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Estimation of Wide-Lane Hardware Delays for single station in GPS system

Abstract

The paper presents study results about determination Wide-Lane Hardware Delays in GPS system. For this purpose GPS data from RYKI reference station were used. Melbourne-Wübbena linear combination were applied for estimation *WHD*. Computations were executed in SciTEC software, which code source was written in Scilab 5.4.1. Firstly, results from SciTEC software show that *WHD* are very stable over few days. In this paper 4 experiments are presented. Accuracy of *WHD* in submitted paper is less than 2 ns. Over few days, magnitude order of mean *SWHD* is ± 1 ns, what corresponds to 0.3 cycle of wavelength in L6 combination. Difference between maximum and minimum value of *SWHD* over 6 days is about ± 2.5 ns. *RWHD* over few days are so very stable, with mean value about −0.154 ns. Standard deviation of daily repeatability *RWHD* parameter is less than 0.07 ns, what corresponds to 2% of wavelength in L6 combination.

Keywords: GPS, Wide-Lane Hardware Delays, Melbourne-Wübbena linear combination, Local Ionosphere Monitoring System.

1. Introduction

One of the major systematic error in precise positioning is determinated ambiguity term. Most of commercial software in satellites geodesy still apply algorithms for estimation integer value of ambiguity. Methods as AFM, LAMBDA or QIF represent group of method, which basic solution of ambiguity is integer parameter [2]. Utilization Ionosphere-Free (L3) linear combination for determinates, especially coordinates, make possible to evaluation ambiguity as a float component. In this aspect, float ambiguity is estimated from difference code and phase observations in L3 combination. Difference between float and fix solution for L3 combination are called Narrow-Lane Hardware Delays. Similar like Ionosphere-Free combination, Melbourne-Wübbena (L6) combination includes new type of instrumental biases in GPS system. These biases are called Wide-Lane Hardware Delays (*WHD*), but sometimes they called Fractional Phase Biases as a part of ambiguity term in phase observations [7, 8]. *WHD* have got two basic types: Satellite WHD (*SWHD*) and Receiver *WHD* (*RWHD*). *SWHD* are defined as difference between float and fix ambiguity of L6 combination and should be determined for each satellite in each available time. Procedure of estimation *SWHD* and *RWHD* are based on single difference technique [1]. Single difference method enables to obtain *SWHD* or *RWHD* only in relative solution, without absolute value. Usually additional constraint should be attached, one for *SWHD* and one for *RWHD*, to determine the absolute value of *WHD*. Typically one bias of *SWHD* and *RHWD* is stable over 24 hours cycle. This approach is utilized in University of Life Sciences (UMB) in Norway. What is important, UMB provide also Narrow-Lane Hardware Delays as a common product of Uncalibrated Hardware Delays (*UHD*). *UHD* product is saved in special Bias Sinex format [12].

CNES Center Analysis proposed zero-difference method for determination *WHD* in two steps: in first process *SWHD* are estimated using data from few days from few stable receivers; in second step *RWHD* are estimated in separate process with known *SWHD* values. In this procedure more than 95% ambiguities are fixed [9, 10].

Very interesting solution of *WHD* is recommended by University of Calgary (UofC). *SWHD* can be solved using phase observations only from one receiver or global network. Daily repeatability of *SWHD* are less than 0.1 cycle for each satellite. In case of *RWHD* more than two receivers are utilized. Especially, pair of receivers should be the same type. Based on proposed solution, *RWHD* are not stable over few days [4, 5].

In this paper, zero difference approach is used to estimation *WHD* biases. GPS observations from RYKI stations were used in calculations. *WHD* were solved in temporal resolution 2 hours, using least square method. SciTEC software was used in computations. Preliminary results from SciTEC software show that *WHD* are very stable over few day. Description of the calculating method for determination *WHD* was explained in first section „Mathematical model for determination *WHD*". Second section of the paper has got information in details about studiy results of *WHD*, and last part of the paper has got some conclusions.

