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The effective area of measurement electrode in volume resistivity and permittivity of solid dielectrics measurements

Abstract

The problems of determination of the effective area of the measurement electrode during the evaluation of volume resistivity and electrical permittivity based on measurements in three electrode system are discussed. The presented analysis shows that because of the inaccuracies of formulas given in the standards concerning the effective area of the measurement electrode, the calculation of the volume resistivity and electrical permittivity might be encumbered with errors. The values of these errors are presented graphically. It has been shown that the factor of the measurement electrode extension at the electrical permittivity measurements also depends on electrical permittivity.

Keywords: dielectric materials, permittivity, resistivity, measurement electrodes.

1. Introduction

The volume resistivity and electrical permittivity of solid dielectrics materials are always measured using the indirect methods [1]. For this purpose, the tested sample is placed between the electrodes which together with the sample form a capacitor and then the volume resistivity or the capacitance of the capacitor is measured. Usually the resistivity measurements are performed in accordance with the IEC 60093 standard [2], while the permittivity measurements in accordance with the IEC 60259 standard [3]. Nowadays very thin metal electrodes (e.g. silver) made using the thin film technology are applied. These electrodes provide the resistivity and permittivity measurements with the smallest errors due to their contact with the dielectric. Usually the measurements with flat plates are made using the three-electrode system [1].

The volume resistivity ρ_v of flat samples is calculated using the equation:

$$\rho_v = R_v \frac{A}{h} , \tag{1}$$

where R_v is the volume resistance, A – effective area of the measurement electrode, h – sample thickness.

The relative permittivity ε_r is calculated using the formula:

$$\varepsilon_r = \frac{C_x}{C_o} , \tag{2}$$

where C_x is the capacitance between the measurement and voltage electrodes, C_o - equivalent geometric capacitance of this capacitor between the same electrodes, when the dielectric is vacuum. For the flat electrodes with guarded electrode:

$$C_o = \varepsilon_o \frac{A}{h} . \tag{3}$$

The effective area of the measurement electrode A is always larger than its geometrical area due to increase of the electric field at the edges of the measuring electrode (Fig. 1). This is called the fringing effect. In order to minimize this effect, a three electrode system is used, with possibly narrow gap g between the measurement electrode (guarded) and the guard ring (guard electrode), but not smaller than 1 mm. Usually g = 2 mm.

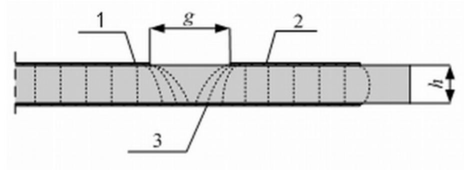

Fig. 1. Distribution of the electric field in volume of dielectric sample in the three electrode system. Electrodes: 1- measurement (guarded), 2- guard, 3- voltage (unguarded)

In practice very often a half of the gap width (g/2) is added to the radius of the circular guarded electrode, i.e. the addition of the gap width g to its diameter d. Similarly, the gap width g is added to each dimension of the rectangular electrode and to the length of the cylindrical electrode [1]. These assumptions were made in the international standard IEC 60093 [2], American standard ASTM D 257 [4], Polish standard PN-E-04403: 1986 [5], concerning the measurements of the volume resistivity, and in the standard IEC 60250: 1969 [3] relating to permittivity measurements of solid dielectrics. In the appendix X2 of the standard ASTM D 257 [4] and in the appendix X2 of the standard ASTM D 150 [6] it is stated that for the calculation of the effective area of the electrode one should not use an increase of the border of guarded electrode equal to g/2, but instead:

$$g/2 - \delta , \tag{4}$$

where:

$$\delta = \frac{2h}{\pi} \ln\cosh\left(\frac{\pi}{4}\frac{g}{h}\right) . \tag{5}$$

The correction of the gap g can also be written in the form:

$$g\left(1 - 2\delta/g\right) = Bg , \tag{6}$$

in which:

$$B = 1 - \frac{4}{\pi}\frac{h}{g} \cdot \ln\cosh\left(\frac{\pi}{4}\frac{g}{h}\right) \tag{7}$$

is the factor which is determining the increase of the spreading margin of the guarded electrode [7].

The equation (7), derived by Amey [7], is correct for the calculation of the resistivity and also the permittivity on condition that the permittivity of the sample $\varepsilon = \varepsilon_r\varepsilon_o$ is much larger than the permittivity of vacuum ε_o. In practice, it is convenient to use a graphical form of this relation [8].

