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# Optimal operation of a process by integrating dynamic economic optimization and model predictive control formulated with empirical model

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In advanced control, a control target tracks the set points and tends to achieve optimal operation of a process. Model predictive control (MPC) is used to track the set points. When the set points correspond to an optimum economic trajectory that is sent from an economic layer, the process will be gradually reaching the optimal operation. This study proposes the integration of an economic layer and MPC layer to solve the problem of different time scale and unreachable set points. Both layers require dynamic models that are subject to objective functions. The prediction output of a model is not always asymptotically equal to the measured output of a process. Therefore, Kalman filter is proposed as a state feedback to the two-layer integration. The proposed controller only considers the linear empirical model and the inherent model is identified by system identification, which is assumed to be an ample representation of the process. A depropanizer process case study has been used for demonstration of the proposed technique. The result shows that the proposed controller tends to improve the profit of the process smoothly and continuously, until the process reaches an asymptotically maximum profit point.

**Key words:** integrate economic optimization and MPC, Matlab-Hysys Interface, Kalman filter, integrate RTO-MPC, depropanizer

## 1. Introduction

The goal of a control strategy involves controlling a process with respect to an optimal trajectory process operation given the presence of a constraint. The achievement of a target involves the construction of control hierarchy that is comprised of various layers. Extant studies examine a widely-known hierarchical control structure [1–4] as shown in Fig. 1a. In the structure, a real-time optimization (RTO) layer provides the optimal operation set points of a process. A model predictive control (MPC) layer brings the process to the optimal

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operating point. However, a drawback of the scheme corresponds to the inconsistency between RTO and MPC layers. The upper layer is the RTO that uses a steady state nonlinear model to calculate the optimal set-points for profit or cost function [2]. The implementation time of the RTO is based on the time required (several hours) for the MPC to move the process to a new stable state because the collected data is used for updating RTO model parameters at a steady state condition. The lower layer MPC is typically based on a dynamic linear model to calculate control moves for the process to reach the set-points from the RTO layer in a time period ranging from seconds to minutes. The performance index problem in MPC is the summation of output error and input move regularization terms in a finite horizon [5]. In case the MPC controller is used to track the set-points from the RTO layer, the objective function of the MPC includes a cost term  $\|u - u_{set}\|$ . This leads to a problem of unreachable set-points because the MPC distributes the offset between the set-points for both  $y_{set}$  and  $u_{set}$ . This in turn causes an inconsistency due to the sampling time difference and unreachable set-points, and thereby results in poor economic performance. The inconsistency is reduced by a layered structure that could consist of three-layers, two-layers, or one-layer.

In a three-layer structure, an additional layer is introduced between RTO and MPC layers. The role of the middle layer involves approximately calculating the set-points for the MPC layer by using the same model at the MPC layer in the cost function [3]. The model in middle layer is transformed from dynamic into steady state version. In a two-layer strategy, the RTO layer is replaced by a dynamic real-time optimization (DRTO) [6, 7] or economic model predictive control (EMPC) layer [8] that leads to the reduction of inconsistency in a different time scale. In a one-layer structure, the RTO layer merges with the MPC layer by integrating the economic function in the RTO layer as a part of the target function in the MPC layer [9, 10]. A main issue of the one-layer method is that the optimization function becomes a complex nonlinear function if the economic problem is nonlinear and subject to a rigorous nonlinear model, and this results in a higher computer load and it is barely possible to synchronize the controller with the process. In the present study, the aforementioned inconsistent problems in conventional control structure are solved by simply using the same linear dynamic model at both layers. A linear discrete empirical model is the most commonly used model in most industries [2, 11]. Therefore, the empirical model is chosen to use for both layers. Multiple linear models are used in case a process is highly nonlinear [12]. The problem is that an empirical model needs a complete representation of state of the process at a given instant of time. Therefore, it is necessary for the proposed controller to involve the presence of a state estimator.

The aim of the present study relates to the integration of an economic optimization layer with a MPC layer in the presence of a state estimator at each

sample time to bring process operation to the optimum point. The economic control layer usually add the tracking term, resulting in the decrease in average profit as the weight of tracking term increases [13]. In this work, the change in the input term is added instead of tracking term. Therefore, the average profit is not affected while changing the weight of the adding term. The proposed controller is applied for the Depropanizer. The results show that the integration of economic optimization and MPC ensures the profit is improving compare to the conventional hierarchical control.

## 2. Methodology

### 2.1. Economic layer versus MPC layer

Even, the two-layered structure suffers from the well-known drawback of different time scales and unreachable set-points. The two-layer approach still has high acceptance because if the optimization function of the economic layer fails to converge, then operators can manually fix known set points such that the process is maintained at a given operation point. In order to correct this issue, the empirical dynamic model is used in both economic layer and MPC layer to be able to synchronize between the process and control. The architecture of the approach is shown in Fig. 1b.

