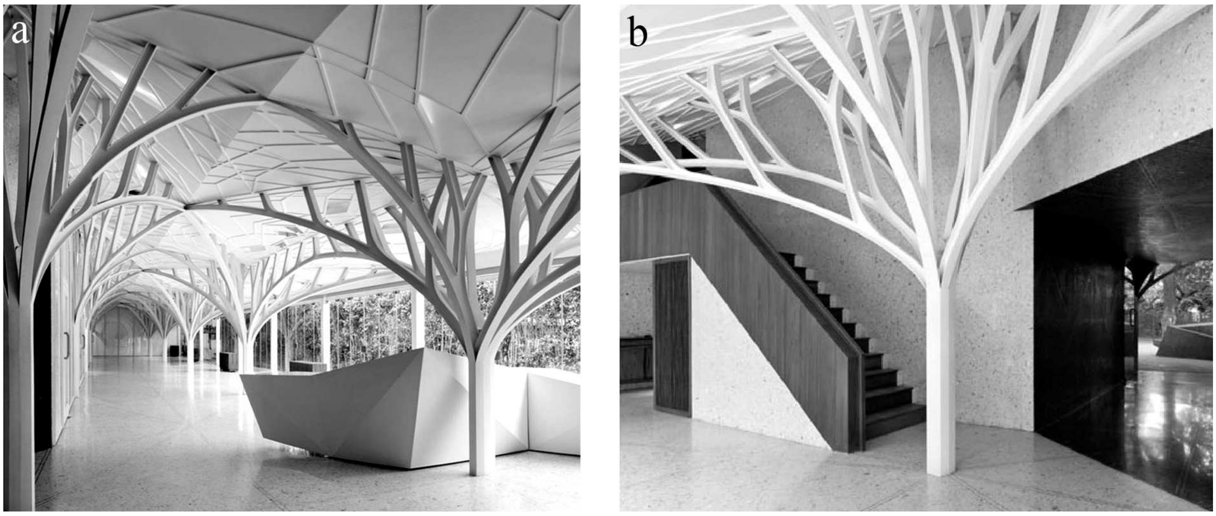

Figure 7: *The Tote Restaurant,* Bombay, design by Serie Architects; a**)** restaurant interior, b**)** arboral support generated by the L-system source: a , b) “Megan Jone’s Blog”, *The Tote. Serie Architects*, 21.04.2012, [accessed on 9 May 2013], < http://meganjonesdesign.wordpress.com/2012/04/21/the-tote-serie-architects/>

3 Summary

The use of non-Euclidean geometry in architecture is currently an important route to developing the optimum structural forms and in the search for effective engineering solutions. Thanks to the developing digital technologies, ever-more sophisticated computer programs are created that help architects find their way in multi-dimensional spaces and model even the most complex structural forms. This is shown through the increasingly bolder projects, including those inspired by non-orientable surfaces, such as the Mobius strip or the Klein bottle. Increasingly, the quality of modern architecture is about the effectiveness of the geometry used, demonstrated not only by the beauty of the spatial form, but also other related technical aspects, which may include material savings or reduction of energy consumption.

In the era of biomimetic trends and directions in architecture, development of known non-Euclidean geometries is necessary to understand the construction of organic structures and biological processes that occur in nature. Even though we know no geometry that would fully describe the natural world, the investigation in the realms of elliptic, hyperbolic and fractal geometry gives rise to interesting and effective architectural solutions. At present, on the path to the “real” geometry mimicking the natural technology, non-Euclidean geometry plays an important role in the search for biomimetic and optimum solutions for the modern architecture.

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GEOMETRIA NIEEUKLIDESOWA W MODELOWANIU SPÓŁCZESNYCH FORM ARCHITEKTONICZNYCH

W poszukiwaniu inspiracji w kształtowaniu przestrzennych struktur architektonicznych, istotne jest analizowanie zależności pomiędzy poszczególnymi elementami strukturalnymi w przestrzeni. Dynamiczny rozwój narzędzi cyfrowych w zastosowaniu do geometrii nieeuklidesowej umożliwia architektom kształtowanie organicznych i jednocześnie logicznych konstrukcyjnie form strukturalnych. W dobie generatywnych metod projektowania i wysoko zaawansowanych programów, możliwe jest modelowanie struktur przestrzennych w geometrii hiperbolicznej, eliptycznej, bądź fraktalnej. Podjęta w referacie tematyka dotyczy wybranych nieeuklidesowych modeli geometrycznych, które są analizowane w generatywnych procesach projektowania form strukturalnych w architekturze.