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Marlena A. Gronowska-Szneler, Jerzy M. Sawicki*

Design criterion for hydrodynamic vortex separators

Gdansk University of Technology, Department of Hydraulic Engineering, Faculty of Civil and Environmental Engineering, G. Narutowicza 11/12, 80-233 Gdansk, Poland

Abstract

The process of designing technical objects involves determination of geometrical parameters that characterize a given device. When device operation is described by differential equations, an inverse problem brings difficulties, as geometrical values being sought condition the solution to the problem. Vortex separators, used for removal of suspension from storm wastewater are usually designed by the 'criterion method'. Firstly, a critical particle is distinguished such that bigger particles are removed from wastewater stream, whereas smaller particles stay in the stream. Next, by comparing values of major forces acting on the critical particle at the most unfavourable point within the separator (usually at outflow cross-section), a force balance is made. The resulting algebraic relation becomes the design criterion.

Keywords: Centrifugal force; Particle trajectory; Sedimentation; Suspended particle; Vortex separator

1 General remarks

Suspensions are the major nonuniform system that occurs both in nature and engineering. One of the most important technological processes concerning suspensions is separation of particles from the carrier medium. As the process is usually carried out in unfavourable conditions and its efficiency is expected to be high, researchers are continuously working on effective solutions. The fundamental element of every technical solution is classical gravitational separation that can

*Corresponding Author. Email adress: jsaw@pg.gda.pl

be enhanced by additional factors, e.g., centrifugal force. The force decelerates transitional motion of suspended particles elongating their settling time, or even delaminates suspension and the carrier medium [15].

Devices that utilize centrifugal force, called rotational separators, come in two types: centrifugal separators and circulative separators, the latter including cyclones and vortex separators. From theoretical point of view their operation is known and understandable. However, methods presented in the literature that can be used for quantitative description of separators operation are questionable [4,10]. Thus, authors of the paper conducted research on a rational model to properly describe operation of these devices. It is especially important as application of most complex solutions (differential equations of suspension motion) is followed by a need to solve inverse problems.

2 Detailed description of the phenomena

From physical point of view, operation of rotational separators is described by [3,12–14]:

- equation of carrier continuity

$$\nabla \cdot \mathbf{u} = 0 \,, \tag{1}$$

- Reynold's equation

$$\frac{D\mathbf{u}}{Dt} = \mathbf{g} - \frac{1}{\rho}\nabla p_e + \nu_e \, \Delta\mathbf{u} \,, \tag{2}$$

- equation of trajectory of suspended characteristic particle

$$\frac{d\mathbf{r}_p}{dt} = \mathbf{v}_p \,, \tag{3}$$

- equation of motion of characteristic particle (Newton's second law)

$$\rho_p V_p \frac{d\mathbf{v}_p}{dt} = (\rho_p - \rho)\, V_p \, \mathbf{g} + \mathbf{F}_{AM} + \mathbf{F}_N + \mathbf{F}_C + \mathbf{F}_{TD} \,, \tag{4}$$

where: $\mathbf{u}$ – carrier velocity, $\mathbf{v}_p$ – particle velocity, ρ, ρ_p – carrier and suspension densities, $\mathbf{g}$ – gravitational acceleration, p_e – effective pressure, ν_e – effective viscosity coefficient (includes turbulence, what can be done using quite numerous models; probably this kind of problem will be solved by means of commercial software; if so, the very popular 'k-ε' model will be of assistance in this case [7,9]), $\mathbf{r}_p$ – particle radius vector, V_p – particle volume, $\mathbf{F}_{AM}$ – associated mass force, $\mathbf{F}_N$ – drag force, $\mathbf{F}_C$ – centrifugal force, $\mathbf{F}_{TD}$ – transversal drift.

Equations (1)–(4) are written in their simple forms, including commonly applied simplifications (suspension is dynamically passive and does not influence the carrier liquid).

3 The need to develop simplified models

The set of Eqs. (1)–(4) needs to be supplemented with essential initial and boundary conditions. These conditions include geometrical and kinematic (liquid discharge) characteristics of separator, as well as initial position of computational suspended particle. Formal difficulties make the solution obtainable only by computer-aided means [1,2,8]. The fundamental element of this solution is trajectory of the considered particle given by its radius vector (liquid velocity field is treated as an additional information only). Sample shape of such a trajectory is presented in Fig. 1 [5].

Analysis of the trajectory course can be used to assess whether device geometry had been chosen properly. If the computational particle of diameter d_c (a boundary between fraction d_{ps} to be separated and fraction d_{pz} that can be suspended in the carrier) settles on separator bottom, device dimensions can be accepted. This theoretical criterion of device efficiency assessment is of 'zero-one' character: when $d_{ps} \leq d_c$ all particles are removed from the carrier, and when $d_{pz} > d_c$ particles are suspended in the carrier (Fig. 2). This characteristic is important, as producers of devices are required to provide information on real rate of particles removal (according to particle diameter), and this rate can only be determined by empirical research on devices already in operation [6].