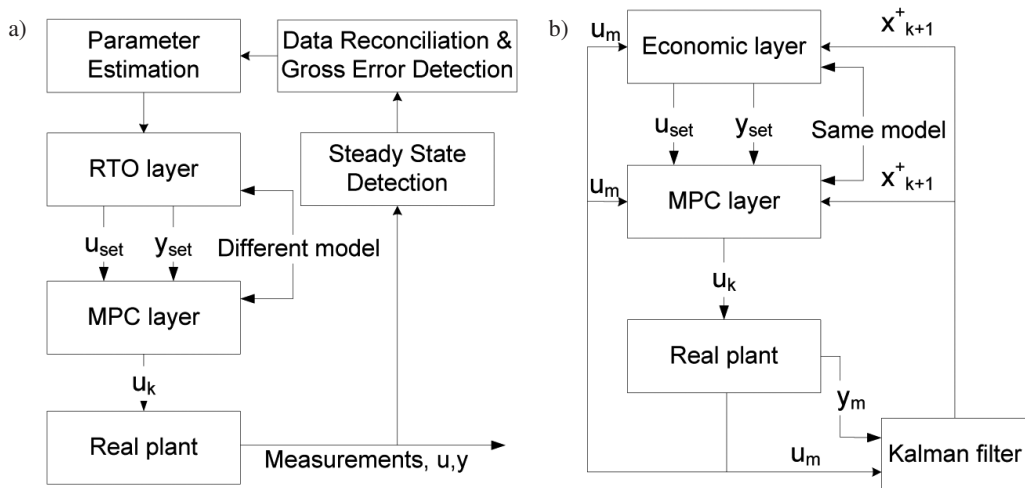


Figure 1: Vertical hierarchies of process control: a) classical control structure, b) proposed control structure

The proposed hierarchy uses an empirical model and especially a discrete state space model that is obtained by system identification. A common discrete

state space model that describes a process is considered, and it is given as follows:

$$\begin{aligned}x_{k+1} &= Ax_k + Bu_k, \\y_{k+1} &= Cx_{k+1},\end{aligned}\tag{1}$$

where  $k$  denotes discrete value of time ( $k = \text{integer}$ ),  $x \in R^{n_x}$  denotes the state vector,  $u \in R^{n_u}$  denotes the manipulated variable vector, and  $y \in R^{n_y}$  denotes the algebraic controlled vector. The discrete state space model corresponds to a one step ahead prediction model. The matrices  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$  are typically termed as a process matrix, an input matrix, and an output matrix, respectively, and determined by system identification.

When an empirical model is used, a potential exists for an offset between the calculated prediction output,  $y_{k+1}$ , and measured output from the process,  $y_m$ , due to lack of accurate state value of process.

## 2.2. State estimation by Kalman filter

The model (1) is applied in the proposed control technique. Matrices  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$  in (1) are fixed once identification of the model of the MPC is accomplished, and thus  $y_{k+1}$  as calculated by equation (1) differs from the measured output,  $y_m$ , at time  $k + 1$  due to the incorrect state estimate. It is only possible to obtain  $y_m$  asymptotically equal to  $y_{k+1}$  through the state estimate of the vector  $x_{k+1}$ . Additionally, in order to obtain the corrected state,  $x_{k+1}^+$ , a discrete-time Kalman filter is employed for the state estimation algorithm for the corrected  $x_{k+1}$ .

The discrete-time Kalman filter equations in [14] are summarized as follows:

Priori estimate state is as follows:

$$x_k^- = Ax_{k-1}^+ + Bu_{k-1}.\tag{2}$$

Priori covariance is as follows:

$$P_k^- = AP_{k-1}^+A' + Q.\tag{3}$$

Kalman gain is as follows:

$$K_k = P_k^-C'(CP_k^-C' + R)^{-1}.\tag{4}$$

Posteriori state estimate is as follows:

$$x_k^+ = x_k^- + K_k(y_{m|k} - Cx_k^-).\tag{5}$$

Posteriori covariance is as follows:

$$P_k^+ = (I - K_kC)P_k^-, \tag{6}$$

where  $x_k^-$  denotes priori estimate state when  $x_k$  is estimated with all the measurements up to time  $k - 1$ ,  $x_k^+$  denotes the posteriori estimate state when all the measurements up to time  $k$  are available for estimate  $x_k$ ,  $\mathbf{Q}$  denotes process noise covariance matrix, and  $\mathbf{R}$  denotes the measurement covariance matrix, the transpose of a matrix or vector is denoted by the prime.