Lisowski and Skopec showed that the factor B can be also expressed by the equation [9]:

$$B = 1 - \frac{H - 1}{\left(1 - \frac{1}{\varepsilon_r}\right)\left(H + 1\right) + \frac{\pi g H}{\varepsilon_r h\left(H - 1\right)}} , \tag{8}$$

where H is determined using the following formula:

$$\pi \frac{g}{h} = H - \frac{1}{H} + 2\ln H . \tag{9}$$

It should be noted that equation (8) shows the factor *B,* and thus the effective area of the measurement electrode, depends not only on the ratio *g/h*, but also on the permittivity. This is not taken into account in any of the existing standards and publications.

The volume resistivity is measured by placing the sample in a direct current electric field. For this field the pulsation $\omega = 0$ and the formula (8) is converted to the following form [9]:

$$B_{(\omega=0)} = \frac{2}{H+1} . \qquad (10)$$

Whereas if $\varepsilon_r \to \infty$, the factor $B_{(\varepsilon_r \to \infty)} = B_{(\omega=0)}$ [9]. According to the equation (10) during the resistivity measurements the factor *B* is not dependant on the dielectric permittivity of the sample.

Figure 2 shows the graphical illustration of the factor *B* as calculated using equation (8) as a function of the ratio of the gap width *g* to sample thickness *h* for different relative permittivity $\varepsilon_r = \varepsilon / \varepsilon_o$. With decreasing of the permittivity ε_r the differences between these characteristics are becoming larger.

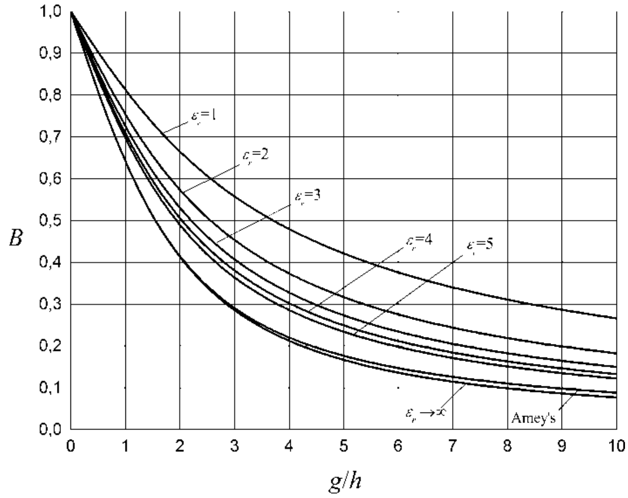

Fig. 2. The factor *B*, calculated from the formula (8) for different relative permittivities, and from Amey's formula (7)

Figure 2 also includes the illustration of the factor *B* calculated from Amey's formula (7). The characteristic determined using the Amey's relation approximately coincides with the characteristic calculated from the equation (8) for the case $\varepsilon_r \to \infty$, i.e. for the measurement of volume resistivity ρ_v.

2. The calculation analysis of effective area of the measurement electrode

For the concentric circular electrodes with the measurement electrode diameter equal to *d*, the effective area of the electrode is calculated using the formula:

$$A = \frac{\pi (d + Bg)^2}{4} . \qquad (11)$$

Its value depends on the factor *B*.

For example, let us analyze the effective area of the measurement electrode when the diameter $d = 25$ mm and the width of the gap $g = 2$ mm. The results of the calculations for different relative permittivity are shown graphically in Figure 3.

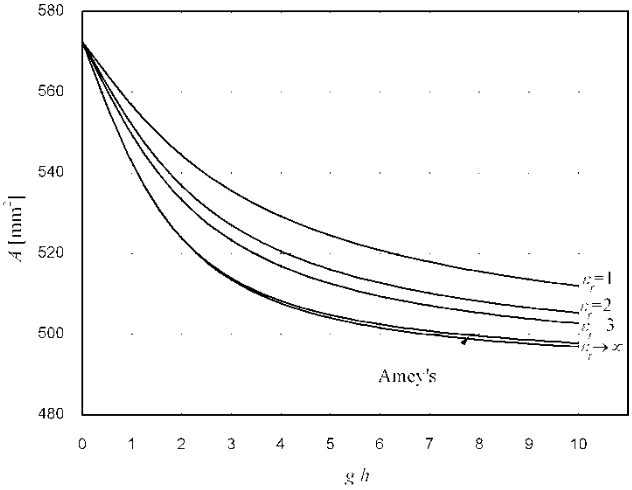

Fig. 3. The effective area *A* of the measurement electrode, calculated from formula (11) , versus *g/h* for different relative sample permittivities , where the factor *B* is calculated from relationship (8) and also from the Amey's formula (7)

Usually in the calculation of the effective area of the measurement electrode it is assumed that the factor $B = 1$, i.e. the gap width *g* is added to the electrode diameter *d*. The relative error introduced by this assumption can be calculated from the relation:

$$\delta A = \frac{A_{B=1} - A_{B\neq 1}}{A_{B=1}} 100 \quad [\%], \qquad (12)$$

where $A_{B=1}$ and $A_{B\neq 1}$ are the effective areas of measuring electrode for $B = 1$ and for $B \neq 1$, determined using the equation (8).
The results of calculation errors of the effective area of the measurement electrode, assuming that the coefficient $B = 1$, are illustrated graphically in Figure 4.

