Article citation info:
ZIEJA M, WAŻNY M, STĘPIEŃ S. Distribution determination of time of exceeding permissible condition as used to determine lifetimes of selected aeronautical devices/systems. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2016; 18 (1): 57–64, http://dx.doi.org/10.17531/ein.2016.1.8.

Mariusz ZIEJA
Mariusz WAŻNY
Sławomir STĘPIEŃ

DISTRIBUTION DETERMINATION OF TIME OF EXCEEDING PERMISSIBLE CONDITION AS USED TO DETERMINE LIFETIMES OF SELECTED AERONAUTICAL DEVICES/SYSTEMS

WYZNACZENIE ROZKŁADU CZASU PRZEKRACZANIA STANU GRANICZNEGO I JEGO ZASTOSOWANIE DO OKREŚLANIA TRWAŁOŚCI WYBRANYCH URZĄDZEŃ LOTNICZYCH*

The paper refers to the modelling of changes in ever-growing deviations from diagnostic parameters that describe health/maintenance status of one from among numerous aircraft systems, i.e. of a sighting system. Any sighting system has been intended, first and foremost, to find a sighting angle and a lead angle, both of them essential and indispensable to fight hostile targets. Destructive factors such as, e.g. ageing processes, that keep affecting the aircraft as a whole throughout its operation, make these angles change: actual values thereof differ from the calculated ones. Such being the case, a considerable error may be introduced in the process of aiming the weapons to, in turn, result in the reduction of values that describe the quality of the sighting process. That is why any sighting system requires specific checks possibly (if need be) followed with some adjustments (based on the findings of these checks) to remove negative effects of any ageing processes that might have affected this system. Determination of the density function of the deviation using difference equations and the Fokker-Planck equation is a basic element of the presented method, which enables next analyses. Innovative elements of the paper are as follows:
– determination of distributions of time of exceeding the permissible (boundary) condition using the density function of the deviation,
– application of distributions of time of exceeding the permissible (boundary) condition for modification of operation/maintenance systems of selected aeronautical devices. The paper has been concluded with a numerical example that proves the application-oriented nature of the issues in question, represented by the earlier conducted assessment of lifetimes of the systems intended to find the sighting and lead angles (ε and β). The in the paper discussed method to assess the lifetime may as well be applied to another systems/devices. It shows a versatile nature and makes a valuable contribution to the methods of maintaining any engineered systems in good condition (i.e. of providing maintenance to any engineered systems).

Keywords: *reliability, life, permissible condition, lead angle, sighting angle, airborne sighting system.*

Praca dotyczy modelowania zmian narastających odchyłek parametrów diagnostycznych charakteryzujących stan techniczny jednego z systemów statku powietrznego, tj. systemu celowniczego. Jednym z głównych zadań systemu celowniczego jest wyznaczenie kątów celowania i wyprzedzenia niezbędnych do zwalczania celów przeciwnika. Oddziaływanie w czasie eksploatacji statku powietrznego czynników destrukcyjnych m.in. procesów starzeniowych, powoduje, że kąty te ulegają zmianie i ich rzeczywiste wartości różnią się od wartości kątów obliczeniowych. Wystąpienie takiej sytuacji powoduje wprowadzenie dość istotnego błędu do procesu celowania i obniża wartość wskaźników charakteryzujących jakość jego przebiegu. Z tego też względu system celowniczy wymaga określonej kontroli i w oparciu o uzyskane wyniki, potencjalnej regulacji mającej na celu usunięcie ujemnych skutków procesów starzeniowych celownika. Podstawowym elementem pracy umożliwiającym dalsze analizy było wyznaczenie funkcji gęstości odchyłki z wykorzystaniem równań różnicowych oraz równania Fokkera-Plancka. Do nowatorskich elementów pracy należy zaliczyć:
– wyznaczenie rozkładu czasu przekroczenia stanu dopuszczalnego (granicznego) z wykorzystaniem funkcji gęstości odchyłki,
– zastosowanie rozkładu czasu osiągania stanu granicznego do modyfikacji systemów eksploatacji urządzeń lotniczych.
Praca podsumowana jest przykładem obliczeniowym przedstawiającym aplikacyjny charakter poruszanej tematyki, odwzorowanej na przykładzie oceny trwałości układów określających kąt celowania i wyprzedzenia (ε i β). Przedstawiona metoda oceny trwałości w niniejszym artykule może być zastosowania do innych urządzeń. Ma ona ogólny charakter i stanowi wkład do metod utrzymania systemów technicznych.