As shown in Fig. 1b, the controller sends the control moves  $u_k$  to the process that when substituted in equation (1) results in  $y_{k+1}$ . Because of unknown state of the process,  $y_{k+1}$  is offset by  $y_m$ . The Kalman filter based on  $y_m$  calculates optimal state estimate,  $x_{k+1}^+$ , which is substituted as  $x_{k+1}$  in equation (1) to obtain  $y_{k+1}^+$  asymptotically equal to  $y_m$ . Therefore, the Kalman filter results in reduce the offset.

### 2.3. MPC layer

**Definition 1** MPC is used to determine the approximate input based on the prediction model (discrete state space model) by optimizing the control law function as follows:

$$J = \sum_{n=1}^{n_y} \|y_{set} - y_{k+n}\|_{W_S}^2 + \sum_{n=1}^{n_u} \|u_{set} - u_{k+n-1}\|_{W_Q}^2 + \sum_{n=1}^{n_u} \|u_{k+n-1} - u_m\|_{W_R}^2 \quad (7)$$

subject to:

discrete state space model (1)

$$x_0 = x_k^+,$$

$$u_{\min} \leq u_{k+n-1} \leq u_{\max},$$

$$\Delta u_{\min} \leq \Delta u_{k+n-1} \leq \Delta u_{\max},$$

where  $n_y$  denotes the prediction horizon,  $n_x$  denotes the control horizon ( $n_x \leq n_y$ ),  $y_{set} \in \mathbf{R}^{n_y}$  denotes the set points, and  $u_m \in \mathbf{R}^{n_u}$  denotes the measured input at each transition. Additionally,  $\|\cdot\|_{\mathbf{R}}^2$  is the square of the two-norm of a vector formulated with diagonal matrix  $\mathbf{R}$  and is defined as follows:

$$\|x\|_{\mathbf{R}}^2 = R_{11}x_1^2 + R_{22}x_2^2 + \dots + R_{nn}x_n^2.$$

$W_S$ ,  $W_Q$  and  $W_R$  denote positive weights since  $\sum_{n=1}^{n_u} \|u_{k+n-1} - u_m\|_{W_R}^2$  is a regularization term that penalizes the control moves.

It is assumed that the discrete state space model (1) is obtained by system identification from a real process.

Thus, Equation (7) is re-expressed as follows:

$$\begin{aligned}
 J = & \left\| r * ones(ny, 1) - \underline{y}_{k+1}^{ny} \right\|_{W_S}^2 + \left\| \underline{u}_k^{nu} - u_{set} * ones(nu, 1) \right\|_{W_Q}^2 \\
 & + \left\| \underline{u}_k^{nu} - u_m * ones(nu, 1) \right\|_{W_R}^2, \quad (8)
 \end{aligned}$$

where

$$\begin{aligned}
 \underline{y}_{k+1}^{ny} &= \begin{bmatrix} \mathbf{y}_{k+1} \\ \mathbf{y}_{k+2} \\ \vdots \\ \mathbf{y}_{k+ny} \end{bmatrix}; & \underline{u}_k^{nu} &= \begin{bmatrix} \mathbf{u}_k \\ \mathbf{u}_{k+1} \\ \vdots \\ \mathbf{u}_{k+nu-1} \end{bmatrix}; \\
 r * ones(ny, 1) &= \begin{bmatrix} \mathbf{r}^1 \\ \vdots \\ \mathbf{r}^{ny} \end{bmatrix}; & u_m * ones(nu, 1) &= \begin{bmatrix} \mathbf{u}_m^1 \\ \vdots \\ \mathbf{u}_m^{nu} \end{bmatrix}; \\
 u_{set} * ones(nu, 1) &= \begin{bmatrix} \mathbf{u}_{set}^1 \\ \vdots \\ \mathbf{u}_{set}^{nu} \end{bmatrix}.
 \end{aligned}$$

The predictive output vector is as follows:

$$\underline{y}_{k+1}^{ny} = P x_k + H \underline{u}_k^{ny}, \quad (9)$$

where:

$$\mathbf{P} = \begin{bmatrix} \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{ny} \end{bmatrix}; \quad \mathbf{H} = \begin{bmatrix} \mathbf{CB} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{CAB} & \mathbf{CB} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{CA}^{ny-1}\mathbf{B} & \mathbf{CA}^{ny-2}\mathbf{B} & \dots & \mathbf{CB} \end{bmatrix}.$$

The MPC controller optimizes J function (8) and obtains vector  $\underline{u}_k^{ny}$  as a root. However, only  $u_k$  is considered in the vector and sent as manipulated variables to the real process.

The MPC typically tracks the output set-point with the first and third terms in Equation (8) only. However, the proposed integration technique includes the second term while tracking the set-point from the RTO or economic control layer because the set points include both input and output.