2. Mathematical model for determination WHD

Melbourne-Wübbena linear combination is used for determination Wide-Lane Hardware Delays. In general conception, L6 combination is free of systematic errors as ionosphere delay, troposphere delay, instrumental biases *DCB*, receiver and satellite clock error and geometrical distance satellite-receiver. Basic equations for Melbourne-Wübbena linear combination is given by [3, 6]:

$$\begin{cases} L_5 = \rho' + \alpha \dfrac{f_1}{f_2} I_1 + \beta \dfrac{f_1}{f_2} I_2 + \lambda_5 N_5 + \alpha \dfrac{f_1}{f_2} DCB + \beta \dfrac{f_1}{f_2} DCB \\ P_5 = \rho' + \delta \dfrac{f_1}{f_2} I_1 + \gamma \dfrac{f_1}{f_2} I_2 + \delta \dfrac{f_1}{f_2} DCB + \gamma \dfrac{f_1}{f_2} DCB \end{cases}, \quad (1)$$

where:

$$\begin{cases} L_5 = \alpha \cdot L_1 + \beta \cdot L_2 \\ P_5 = \delta \cdot P_1 + \gamma \cdot P_2 \\ \alpha = \dfrac{f_1}{f_1 - f_2} \\ \beta = -\dfrac{f_2}{f_1 - f_2} \\ \delta = -\dfrac{f_1}{f_1 + f_2} \\ \gamma = -\dfrac{f_2}{f_1 + f_2} \end{cases}, \quad (2)$$

L_5, P_5- L6 combination for phase and code observations,
ρ'- non dispersive term,
I_1, I_2- ionosphere delay on 1st and 2nd frequency,

$$I_1 = \frac{40.3}{f_1^2} TEC ,$$

$$I_2 = \frac{40.3}{f_2^2} TEC ,$$

TEC- Total Electron Content,
f_1,f_2- 1st and 2nd frequency in GPS system,
λ_5- wavelength in L6 combination,
N_5- ambiguity term,
DCB- instrumental biases in GPS system.

If ionosphere delays I_1 and I_2 will be expressed by *TEC* function, then:

$$\begin{cases} L_5 = \rho' + A + \lambda_5 \cdot N_5 + B \\ P_5 = \rho' + C + D \end{cases} \quad (3)$$

where:

$$\begin{cases} A = \frac{40.3TEC}{f_1 - f_2} \cdot \frac{f_1}{f_2}\left(\frac{1}{f_1} - \frac{1}{f_2}\right) \\ B = \frac{f_1 DCB}{f_1 - f_2}\left(\frac{f_1}{f_2} - 1\right) \\ C = \frac{40.3TEC}{f_1 + f_2} \cdot \frac{f_1}{f_2}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) \\ D = \frac{f_1 DCB}{f_1 + f_2}\left(\frac{f_1}{f_2} + 1\right) \end{cases} . \quad (4)$$

In next step, difference between phase and code observations for L6 combination should be written:

$$L_6 = L_5 - P_5 = \left(A - C\right) + \lambda_5 \cdot N_5 + \left(B - D\right). \quad (5)$$

Differences (*A–C*) and (*B–D*) can be solved, as follows:

$$\begin{cases} A - C = \frac{40.3TEC}{f_1 - f_2} \cdot \frac{f_1}{f_2}\left(\frac{f_2 - f_1}{f_1 \cdot f_2}\right) + \frac{40.3TEC}{f_1 + f_2} \cdot \frac{f_1}{f_2}\left(\frac{f_2 + f_1}{f_1 \cdot f_2}\right) = 0 \\ B - D = \frac{f_1 DCB}{f_1 - f_2} \cdot \left(\frac{f_1 - f_2}{f_2}\right) - \frac{f_1 DCB}{f_1 + f_2} \cdot \left(\frac{f_1 + f_2}{f_2}\right) = 0 \end{cases} .(6)$$

Based on equation (6), final equation of Melbourne-Wübbena linear combination will be given by:

$$L_6 = L_5 - P_5 = \lambda_5 \cdot N_5 . \quad (7)$$

What is important, sum of coefficients *α*, *β*, *δ*, *γ* in L6 combination should be equal to 0:

$$\alpha + \beta + \delta + \gamma = 0 , \quad (8)$$

and underlines that main target of L6 combination is obtained.