Słowa kluczowe: *niezawodność, trwałość, stan dopuszczalny, kąt wyprzedzenia, kąt celowania, celownik lotniczy.*

1. Introduction

The issues of providing maintenance to any engineered systems require to be approached from many aspects. The reasons are: many and various areas they are functioning in, and many and various factors that considerably affect processes of providing maintenance to the engineered systems [1, 21, 32, 35, 36, 42]. The available literature

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl*** on the designing of strategies of how to provide maintenance to engineered objects delivers numerous classifications of models in question [2, 4, 5, 19, 26, 29]. Among them, there are some of extremely great significance, i.e. ones where the system's renewal is based on checking some specific diagnostic parameters. These are models of the so-called condition-based maintenance [7, 8, 12, 17, 22]. Furthermore, there are many studies where authors have assumed that a failure to a system is a rapidly occurring event. In the 1970s an idea of the Delay-Time Analysis (DTA) was developed. A good deal of publications on the modelling and implementation of the DTA concept have been issued up to the present [20, 25, 40]. The maintenance-dedicated literature proves both the significance of keeping devices/systems operational and the ever growing costs of maintaining this serviceability. At the same time, it should be emphasized here that there is a good many works in the available literature that deal with the problems of the environment/ambient conditions, ageing and wear-and-tear processes, etc. that affect the functioning of any engineered system [14, 31, 33, 37].

Devices/systems used in military aircraft are usually highly technologically advanced. This is the reason why formulation/generation of optimum operational models of these systems proves a complicated task. Methods based on changes in diagnostic parameters prove then extremely useful for the assessment of reliability and life of aeronautical devices/systems [24, 38, 39, 41, 43, 44]. The primary objective of a military aircraft is to perform a specified mission, in the course of which it quite often happens that air warfare agents have to be used. Effectiveness of applying them depends on many and various factors, just to mention:

1) health/maintenance status of an aircraft weapon system,
2) conditions for performing the combat mission in question,
3) the kind of a target to be attacked,
4) pilot's skill, etc.

Since the range of topics in this area is really wide, the main focus of attention is analysis of health/maintenance status of a selected subsystem of an air weapon system, in the case given consideration, a sighting system which remained under examination throughout its operation.

The sighting system is considered serviceable (fit for use), if check parameters remain within the tolerance limits (interval). If not, the system should be subjected to maintenance to restore nominal values of its functional qualities/parameters. Therefore, the essence of the whole process of the sighting system's operation resolves itself into that diagnostic parameters are not permitted to exceed the specified level of error, which makes the system fully and successfully used. Destructive processes affecting the sighting system are unavoidable. What results is the loss of nominal values by the diagnostic parameters. Hence, it is essential to determine the moment values of the diagnostic parameters reach the permissible (boundary) level. Such being the situation, it is necessary to interfere in the system's structure to remove negative effects of destructive factors affecting it.

The sighting marker displayed on the reflector of a sight head is probably the most essential functional parameter of a sighting system. While aiming at a target, the pilot/operator is expected to make the sighting marker stay in alignment with the target. Its position is defined with two angles, i.e. a sighting angle and a lead angle. Hence, final effects of the aiming process are conditioned by the accuracy of values found for these angles.

Let us assume that deviation from the analytical value can be found in the following way:

$$\Delta x_0 = \left| X - W_0 \right| = \left| \left[X - \bar{X} \right] + \left[\bar{X} - \bar{X}_1 \right] + \left[\bar{X}_1 - W_0 \right] \right|, \quad (1)$$

where:

X – analytical value of the lead angle at the final instant of the aiming process;

W_0 – a real-time value of the sighting angle or the lead angle found at the final instant of the aiming process;

$\left[X - \bar{X} \right]$ – an error of the model of calculating the sighting angle or the lead angle;

$\left[\bar{X} - \bar{X}_1 \right]$ – a transferred error of data for calculating the angle of interest;

$\left[\bar{X}_1 - W_0 \right]$ – an error generated by the algorithm for calculating the $\bar{X}_1$ function.

Destructive processes result, among other things, in some change in W_0. Hence, deviation effected by these processes can be determined in the following way:

$$Z_{(t)} = \left| W_{(t)} - W_0 \right|, \quad (2)$$

where:

$W_{(t)}$ – destructive-processes-affected value of an angle;

$Z_{(t)}$ – deviations described as an increasing stochastic process.

Increments in the value of deviation result from the deterioration in the health/ maintenance status of assemblies and units of a system/device due to destructive processes of the ageing, wear-and-tear, and fatigue nature that affect the system's/device's components, assemblies, etc. Deterioration in health of the system's assemblies/units is not always directly indicated and warned of, which makes any evaluation thereof rather difficult; hence the idea of applying the ever growing deviations in diagnostic parameters or operating characteristics of the system/device in question to estimate probability that a failure occurs within the interval (0, *t*) by means of a reliability function determined on the basis of the distribution of time of exceeding the permissible (boundary) condition.

The sight to be used on an aircraft has to be set up in some correct position (aligned to its axis), so that the optical line of the sight points to the null position. Destructive effect of ageing processes results in the deviation of the optical line of the sight from the null position. Therefore, relationship (2) can be written down in the following form:

$$z = \left| Z - Z_0 \right|, \quad (3)$$

where:

z – deviation from the null position of the line of sighting, treated as a diagnostic parameter;

Z – position of the line of sighting as evaluated with account taken of effects of destructive processes;

Z_0 – required value of the null position of the line of sighting.