#### 2.4. Economic optimization control layer

The cost function of the economic layer formulated with the empirical model is given as follows:

$$J_{eco} = \Delta V_{eco} + REG, \quad (10)$$

where

$$\Delta V_{eco} = - \sum_{n=1}^N l(x_{k+n}, u_{k+n-1}) - Nl(x_k^+, u_m)$$

and

$$REG = \sum_{n=1}^N \|y_{k+n} - y_k^+\|_{R_1}^2 + \sum_{n=1}^N \|u_{k+n-1} - u_m\|_{R_2}^2,$$

$$\begin{aligned}
 J_{eco} = & - \sum_{n=1}^N (l(x_{k+n}, u_{k+n-1}) - Nl(x_k^+, u_m)) \\
 & + \sum_{n=1}^N \|(y_{k+n} - y_k^+)\|_{R_1}^2 + \sum_{n=1}^N \|(u_{k+n-1} - u_m)\|_{R_2}^2
 \end{aligned} \quad (11)$$

which subject to the following:

discrete state space model (1)

$$\begin{aligned}
 x_0 &= x_k^+, \\
 0 \leq \alpha(|x_{k+1} - x_k^+|) &\leq \sum_{n=1}^N l(x_{k+n}, u_{k+n-1}) - Nl(x_k^+, u_m), \\
 u_{\min} &\leq u_{k+n-1} \leq u_{\max} \\
 \Delta u_{\min} &\leq \Delta u_{k+n-1} \leq \Delta u_{\max}, \\
 y_{\min} &\leq y_{k+n} \leq y_{\max},
 \end{aligned}$$

where  $N$  denotes prediction horizon,  $\Delta V_{eco}$  denotes the difference between prediction and current economic terms,  $l(x_k^+, u_m)$  is the current economic term that is calculated based on measured input and estimated state, and  $l(x_{k+1}, u_k)$  denotes the prediction economic term. Additionally,  $REG$  denotes the regularization term,  $x_k^+$ ,  $u_m$  denote current estimated state and measured input, respectively, at each iteration,  $y_k^+ = Cx_k^+$  denotes the estimated output, and  $R_1$  and  $R_2$  denote positive weights. A continuous non-decreasing scalar function is defined as  $\alpha(\cdot)$ .

## 2.5. Two layer control hierarchy algorithm

### *Integrating two layers with the Kalman filter*

The Kalman filter is integrated to obtain the asymptotically corrected state of the process. The method for integrating two layers with the Kalman filter is as follows:

Specifically,  $x_k^+$  in (5) substitutes  $x_k$  in (9) at each iteration as follows:

$$\underline{y}_{k+1}^{ny} = Px_k^+ + H \underline{u}_k^{ny}. \quad (12)$$

Thus,  $y_{k+1}$  in the vector  $\underline{y}_{k+1}^{ny}$  corresponds to the prior output,  $y_{k+1}^-$ . After sufficient iterations,  $y_{k+1}^-$  is asymptotically equal to the output from the real process,  $y_m$ .

Additionally,  $x_k^+$  in (5) also substitutes  $x_k^+$  in (11) at each iteration.

### *Implementation strategy*

The economic optimization layer and MPC obtain the state estimation from the Kalman filter  $x_k^+$  in each iteration. First, MPC is used to track a given set point to obtain the process operation at stability, resulting in the state estimation at this time is more accurate. Subsequently, the economic optimization layer is switched on to address the economic function. The economic optimization layer sends the first element in the result to the MPC layer as set points consequently after solving an optimization problem. Subsequently, the MPC controller forces the process to follow the optimal trajectory.

## 3. Results and discussions

### 3.1. Case study

#### 3.1.1. Process description

Figure 2 shows the dynamic simulation of the Depropanizer column process that was described in a previous study [15]. The Depropanizer column is designed with 24 stages and the feed stream position at stage 12. The pressure at the top and at the reboiler corresponds to 1925 kPa and 2070 kPa, respectively, and the feed composition is shown in Fig. 2. The controlled variables correspond to the purity of top and bottom products based on propane composition while the manipulated variables correspond to the reflux flow rate and boil up flow rate. The control strategy is designed in Simulink-Matlab to control the dynamic process simulation at optimal economic operation by using a Matlab-Hysys interface. The interface code is from a toolbox used in a previous study [16].