Ambiguity term in equation (7) includes very hard to determination *WHD*:

$$N_5 = N_5^0 + SWHD + RWHD , \quad (9)$$

where:

N_5^0 – fix ambiguity,
SWHD- Satellite *WHD*,
RWHD- Receiver *WHD*.

Ambiguity N_5^0 is integer value and can be expressed in cycle unit (1 cycle corresponds more than 86 cm). *SWHD* and *RHWD* are constant over measurements period. Their mean errors should be less than 1 cycle, but in short time can jump to 2 or 3 cycles.

Equation (7) represents float solution of ambiguity N_5 and it's unique value for each satellite-receiver pair from 2 hours data. In first step, fix ambiguity N_5^0 is estimated, using Kalman filter method. Ambiguity N_5^0 is very stable and corresponds for integer solution of ambiguity N_5. *WHD* are realized from difference between float and fix solution, as below:

$$N_5 - N_5^0 = SWHD + RWHD . \quad (10)$$

In second step, *WHD* are determined from equation 10 and their values are respected as a measurements noise. Least square method is proposed for solution *WHD* [11]:

$$A \cdot X + L = V , \quad (11)$$

where:

A – matrix with dimension (n, m), matrix include values 1 and 0,
X – vector with unknown biases,
L – vector of observations,
V – vector of residuals.

Matrix *A* has got rank deficient, equals one, what causes problem in determination *X* vector. To eliminate this effect, one or more constraints should be added to the matrix *A*. Author proposed to apply one constraint, that reference sum of *SWHD* is stable and equals 0:

$$\sum_{1}^{m} SWHD = 0 , \quad (12)$$

where:

m – number of unknown parameters.

Constraint from equation (12) assures that all *WHD* are stable in relation to the gravity center of *SWHD*. Moreover, this constraint enables the estimation *RWHD* as a resultant value of difference fix and float ambiguity. *SWHD* in presented approach are a systematic errors, but should be regarded as a random errors (similar as residuals).

After upper procedure, matrix *A* has got full rank and can be used in further computations. Now, *X* vector is estimated from normal equations frame, as below:

$$X = \left(A^T \cdot A\right)^{-1} \cdot A^T \cdot L . \quad (13)$$

X vector is obtained in temporal resolution 2 hours, what creates 12 observations sessions. After each sessions *WHD* are saved and arithmetical value of each biases are calculated. Weight matrix is not be used in numerical computations. Mean errors of unknown parameters are solved:

$$\begin{cases} Cx = m0^2 \cdot \left(A^T \cdot A\right)^{-1} \\ mx = \sqrt{Cx} \end{cases} , \quad (14)$$

where:

Cx – covariance matrix of mean errors,
*m*0 – standard deviation,
mx – mean errors of unknown parameters.

Computations were executed in SciTEC software, which code source was written in Scilab 5.4.1. SciTEC is open source toolbox and currently version 1.0.0 (for GPS system) is utilized in computations. SciTEC has got module *WHD*, which make possible the visualization *WHD* in graphic window and additionally print *WHD* biases in *DCB* file format. *WHD* biases is a product of Local Ionosphere Monitoring System in Ryki District (short name LIMS_RYKI). In LIMS_RYKI, *WHD* biases are solved as a optionally product of *DCB* with abbreviation ”*WHD”. Basic conception of LIMS_RYKI is estimation *TEC* parameters for determination of ionosphere state over Ryki District. Ionosphere TEC maps are available at website http://ztsdeblin.16mb.com/produkty. html and also final results of *TEC* are saved in IONEX file. What is important, Narrow-Lane

Hardware Delays can be obtained in LIMS_RYKI as a sub-product of Uncalibrated Hardware Delays.