2. How to find the density function of the null position of the line of sighting

The following assumptions have been made in the model proposed to evaluate stability of the null position of directions of sighting and allowance:

1) the system's health/maintenance status is determined with one diagnostic parameter "z" in the form of deviation from the initial (zero) value:

$$z = \left| Z - Z_{nom} \right| , \tag{4}$$

where: Z_{nom} – nominal value (null position) of the diagnostic parameter;

2) deviation of the diagnostic parameter changes throughout the whole operational phase of the aircraft, i.e. while in the air and during parking;
3) the "z" parameter is non-decreasing;
4) the rate of changes in the diagnostic parameter can be described with the following relationship:

$$\frac{dz}{dt} = c , \tag{5}$$

where:

c – random variable that describes changes affected by features of the system's components;

t – calendar-based time.

The dynamics of the rate of changes in the deviation of "z" can be described, when approached stochastically, with the following difference equation:

$$U_{z,t+\Delta t} = \left(1 - P\right) U_{z,t} + P U_{z-\Delta z,t} , \tag{6}$$

where:

$U_{z,\,t}$ – probability that the diagnostic parameter takes value z at time instant t;

P – probability that value of the deviation increases throughout time interval Δt by the amount of Δz;

Δz – increment in the deviation.

For probability equal to $P = 1$, eq (6) can be written down - with the functional notation applied - in the following form:

$$u\left(z, t + \Delta t\right) = u\left(z - \Delta z, t\right), \tag{7}$$

where: $u(z,t)$ – density function of values of deviations of the diagnostic parameter as affected by time.

Eq (7) should be read in the following way: probability that at time instant t value of the deviation was z - Δz and throughout time interval Δt increased by Δz. Eq (7) is now to be rearranged into a partial differential equation. Therefore, the following approximations are to be introduced now [10, 13, 28, 30]:

$$\begin{aligned} u\left(z, t + \Delta t\right) &= u\left(z,t\right) + \frac{\partial u\left(z,t\right)}{\partial t}\Delta t, \\ u\left(z - \Delta z, t\right) &= u\left(z,t\right) - \frac{\partial u\left(z,t\right)}{\partial z}\Delta z + \frac{1}{2}\frac{\partial^2 u\left(z,t\right)}{\partial z^2}\left(\Delta z\right)^2 . \end{aligned} \tag{8}$$

Now, if we apply eq (8) to eq (7), the latter takes the form:

$$\frac{\partial u\left(z,t\right)}{\partial t} = -b\frac{\partial u\left(z,t\right)}{\partial z} + \frac{1}{2} a \frac{\partial^2 u\left(z,t\right)}{\partial z^2} , \tag{9}$$

where:

$b = E[c]$– an average increment in the diagnostic parameter's deviation per time unit;

$b = E[c^2]$ – a mean square of the increment in the diagnostic parameter's deviation per time unit.

Let us find a particular solution to eq (9), such that for $t \to 0$ is convergent to the so-called Dirac function, i.e. $u(z, t) \to 0$ for $z \neq 0$ and $u(0, t) \to +\infty$ but in such a way that the integral of function u equals '1' for all $t > 0$.

Solution to eq (9) for the above-formulated condition takes the form [10, 13, 28, 30]:

$$u\left(z,t\right) = \frac{1}{\sqrt{2\pi A\left(t\right)}} e^{-\frac{\left(z - B(t)\right)^2}{2A(t)}} , \tag{10}$$

where:

$$\begin{aligned} B\left(t\right) &= \int_0^t b dt = bt = \bar{c}t, \\ A\left(t\right) &= \int_0^t a dt = at = \bar{c}^2 t. \end{aligned} \tag{11}$$

The density function of the increase in value of the diagnostic parameter's deviation can be directly applied to evaluate reliability of the system/device in question.

3. How to find distribution of time of exceeding the permissible (boundary) condition

Probability that the diagnostic parameter exceeds the permissible (boundary) value can be presented – using the density function of changes in the diagnostic parameter's deviation (10) – in the following form [3]:

$$Q\left(t; z_g\right) = \int_{z_g}^{\infty} \frac{1}{\sqrt{2\pi at}} e^{-\frac{\left(z - bt\right)^2}{2at}} dz , \tag{12}$$

The density function of the distribution of time the permissible value z_g has been exceeded for the first time takes the following form:

$$f\left(t\right) = \frac{\partial}{\partial t} Q\left(t; z_g\right). \tag{13}$$

With account taken of eq (10), the following is arrived at:

$$f\left(t\right) = \frac{\partial}{\partial t} \int_{z_g}^{\infty} \frac{1}{\sqrt{2\pi at}} e^{-\frac{\left(z - bt\right)^2}{2at}} dz . \tag{14}$$