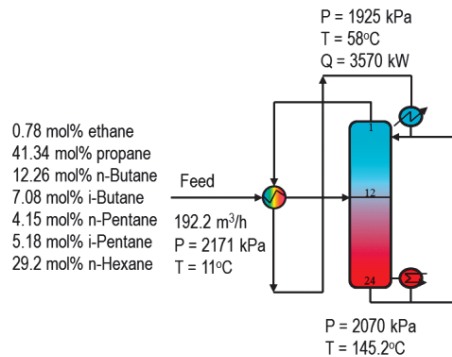


Figure 2: Dynamic depropanizer process simulation

### 3.1.2. Implementation of the proposed hierarchy at the Depropanizer distillation column

Assumption: The first principle model of the dynamic simulation is not known. The economic cost functions are known and are sent from planning and scheduling modes.

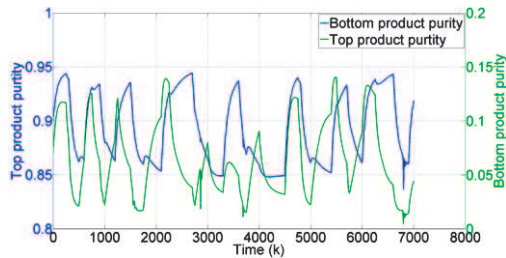
In this application, a nonlinear mathematical of dynamic simulation of Depropanizer process is not available because identifying a nonlinear process is a very difficult task. However, the linear system identification is easier to implement than nonlinear model. Moreover, the linear model sufficiently accurate to describe the nonlinear process in a certain region. Therefore, in this case study, the model is used for economic control layer and MPC layer, which is obtained by linear system identification method.

#### a) Model identification

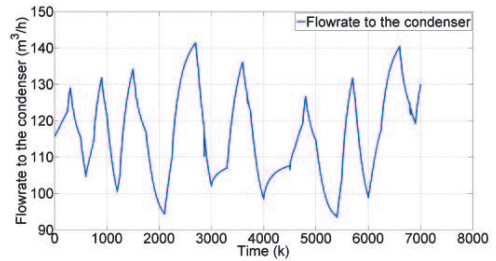
The models used for the hierarchy are identified by closed-loop system identification. Two PI controllers are designed in Matlab-Simulink to manipulate reflux flowrate and boil-up flowrate to bring the controlled variables (propane composition in the top and bottom products) to the given set-points. Pseudo-Random Bit Sequence (PRBS) signals are designed to generate bias to the set-points (the current set points position corresponds to 0.9 for the top and 0.07 for the bottom). The 7000 data points including manipulated variables, controlled variables, and flowrate to the condenser are obtained for model identification and validation as shown in Fig. 3a, b, c.

Two discrete state space models are identified by the subspace method (n4sid) [17, 18]. The n4sid code is ready in Matlab toolbox, is used to obtain the models. The extensive discussion of n4sid will not be presented because out of scope of the target of this paper.

a) Data on product purities



b) Flowrate with respect to the condenser data



c) Manipulated data variables

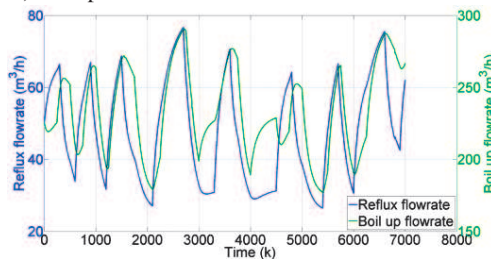


Figure 3

The identified matrices for the first discrete state space model (model 1) include the following inputs: reflux flowrate ( $u_1$ ) and boil up flowrate ( $u_2$ ); and the following outputs: propane composition of the top product ( $y_1$ ) and bottom product ( $y_2$ ).

$$A1 = \begin{bmatrix} 2.2077 & 0.1053 & 1 & 0 & 0 & 0 & 0 & 0 \\ -0.3009 & 2.2246 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1.4570 & -0.2570 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0.7212 & -1.7152 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0.1268 & 0.2106 & 0 & 0 & 0 & 0 & 1 & 0 \\ -0.5622 & 0.5392 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0.1191 & -0.0591 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.1363 & -0.0503 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$B1 = \begin{bmatrix} 0.0005 & 0 \\ 0.0009 & -0.0005 \\ -0.0008 & 0 \\ -0.0015 & 0.0006 \\ 0.0004 & 0 \\ 0.0006 & 0 \\ -0.0001 & 0 \\ 0 & -0.0001 \end{bmatrix}; \quad C1' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

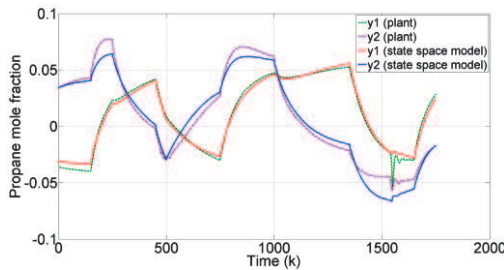
The other identified matrices for the second discrete state space model (model 2) include the following inputs: reflux flowrate and boil up flowrate; and the following output: flowrate to the condenser ( $F_C$ ).