The above described procedure can be applied to simulate operation of a particular device. However, majority of users is interested in designing a new object, and not simulating an already existing one. To design a new object, an inverse problem needs to be solved – what system geometry to choose so that device works as planned (a solution to system of equations).

In most cases, theory of equations of the mathematical physics does not specify methods to describe system geometry (and/or initial and boundary conditions) in such a way to acquire assumed solution. Thus, researchers and engineers are usually forced to apply optimisation methods or to rely on their intuition and trial and error method (which are not very convenient for small and local technical companies). Thus, it is desired to find intermediate simplified methods that

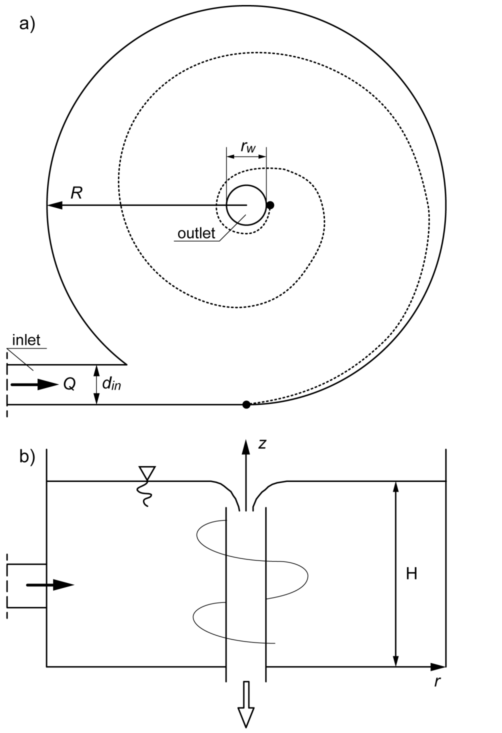

Figure 1: Sample trajectory of a characteristic suspended particle: $r_w = 0.02$ m, $d_{in} = 0.075$ m, $H = 0.20$ m, $R = 0.40$ m.

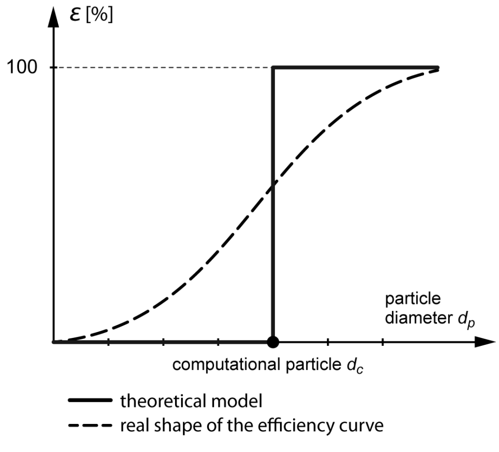

Figure 2: Predicted efficiency, ε, of suspension removal.

can be helpful when solving inverse problems, as well as become a separate design tool. Such a method to design vortex separators is presented further.

4 Possibilities of developing a simplified model

The source of formal difficulties in Eqs. (1)–(4) describing operation of vortex separators are relations (1) and (2) expressing motion of the carrier liquid. Their solution in differential version has numerical form, the fact forcing an analogical method to solve Eqs. (3) and (4).

Application of an approximate model of carrier liquid velocity field gives a possibility to adapt one of simplified versions of Eqs. (3) and (4), even ones that can be solved by analytical means. As a result, one could obtain final algebraic relations for separators dimensioning, as well as choosing geometry on subsequent steps of computational fluid dynamics (CFD) design procedure. Such methodology is often used in engineering especially for flow-through objects [10,11].

5 Proposition of a simplified model

Authors of the paper described velocity field of the carrier liquid in vortex separator by kinematic relations with basic characteristics derived from observations of the phenomenon. Simultaneously, these relations fulfil certain theoretical requirements that ensure physical logics of description. Radial velocity is the velocity averaged over depth, H_e, of flow active zone (vertical velocity is neglected, $u_z = 0$)

$$u_r = -\frac{Q}{2\pi r H_e} \ , \tag{5}$$

$$H_e = 0.5\,(d_{in} + H) \ , \tag{6}$$

where: Q – flow discharge, r – radial coordinate, d_{in} – inlet diameter, H – total water depth. Tangential velocity is expressed by the relation

$$u_t = \frac{B}{r^{0.65}} \ . \tag{7}$$

Multiplier B is determined from condition of equality between energy flux delivered by liquid inflowing to the device and energy flux dissipated by rotating liquid [11–13]:

$$B = 4.63\,Q\left[H d_{in}^4 \left(r_w^{-0.95} - 1.41 R^{-0.95}\right)^{-1/3}\right] \ , \tag{8}$$

where: r_w – outlet diameter, R – separator radius.