Therefore,

$$f\left(t\right) = \int_{z_g}^{\infty} \left\{ \frac{\partial}{\partial t} \left[\frac{1}{\sqrt{2\pi at}} e^{-\frac{\left(z - bt\right)^2}{2at}} \right] \right\} dz . \tag{15}$$

Having accepted definition (10), we get:

$$f\left(t\right) = \int_{z_g}^{\infty} \left\{ \frac{\partial}{\partial t} u\left(z,t\right) \right\} dz . \tag{16}$$

Furthermore, the time derivative of function (10) takes the following form:

$$\frac{\partial}{\partial t}\left[u(z,t)\right]=u(z,t)\left(\frac{z^2-b^2t^2-at}{2at^2}\right). \tag{17}$$

Eq (17) has been substituted into eq (16):

$$f(t)_{z_g}=\int_{z_g}^{\infty}\left[u(z,t)\left(\frac{z^2-b^2t^2-at}{2at^2}\right)\right]dz. \tag{18}$$

Now, we are looking for the antiderivative of the function for the integrand in eq (18). We predict that function of the following form:

$$w(z,t)=u(z,t)\left(-\frac{z+bt}{2t}\right),$$

is the antiderivative of the function for the integrand in eq (14).
Let us make a check:

$$\frac{\partial}{\partial z}\left[u(z,t)\left(-\frac{z+bt}{2t}\right)\right]=-u(z,t)\left(-\frac{z-bt}{at}\right)\left(-\frac{z+bt}{2t}\right)+u(z,t)\left(-\frac{1}{2t}\right)=$$
$$=u(z,t)\left[\frac{(z-bt)(z+bt)}{2at^2}-\frac{1}{2t}\right]=u(z,t)\left[\frac{z^2-b^2t^2-at}{2at^2}\right]. \tag{19}$$

Hence the inference that the antiderivative of the function against the integrand in eq (18) takes the form:

$$w(z,t)=u(z,t)\left(-\frac{(z+bt)}{2t}\right) \tag{20}$$

Therefore, if the integral (18) is calculated, we arrive at what follows:

$$f(t)_{z_g}=u(z,t)\left(-\frac{z+bt}{2t}\right)\Bigg|_{z_g}^{\infty}=\frac{z_g+bt}{2t}\frac{1}{\sqrt{2\pi at}}e^{-\frac{(z_g-bt)^2}{2at}}. \tag{21}$$

Relationship (21) determines the density function of time the boundary (permissible) condition has been exceeded for the first time by the diagnostic parameter's deviation.

4. Evaluation of lifetimes of some selected structural units of the sight

A formula for the reliability of the aircraft system's/device's unit takes the form [11]:

$$R(t)=1-\int_0^t f(t)_{z_g}\,dt, \tag{22}$$

where the density function $f(t)_{z_g}$ is defined with eq (21).

On the other hand, the unreliability of the aircraft system's/device's unit can be found from the following relationship:

$$Q(t)=\int_0^t \frac{z_g+bt}{2t}\cdot\frac{1}{\sqrt{2\pi at}}e^{-\frac{(z_g-bt)^2}{2at}}dt. \tag{23}$$

The integral (23) should be reduced to some simpler form. It can be noticed that the integrand can be written down in the following form:

$$\frac{z_g+bt}{2t}\cdot\frac{1}{\sqrt{2\pi at}}e^{-\frac{(z_g-bt)^2}{2at}}=\frac{z_g+bt}{2t}\cdot\frac{1}{\sqrt{2\pi at}}e^{-\frac{(bt-z_g)^2}{2at}}, \tag{24}$$

the problem can then be reduced to solving the indefinite integral:

$$\int\frac{(z_g+bt)}{2t}\cdot\frac{1}{\sqrt{2\pi at}}e^{-\frac{(bt-z_g)^2}{2at}}dt. \tag{25}$$

After substitution

$$\frac{(bt-z_g)^2}{2at}=u,$$

the integral (25) takes the form:

$$\int\frac{z_g+bt}{2t}\cdot\frac{1}{\sqrt{2\pi at}}e^{-u}\cdot\frac{2at^2}{(bt+z_g)(bt-z_g)}du=\frac{1}{2\sqrt{\pi}}\int\frac{1}{\sqrt{u}}e^{-u}du. \tag{26}$$

Then, another substitution should be made:

$$\sqrt{u}=w,$$
$$du=2w\,dw.$$

Taking account of the above written relationships the integral (23) can be written in the following form:

$$\frac{1}{2\sqrt{\pi}}\int\frac{1}{w}e^{-w^2}2w\,dw=\frac{1}{\sqrt{\pi}}\int e^{-w^2}dw. \tag{27}$$