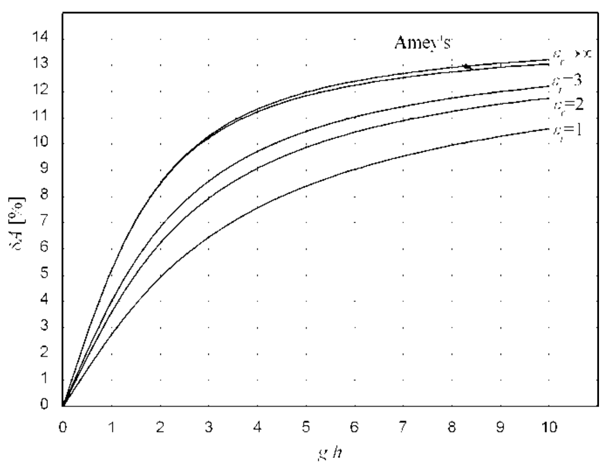

Fig. 4. The relative calculation errors of the effective measurement electrode area caused by assuming the factor $B = 1$

For $\varepsilon_r \to \infty$ the relative error of the calculation of the effective area of the measurement electrode, due to the assumption of $B = 1$, reaches even up to 13% regardless whether the coefficient *B* was determined using the Amey's relation (7) or equation (8).

If the calculation of the effective area is executed using *B* determined by Amey's relation (7) instead of the equation (8), the relative error of this calculation:

$$\delta A = \frac{A_{B(\text{Amey})} - A_{B(8)}}{A_{B(\text{Amey})}} 100 \quad [\%] , \qquad (13)$$

where: $A_{B(\mathrm{Amey})}$ i $A_{B(8)}$ are the effective areas of measurement electrode for B determined by the Amey's formula (7) and B using the equation (8).

The results of the calculations are illustrated graphically in Figure 5. It can be seen that the maximum relative error of the calculation of the effective area of measurement electrode might be ca. 1.5%.

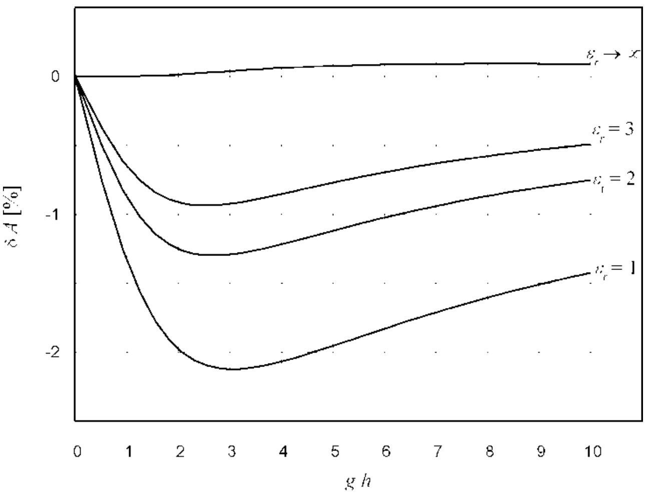

Fig. 5. The relative calculate error of the effective area of the measurement electrode caused by the factor B determined by the Amey's formula (7) instead of equation (8)

3. Conclusions

The effective area of the measurement electrode applied to a flat sample, guarded by the ring electrode, should be calculated using the formula (11). Similarly, during the calculations of the the effective area of rectangular and cylindrical electrodes, Bg should be added to the geometric dimension.

Factor B, which takes into account the enlargement of the effective area of the measurement electrode during the calculation of the volume resistivity, should be determined using the relation (7) or (10), and during the calculation of the permittivity - using the equation (8), wherein the coefficient H should be computed using the formula (9). It should be noted that the coefficient B depends on the value of the permittivity during the permittivity measurements. It was noted that the Amey's formula, commonly used to calculate the effective area of the measurement electrode during the measurements of low permittivity, can cause several percent of measuring errors. However, the biggest errors, even over 10%, occur when the factor B = 1 was assumed.

This work was carried out as a statutory project supported by the Ministry of Science and Higher Education, Warsaw, Poland

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Received: 09.12.2014 *Paper reviewed* *Accepted: 05.01.2015*

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