$$\mathbf{A2} = \begin{bmatrix} 0 & 0 & 0 & -0.2531 \\ 0.25 & 0 & 0 & 0.5094 \\ 0 & 1 & 0 & -1.2894 \\ 0 & 0 & 1 & 1.8398 \end{bmatrix}; \quad \mathbf{B2} = \begin{bmatrix} 0.0798 & 0.0756 \\ -0.3356 & 0.1645 \\ 0.0980 & -0.5788 \\ 0.2215 & 0.3967 \end{bmatrix}; \quad \mathbf{C2}' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.5 \end{bmatrix}.$$

The model 1 and model 2 have the same inputs, therefore these can be combined to become one model with 3 outputs and 2 inputs.

Fig. 4a, b show the comparison between the prediction plots from the models and the remaining 25% data used for validation from the dynamic simulation process. In Fig. 4a, b, it is also observed that the behavior of the dynamic simulation process is accurately predicted by model 1 and model 2. Fitting percentages calculated by the normalized root mean square error method (NRMSE) Matlab toolbox correspond to 89.25%, 80.33%, and 92.36% for  $y_1$ ,  $y_2$ , and  $F_C$  respectively.

a) Validation for model 1



b) Validation for model 2

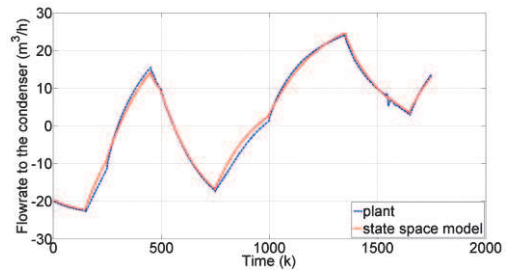


Figure 4

### b) Controller design scheme

The matrices  $\mathbf{A1}$ ,  $\mathbf{B1}$ , and  $\mathbf{C1}$  and  $\mathbf{A2}$ ,  $\mathbf{B2}$ , and  $\mathbf{C2}$  are substituted for  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$ , respectively, in the model (1). Physical constraints on the equipment as well as control actuators and targets from planning and scheduling layers cause the control actions to be bounded as follows: reflux flowrate ( $u_1$ ) is from 10 m<sup>3</sup>/h to 110 m<sup>3</sup>/h, boil-up flowrate ( $u_2$ ) is from 150 m<sup>3</sup>/h to 300 m<sup>3</sup>/h, the purity of the top product ( $y_1$ ) is from 0.86 to 0.94, and the purity of the bottom product ( $y_2$ ) is from 0.03 to 0.11. Both purity products are calculated based on the propane composition.

- The economic cost function with tuning parameters is presented in form of Matlab code as follows:

$$\begin{aligned}
 J_{eco} = & \text{sum} \left( - \sum_{n=1}^{10} (l(x_{k+n}, u_{k+n-1}) - 10 \times l(x_k^+, u_m)) \right) \\
 & + 100 \times \text{norm} \left( \text{repmat}(y_k^+, 10, 1) - \underline{y}_{k+1}^{10} \right)^2 + \dots \\
 & + 40 \times \text{norm} \left( \underline{u}_k^{10} - \text{repmat}(u_m, 10, 1) \right)^2. \quad (13)
 \end{aligned}$$

- The MPC cost function with tuning parameters is presented in form of Matlab code as follows:

$$\begin{aligned}
 J = & \text{norm} \left( \text{repmat}(y_{set}, 10, 1) - \underline{y}_{k+1}^{10} \right)^2 \\
 & + \text{norm} \left( \underline{u}_k^6 - \text{repmat}(u_{set}, 6, 1) \right)^2 + \dots \\
 & + \text{norm} \left( \underline{u}_k^6 - \text{repmat}(u_m, 6, 1) \right)^2, \quad (14)
 \end{aligned}$$

where:  $y_{set}$  : output\_setpoints;  $u_{set}$  : input\_setpoints;  $u_m$  : measured\_inputs.

The output and input set-points are from RTO layer or economic control layer, which are sent to MPC layer for tracking purpose.