After substitution:

$$w^2=\frac{y^2}{2},$$

$$dw=\frac{y}{\sqrt{2}},$$

the integral of the form:

$$\frac{1}{\sqrt{2\pi}}\int e^{-\frac{y^2}{2}}dy. \tag{28}$$

has been arrived at, where:

$$y = \frac{bt - z_g}{\sqrt{at}}.$$

If the results found are introduced into eq (22), the following formula for reliability is arrived at, providing that the limits of integration are properly written down:

$$R(t) = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{bt - z_g}{\sqrt{at}}} e^{-\frac{y^2}{2}} dy. \tag{29}$$

Cumulative distribution function for the standard normal distribution takes the form [27]:

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy. \tag{30}$$

With eq (29) taken into account, the final form of the formula for reliability of the aircraft's structural component can be expressed with the following relationship:

$$R^*(t) = 1 - \Phi\left(\frac{b^* t - z_g}{\sqrt{a^* t}}\right), \tag{31}$$

where:

b^* and a^*– coefficients estimated on the basis of data received from the aircraft operation-and-maintenance processes.

Therefore, the probability that an aircraft's system/device suffers a failure can be found from the following relationship:

$$Q^* = 1 - R^*(t) = \Phi(\gamma), \tag{32}$$

where:

$$-\gamma = \frac{b^* t - z_g}{\sqrt{a^* t}}. \tag{33}$$

Eq (33) is multiplied by '−1'. This is the way to pass from the negative semi-axis to the positive one; now, the 'reliability' can be referred to instead of the 'unreliability'.

$$\gamma^* = \frac{z_g - b^* t}{\sqrt{a^* t}}. \tag{34}$$

To settle the reliability level, one has to find the γ* value from the normal distribution tables. Then, with the γ*value known, stability of the null system can be determined:

$$T = \frac{\left(2b^* z_g + (\gamma^*)^2 a^*\right) - \sqrt{\left(2b^* z_g + (\gamma^*)^2 a^*\right)^2 - 4b^{*2} z_g^2}}{2b^{*2}}. \tag{35}$$

To make use of eq (35), one needs to find (estimate) values of constants in this formula. Therefore, it is assumed that observation of the system/device in question throughout the whole operational phase (i.e. operation and maintenance) thereof has delivered data on the growth of the diagnostic parameter's deviation, in the following form:

$$\left[(z_0, t_0), (z_1, t_1), (z_2, t_2), \ldots, (z_n, t_n)\right]. \tag{36}$$

The best way to find values of 'a' and 'b' for the data at hand is a method that uses the likelihood function. The general-case form of this function can be presented as the following relationship [6, 27]:

$$L = \prod_{k=0}^{n-1} g(t_k, z_k, \theta_1, \theta_2, \ldots, \theta_m), \tag{37}$$

where:

$g(t_k, z_k, \theta_1, \theta_2, \ldots, \theta_m)$ – density function of the total probability of variable z;

$(\theta_1, \theta_2, \ldots, \theta_m)$ – parameters of the density function;

z_k – measured values of the parameter z consumption at time instants $(t_1, t_2, \ldots, t_k)$, respectively.

Finding estimates $\theta_1^*, \theta_2^*, \ldots, \theta_m^*$ of unknown parameters $\theta_1, \theta_2, \ldots, \theta_m$ with the maximum likelihood method means nothing but having solved equations of the following form:

$$\frac{\partial \ln L}{\partial \theta_j} = 0, \tag{38}$$

where:

$j = 1, 2, \ldots, m$;

m – the number of parameters that describe a given engineered object.

Such being the case, finding estimates b^* and a^* of unknown parameters b and a with the maximum likelihood method means having solved the following system of equations [6,27]:

$$\begin{cases} \dfrac{\partial \ln L}{\partial b} = 0 \\ \dfrac{\partial \ln L}{\partial a} = 0 \end{cases}. \tag{39}$$

Having solved the system of equations (39) the b^* and a^* are found:

$$b^* = \frac{z_n}{t_n}, \tag{40}$$

$$a^* = \frac{1}{n}\sum_{k=0}^{n-1}\frac{\left[\left(z_{k+1}-z_k\right)-b^*\left(t_{k+1}-t_k\right)\right]^2}{\left(t_{k+1}-t_k\right)}. \quad (41)$$

5. A numerical case and final remarks

A sighting head is one of the major components of an airborne sighting system. A sighting marker is displayed thereon. At the stage of the manufacture all the components are adjusted to have them furnished with nominal values to, in turn, perform a combat mission with the smallest error possible.

In the course of operation a check of the system's components being adjusted against each other is performed, i.e. two parameters ε and β are analysed. These are parameters that describe co-ordinates of the sighting marker's position for some pre-set conditions of the system's operation. With the operation-delivered data on these co-ordinates it has been confirmed that values of these parameters change with time of the system's operation (*Fig. 1*).