Design criterion is defined by a requirement that centrifugal force $\mathbf{F}_C$ acting on the computational particle and keeping it inside the device is not smaller than the sum of drag force $\mathbf{F}_N$ and transversal drift $\mathbf{F}_{TD}$ (that direct suspended particles towards outlet). This means that, in case of horizontal directions, associated mass force $\mathbf{F}_{AM}$ and inertial force are removed from Eq. (4). The requirement is stated for $r = r_w$ point (at the outlet) where forces reach their maximum values (drag force expressed by Newton's formula):

$$\rho_p V_p \frac{u_t^2}{r} \geq \rho V_p \frac{u_t^2}{r} + C_D F_C \frac{\rho u_r^2}{2} \ , \tag{9}$$

where: C_D – drag force coefficient. Taking into account Eq. (6) it is assumed that $H_e \approx 0.5H$. Transformation of Eq. (9) for spherical particles ($C_D = 0.44$) yields a simple requirement

$$\frac{0.0016 \ d_{in}^{8/3}}{H^{4/3} \ r_w^{1/3} \ d_c} + 1 \leq \frac{\rho_p}{\rho} \ . \tag{10}$$

Relationship (10) is a convenient criterion, new and original, allowing for subsequent corrections of separator geometry while using CFD methods. It can also be used as a separate design criterion. Moreover, it allows to analyze conditions of separator operation. The relation explains why cyclones designed for removal of dust from air ($\rho_p/\rho_{air} \approx 2700$) work better than separators for storm wastewater treatment ($\rho_p/\rho_{water} \approx 2.7$), as well as why these devices cannot be successfully used to remove residual fish food from ponds ($\rho_p/\rho_{water} \approx 1.4$) [16].

6 Conclusions

Operation of vortex separators can be described by solving a set of specific differential equations (CFD). This method is difficult and time consuming, however, it allows for a detailed simulation of the considered process. In case of designing new objects one needs to solve a typical inverse problem. Existing methods for correcting subsequent approximations of system geometry so that assumed requirements are reached are complex and time consuming. The paper presents a new physical criterion in the form of simple algebraic relation (10) that includes basic dimensions of vortex separator. The relation can be used both as an additional tool with CFD methods and as an original separate design criterion.

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References

[1] Dyakowski T., Kraipech W., Nowakowski A.P., Williams R.: *A three dimensional simulation of hydrocyclone behaviour.* In: Proc. 2nd Int. Conf. on CFD, Melbourne 1999, 205–210.

[2] Frith P.C.W., Duggins R.K.: *Flow modulation in turbulent vortex chambers.* In: Proc. 9th Australian Fluid Mech. Conf., Auckland 1986, 1–10.

[3] Gronowska M.A.: *Specification of forces in rotational separator.* AHEEM **59**(2012), 1-2, 49–62.

[4] Gronowska M.A., Sawicki, J.M.: *Study of rotational separators operation and design.* In: Technical Progress in Sanitary Engineering, (J.M. Sawicki, K. Weinerowska-Bords, Eds.), Gdansk University of Technology Publishers, Gdansk 2011, 40–48.

[5] Gronowska M.A., Sawicki J.M., Zima P.: *Motion of suspended particles in vortex separator.* In: Proc. 13th Int. Symp. on Water Management and Hydraulic Engineering, Bratislava 2013, 171–176.

[6] Kotowski A., Wójtowicz P.: *Analysis of hydraulic parameters of conical vortex separators.* Pol. J. Env. Studies **19**(2015), 4, 749–756.

[7] Launder B.E., Spalding D.B.: *Lectures in mathematical models of turbulence.* Academic Press, London 1972.

[8] Martignoni W.P., Bernardo S., Quintani C.L.: *Evaluation of cyclone geometry and its influence on performance parameters by CFD.* Braz. J. Chem Eng. **24**(2007), 83–94.

[9] Pozorski J.: *On general-purpose turbulence models in CFD.* Transactions IFFM **127**(2015), 45–62.

[10] Rhodes M.: *Introduction to Particle Technology.* Wiley & Sons, Chichester 1998.

[11] Sawicki J.M.: *Aerated grit chambers hydraulic design equation.* J. Environ. Eng. **130**(2004), 9, 1050–1058.

[12] Sawicki J.M.: *Transversal pressure effect in circulative separator.* AHEEM **59**(2012), 1-2, 3–12.

[13] Slattery J.C.: *Advanced Transport Phenomena.* Cambridge University Press, Cambridge 1999.

[14] Soo L.: *Fluid Dynamics of Multiphase Systems.* Blaisdell Publ., London 1966.

[15] Stairmand C.J.: *The design and performance of cyclone separators.* Trans. Inst. Chem. Eng. **29**(1951), 356–373.

[16] Veerapen J.P., Lowry B.J. and Couturier M.F.: *Design methodology for the swirl separator.* Aquacultural Eng. **33**(2005), 21–45.