3. Experiment and Results

Observations data from RYKI reference stations were used in experiments. RYKI station is a part of ASG-EUPOS system in Lubelskie Voivodeship area and has been installed on the roof in Ryki Prefect. Technical infrastructure of RYKI stations is created by Trimble NetRS equipment, which can received and collected only GPS observations. *C*1 and *P*2 for code observations and L1 and L2 for phase observations are included in RINEX format. Trimble NetRS is receiver of *C*1/*P*2 class, which utilizes cross-correlation technique for repair precise code P on 2nd frequency. In case of P code on 1st frequency, instrumental biases *DCB P*1*C*1 should be added to *C*1 code for creation artificial pseudocode P1, as follows:

$$P1 = C1 + DCB_{P1C1}, \qquad (15)$$

where:

DCB_{P1C1} – instrumental biases between code *P*1 and *C*1 on 1st frequency in GPS system.

Artificial code *P*1 has got accuracy of pseudorange just like code *C*1, but instrumental biases DCB_{P1C1} reduces noise on code *P*1. In practice only DCB_{P1C1} for satellite is added to *C*1 code, because DCB_{P1C1} for receiver of *C*1/*P*2 class is not available.

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GPS WHD ESTIMATION BY TST FOR DAY 320, 2014                     16-NOV-14 15:42
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WIDE HARDWARE DELAYS FOR SATELLITES AND RECEIVERS:

PRN / STATION NAME        VALUE (NS)  RMS (NS)
***   ****************    *****.***   *****.***
G01                          -0.384      1.412
G02                           1.173      1.198
G04                          -0.369      0.935
G05                          -0.587      1.169
G06                           0.043      0.951
G07                          -1.864      1.028
G08                           0.748      1.036
G09                          -0.054      0.884
G10                           1.013      1.075
G11                           0.179      1.014
G12                           0.008      1.093
G13                           0.384      0.994
G14                           1.163      1.322
G15                           0.186      1.275
G16                           0.224      1.045
G17                           0.296      1.009
G18                           2.232      1.288
G19                           0.206      0.845
G20                          -1.645      1.038
G21                           0.250      1.003
G22                           0.276      1.165
G23                          -0.750      0.879
G24                          -0.538      1.052
G25                          -0.486      1.078
G26                          -0.577      1.069
G27                          -1.009      1.102
G28                          -0.456      1.020
G29                          -0.578      1.229
G30                           0.847      0.947
G31                           0.047      1.070
G32                           0.023      1.001
G     RYKI                   -0.077      0.737
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Fig. 1. *WHD* for 320 day of 2014 year

WHD biases can be represented in text file (see Figure 1). *WHD* file, similar as *DCB* file, includes 2 part:
- header part (information about type of *WHD* estimated, in presented case *WHD* biases are estimated using GPS data),
- section with data (magnitude and mean error of *WHD* for each satellite and receiver, in presented case for RYKI station).

Figure 1 presents *WHD* biases with mean errors for 320 day of 2014 year. For *SWHD* only 31 biases are showed, without *SWHD* for SV3 satellite, and one bias for receiver. Maximum value of *SWHD* has got satellite G18 and minimum value satellite G07, respectively. Difference between both values is about 4 ns.

Accuracy of *SWHD* is less than 2 ns with minimum error for satellite G019 (e.g. 0.845 ns) and maximum error for satellite G01 (e.g. 1.412 ns). Generally, mean error of all *SWHD* is more than 1 ns (e.g. 1.072 ns). In case of *RWHD*, magnitude is less than 0.1 ns, with standard deviation less than 1 ns. More details about *WHD* characteristic over few days can be find in Figure 2 and 3 and also in Table 1.

Figure 2 presents *SWHD* biases for all satellites for 6 days of 2014 year. Magnitude order of *SWHD* are less than ± 2.5 ns (nearly to ± 0.75 m). More than 90% available satellites over 6 days have got value between ± 1.5 ns. What is important, each *SWHD* are less than 1 cycle in L6 combination. Based on Figure 2, *SWHD* biases are very stable with comparison to wavelength of L6 combination.