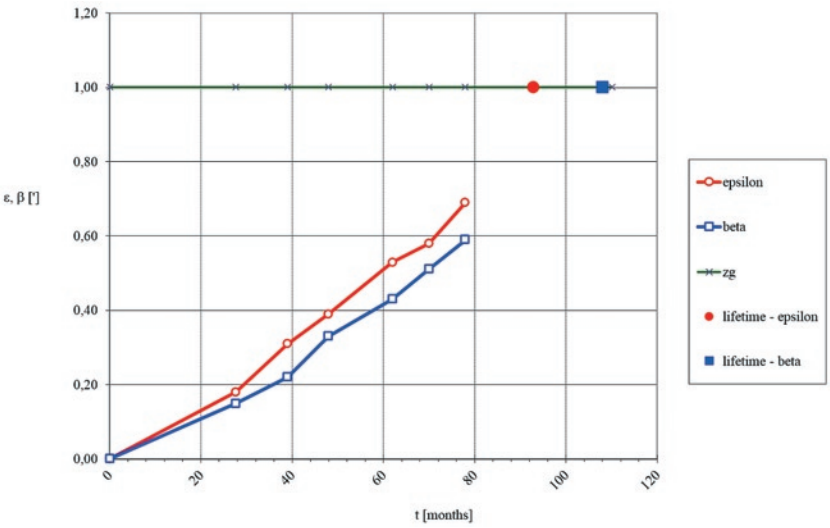

Fig. 1. Graphically presented changes in values of diagnostic parameters of the sighting head against time of its operation

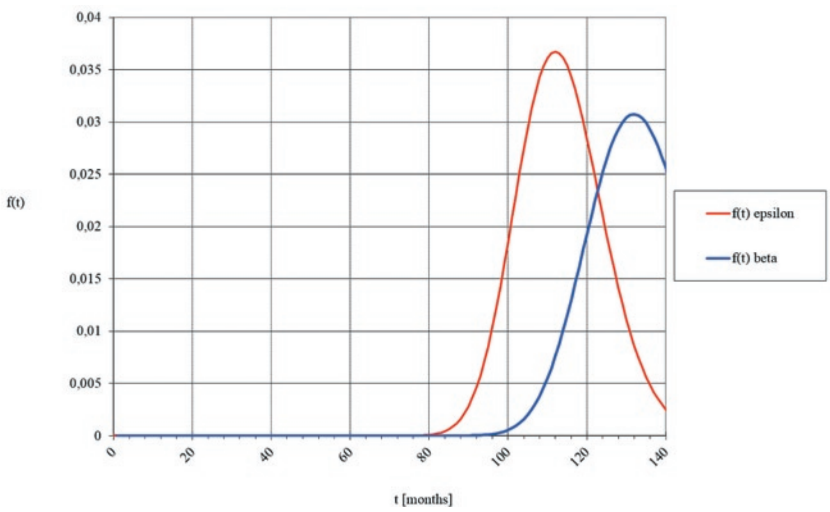

Fig. 2. A graphical form of the density function of time the deviation increases up to the boundary value

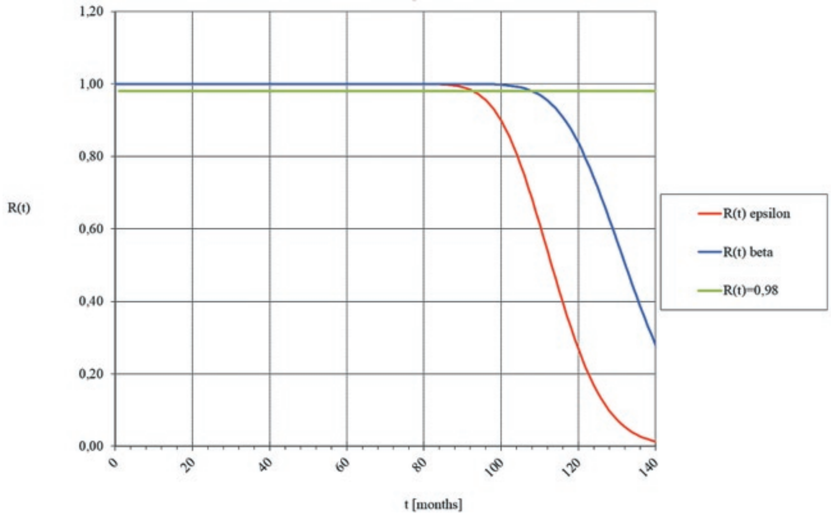

Fig. 3. A graphically presented form of function of sighting head's reliability with account taken of the diagnostics parameters under consideration

Numerical verification of the method in question, based on the data shown in Fig. 1, consisted in finding values of the density function coefficients '*a*' and '*b*' for both the diagnostic parameters; respectively, these values were as follows:

$$a_\varepsilon^* = 0.009, \quad b_\varepsilon^* = 0.0076, \quad a_\beta^* = 0.0001, \quad b_\beta^* = 0.0001. \quad (42)$$

With the reliability level assumed to be $R^*(t) = 0.98$, value of the diagnostic parameter has been found from the normal distribution tables: $\gamma^* = 2.32$. Then, the z_g parameter has been found using technical documentation dedicated to maintenance services (e.g. Maintenance Hanbdbook/Manual); what is to be found there is information on permissible values of deviations of the above-mentioned diagnostic parameters.