### 3.2. Results and discussions

The economic objective function is defined as follows:

$$\text{Profit: } P = p_D F_D + p_B F_B - p_S u_2 - p_F F, \quad (15)$$

where:  $F$  denotes flowrate ( $\text{m}^3/\text{h}$ ) of the feed stream of the column, and the price of  $F$  corresponds to  $p_F$  ( $\$/\text{m}^3$ ). Additionally,  $F_C$  denotes the flowrate ( $\text{m}^3/\text{h}$ ) to condenser,  $F_D$  denotes the flowrate ( $\text{m}^3/\text{h}$ ) of the top product that is sold at the commercial price  $p_D$  ( $\$/\text{m}^3$ ),  $F_B$  denotes product flowrate ( $\text{m}^3/\text{h}$ ) at the bottom of the column with a commercial value  $p_B$   $\$/\text{m}^3$ ,  $p_S$  denotes the cost ( $\$/\text{m}^3$ ) of the boil up flowrate  $u_2$ , and  $u = [u_1 \ u_2]$ ;  $y = [y_1 \ y_2]$ . The control action includes three controlled variables ( $F_C$ ,  $y_1$ ,  $y_2$ ) and two manipulated variables ( $u_1$ ,  $u_2$ ). The column is maintained at a given operation point by the MPC controller and after this, the economic control layer is executed at time  $t = 400$  (k). Time constant,  $T_s = 0.5$  s.

**Case study:**  $p_D = (10 \cdot y_1)^3$   $\$/\text{m}^3$ ;  $p_B = (10 \cdot (1 - y_2)^3) \cdot 0.5$   $\$/\text{m}^3$ ;  $p_F = 400$   $\$/\text{m}^3$ ;  $p_S = 20$   $\$/\text{m}^3$ ;  $u_{\min} = [10 \ 150]$ ;  $u_{\max} = [110 \ 277.15]$ ,  $y_{\min} = [0.86 \ 0.03]$ ,  $y_{\max} = [0.94 \ 0.11]$ .

The optimum points are calculated by Real Time Optimizer (RTO) in steady state simulation Hysys as following:  $F_C = 142.9$  ( $\text{m}^3/\text{h}$ ),  $R = 75.15$  ( $\text{m}^3/\text{h}$ ) and

$S = 277.1$  ( $\text{m}^3/\text{h}$ ). The optimum points are sent to MPC as set-points. The profit significantly increases and reaches a stable operation approximately 400 times (k) after the MPC controller commences. As shown in Fig. 5b and Fig. 5d, the process is stable at  $F_C = 142.1$  ( $\text{m}^3/\text{h}$ ) and  $u_1 = 76.33$  ( $\text{m}^3/\text{h}$ ) and  $u_2 = 296.7$  ( $\text{m}^3/\text{h}$ ). Thereby, the optimum points cannot be reached because of unreachable set-points problem as aforementioned inconsistent problems. Moreover, MPC does not conduct with the output constraints. Thereby,  $y_1$  is 0.948 as shown in Fig. 5c, which is violating of output constraints. At the time is 400 (k), the economic controller is switched on, which sends the set-points to MPC layer instead of the optimum points from RTO. From 400 (k) of time, the profit starts increasing gradually because the set-points from the RTO are not actual the optimum points of the process.

The main profit function as denoted by equation (15) corresponds to non-linear function, is used in economic control layer. The profit function (15) is affected by the product flowrate as well as by the purity of products. Based on the assumption  $p_D = (10 \cdot y_1)^3$   $\$/\text{m}^3$ ;  $p_B = (10 \cdot (1 - y_2)^3) \cdot 0.5$   $\$/\text{m}^3$ , the price of products increases when the product is purer. Figure 5c shows that the product purities are kept almost constant when the bound is reached. Although it is necessary to increase the top product flowrate, the top product flowrate decreases as