With the relationships derived above and values found, time after which values of deviations of the diagnostic parameters in question exceed the boundary condition has been calculated. For the case given consideration, the time in question is as follows, respectively:

$$t_\varepsilon^* = 93\,[\text{months}], \quad t_\beta^* = 108\,[\text{months}]. \quad (43)$$

Furthermore, using the above presented data, graphical forms of: the density function of the time the deviation keeps growing up to finally reach the boundary value (Fig. 2), and the reliability function $R(t)$ for the analysed parameters (Fig. 3) have been found.

The discussed method of estimating effects of destructive processes upon the availability of airborne sighting systems seems correct and proper. The presented numerical case has both enabled verification of the formulated model and emphasised application-oriented advantages of the developed method. The in this way obtained results enable:

1) The assessment of residual life of the sighting system with the required reliability level maintained;

2) Estimation of the system's reliability on the basis of a group of parameters recorded in the course of the system's operation:
3) Estimation of the system's reliability on the basis of a selected diagnostic parameter;
4) Verification of the process of operating the sighting system (making correction) to maintain the suitable level of reliability between particular checks.

The above-presented method may prove useful in further efforts to make the process of operating and maintaining the aircraft furnished with sighting systems more efficient.

Considering the fact the method proves extremely versatile, it may be successfully applied to determine residual life of any engineered object, the health/maintenance status of which is found using values of diagnostic parameters.

References

1. Augustynowicz J, Dudek D, Dudek K, Figiel A. Prognozowanie okresu bezpiecznej eksploatacji maszyn górniczych. Rozważania o degradacji obiektu. Górnictwo i Geoinżynieria 2007; 31(2): 55-66.
2. Birolini A. Quality and Reliability of Technical Systems. Theory – Practice – Management. Berlin: Springer Verlag, 1984.
3. Bobrowski D. Modele i metody matematyczne teorii niezawodności. Warszawa: WNT, 1985.
4. Casciati F, Roberts B. Mathematical Models for Structural Reliability Analysis. Boca Raton/New York/London/Tokyo: CRC Press, 1996.
5. Cho I D, Parlar M. A survey of maintenance models for multi-unit systems. European Journal of Operational Research 1991; 51, http://dx.doi.org/10.1016/0377-2217(91)90141-h.
6. DeLurgio SA. Forecasting principles and applications. University of Missouri-Kansas City: Irwin/McGraw-Hill, 1998.
7. Dhillon BS. Design Reliability. Fundamentals and Applications. Boca Raton/New York/London/Washington: CRC Press, 1999, http://dx.doi.org/10.1201/9781420050141.
8. Dhillon BS. Mechanical Reliability: Theory, Models and Applications. Washington: AIAA Education Series, 1988.
9. Dinesh Kumar U, Knezevic J, Crocker J, El-Haram, M. Reliability, Maintenance and Logistic Support - A Life Cycle Approach. Boston: Kluwer Academic Publishers, 2000, http://dx.doi.org/10.1007/978-1-4615-4655-9.
10. Franck TD. Nonlinear Fokker-Planck Equations. Fundamentals and Applications. Berlin Heildenberg: Springer-Verlag, 2005.
11. Gercbach IB, Kordoński CB. Modele niezawodnościowe obiektów technicznych. Warszawa: WNT, 1968.
12. Girtler J. Zastosowanie diagnostyki do decyzyjnego sterowania procesem eksploatacji urządzeń. Diagnostyka 2006; 2(38):151-158.
13. Grasman J, Herwaarden OA. Asymptotic Methods for the Fokker-Planck Equation and the Exit Problem in Applications. Berlin Heildenberg: Springer-Verlag, 1999, http://dx.doi.org/10.1007/978-3-662-03857-4.
14. Idziaszek Z, Grzesik N. Object characteristics deterioration effect on task reliability - outline method of estimation and prognosis. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2014; 16 (3): 433-440.
15. Kececioglu DB. Maintainability, Availability & Operational Readiness Engineering Handbook. Lancaster: DEStech Publications, 2003.
16. Kececioglu DB. Reliability Engineering Handbook. Lancaster: DEStech Publications, 2002.
17. Kinnison H, Siddiqui T. Aviation Maintenance Management. New York: The McGraw-Hill Companies, Inc. 2013.
18. Knezevic J. Systems Maintainability. London: Chapman & Hall, 1997.
19. Kołowrocki K, Soszyńska Budny J. Reliability and Safety of Complex Technical Systems and Processes. London: Springer-Verlag, 2011, http://dx.doi.org/10.1007/978-0-85729-694-8.
20. Kovalenko IN, Kuznetsov NY, Pegg PA. Mathematical Theory of Reliability Of Time Dependent Systems with practical Applications. Chichester: John Wiley & Sons, 1997.
21. Legutko S. Development trends in machines operation maintenance. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2009; 2(42): 8-16.
22. Moubray J. Reliability-centered Maintenance II. New York: Industrial Press Inc., 1997.
23. Narayan V. Effective Maintenance Management, New York: Industrial Press Inc., 2012.
24. Nechval KN, Nechval NA, Berzins G, Purgailis M. Planning inspections in service of fatigue-sensitive aircraft structure components under crack propagation. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2007; 4: 3-8.
25. Nowakowski T. Niezawodność systemów logistycznych. Wrocław: Oficyna Wydawnicza Politechniki Wrocławskiej, 2011.
26. Pham H, Wang H. Imperfect maintenance. European Journal of Operational Research 1996; 94, http://dx.doi.org/10.1016/s0377-2217(96)00099-9.
27. Pham H. Handbook of Engineering Statistics. London: Springer-Verlag 2006, http://dx.doi.org/10.1007/978-1-84628-288-1.
28. Risken H. The Fokker-Planck Equation. Methods of Solution and Applications. London: Springer-Verlag, 1984, http://dx.doi.org/10.1007/978-3-642-96807-5.
29. Scarf PA. On the application of mathematical models in maintenance. European Journal of Operational Research 1997; 99, http://dx.doi.org/10.1016/s0377-2217(96)00316-5.
30. Soize C. The Fokker-Planck Equation for Stochastic Dynamical Systems and Its Explicit Steady State Solutions. World Scientific Publishing, 1994.
31. Sugier J, Anders GJ. Modelling and evaluation of deterioration process with maintenance activities. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2013; 15 (4): 305-311.
32. Szpytko J. Integrated Decision Making supporting the exploitation and control of transport devices. Kraków: Uczelniane Wydawnictwa Naukowo-Dydaktyczne 2004.
33. Tomaszek H, Ważny M. Zarys metody oceny trwałości na zużycie powierzchniowe elementu konstrukcji z wykorzystaniem rozkładu czasu przekroczenia stanu granicznego (dopuszczalnego). Radom: Zagadnienia Eksploatacji Maszyn 2008; 155(3).
34. Tomaszek H, Wróblewski H. Podstawy oceny efektywności eksploatacji systemów uzbrojenia lotniczego. Warszawa: Dom Wydawniczy Bellona, 2001.
35. Tomaszek H, Żurek J, Jasztal M. Prognozowanie uszkodzeń zagrażających bezpieczeństwu lotów statku powietrznego. Radom: Wydawnictwo Naukowe JTE, 2008.

36. Tomczyk W. Uwarunkowania racjonalnego procesu użytkowania maszyn i urządzeń rolniczych. Inżynieria Rolnicza 2005; 7: 359-366.
37. Wang H. A survey of maintenance policies of deteriorating systems. European Journal of Operational Research 2002; 139, http://dx.doi.org/10.1016/s0377-2217(01)00197-7.
38. Ważny M. The method of determining the time concerning the operation of a chosen navigation and aiming device in the operation system. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2008; 38(2): 4-11.
39. Ważny M. The metod for assessing residual durability of selected of devices in avionics system. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2009; 43(3): 55-64.
40. Werbińska-Wojciechowska S. Time resource problem in logistics systems dependability modelling. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2013; 15(4): 427-433.
41. Zieja M. Metoda oceny trwałości wybranych urządzeń lotniczych wojskowych statków powietrznych. Problemy utrzymania systemów technicznych. Warszawa: Oficyna Wydawnicza Politechniki Warszawskiej, 2014: 151-160.
42. Zio E. Computational Methods For Reliability and Risk Analysis. Singapore: World Scientific Publishing, 2009, http://dx.doi.org/10.1142/7190.
43. Żurek J, Tomaszek H, Zieja M.: The reliability estimation of structural components with some selected failure model. 11th International Probabilistic safety Assessment and management Conference and the Annual European Safety and Reliability Conference 2012, Curran Associates, Inc., 2012:1741-1750.
44. Żurek J.; Tomaszek H.; Zieja M.: Analysis of structural component's lifetime distribution considered from the of wearing with the characteristic function applied. Safety, Reliability and Risk Analysis: Beyond the Horizon – Steenbergenet al. London: Taylor & Francis Group, 2014: 2597-2602.

Mariusz ZIEJA
Air Force Institute of Technology
ul. Księcia Bolesława 6, 01-949 Warszawa 96, Poland

Mariusz WAŻNY
Sławomir STĘPIEŃ
Military University of Technology
ul. Kaliskiego 2, 00-908 Warszawa 49, Poland

E-mail: mariusz.zieja@itwl.pl,
mwazny@wat.edu.pl, sstepien@wat.edu.pl