a) Optimum profit profile

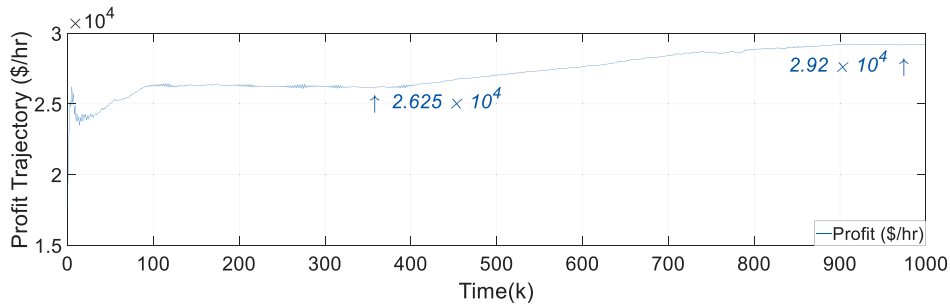
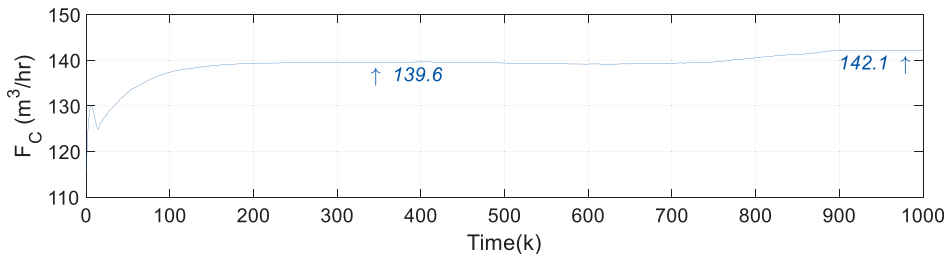
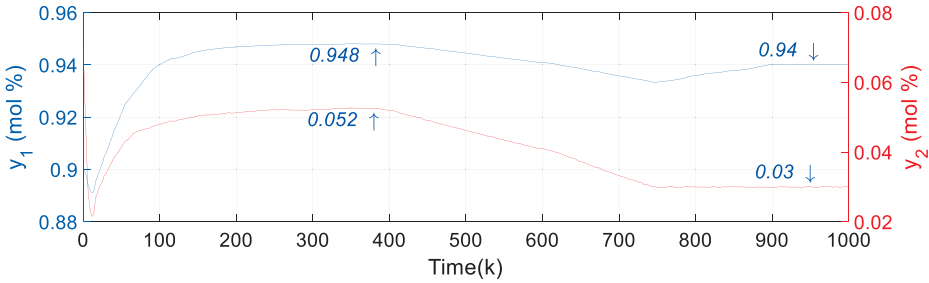
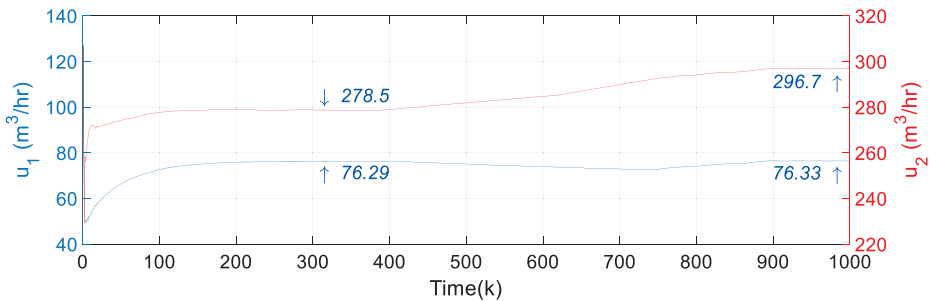
b) Flowrate to condenser,  $F_C$ 

Figure 5

c) Product purities ( $y_1$ : top product purity,  $y_2$ : bottom product purity)



d) Manipulated variables (reflux flowrate,  $u_1$  and boil-up flowrate,  $u_2$ )



e) Output flowrates (D: top product flowrate, B: bottom product flowrate)

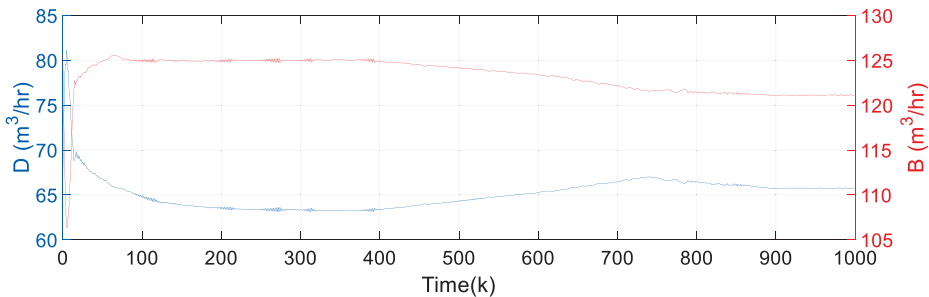


Figure 5

shown in Fig. 5e. This is explained as follows: an increase in both product purity and product flowrate increases the profit. However, when the product purity increases, the top product flowrate decreases and vice versa. In this case, the effect of the product purity on the profit value exceeds that of the product flowrate. Figure 5d shows reflux and boil up flowrates that are manipulated to gradually increase to increase the product purities. As shown in Fig. 5a, the controller tends to improve the economic function (15) smoothly and continuously until the process reaches an asymptotically maximum point and attempts to keep it almost stable at that point.

#### 4. Conclusions

This study, the two-layer hierarchy is presented by integrating economic optimization layer and MPC by using the same model formulated as empirical model. Moreover, the empirical model need to estimate the process state, and thus the two-layer architecture is integrated with the Kalman filter to avoid the offset. The proposed control structure is successfully coded in Matlab-Simulink which is able to bring the dynamic simulation process in Hysys to the optimal profit operation. The results show that the set points from RTO (conventional control structure) are not actually the optimum points. Because the profit gradually increases after the economic control layer is active. As more detail in the case study, the proposed control hierarchy is able to increase the profit of the process higher than classical control structure about 2950 \$/hr.

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