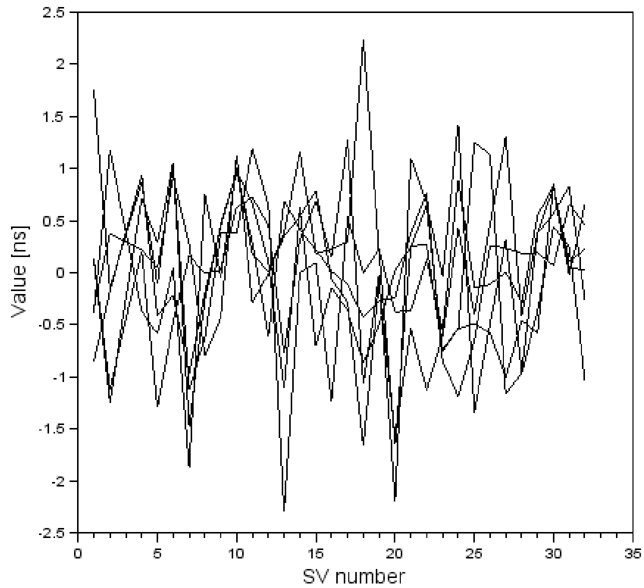

Fig. 2. *SWHD* from 320 to 325 day of 2014 year

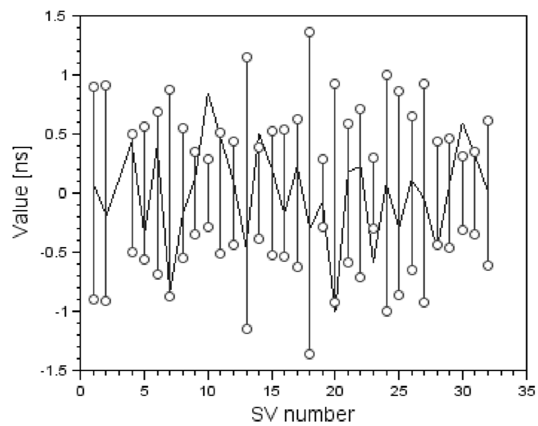

Fig. 3. Mean *SWHD* over 6 days of 2014 year

Mean values of *SWHD* over 6 days are presented in Figure 3. Magnitude order of mean *SWHD* is ± 1 ns, what corresponds to 0.3 cycle of wavelength in L6 combination. Mean *SWHD* over few days are so very stable. Maximum value of mean *SWHD* has got G10 satellite (e.g. 0.845 ns) and minimum value G20 satellite (e.g. -1.004 ns), respectively. Mean value of standard deviation equals to 0.628 ns, with maximum value for G18 satellite (e.g. 1.362 ns) and minimum for G21 satellite (e.g. 0.281 ns).

Tab. 1. RWHD from 320 to 325 day of 2014 year

Day of Year (DOY)	*RWHD*, ns	Mean error of *RWHD*, ns
320	− 0.077	±0.737
321	− 0.252	±0.778
322	− 0.111	±0.760
323	− 0.210	±0.788
324	− 0.101	±0.778
325	− 0.173	±0.778
Mean value	− 0.154	±0.770

Daily repeatability for *RWHD* is presented in Table 1. *RWHD* is very stable over few days and mean value of *RWHD* is about -0.154 ns, with mean error ± 0.770. Minimum value of *RWHD* was determinated in 320 day and maximum value in 321 day of 2014 year. Standard deviation of daily repeatability *RWHD* parameter is less than 0.07 ns, what corresponds to 2% of wavelength in L6 combination.

4. Conclusions

In this paper, new type of instrumental biases in GPS system were presented. Wide-Lane Hardware Delays, sometimes called Fractional Phase Biases, are part of float ambiguity and should be estimated in process of coordinates determination. *WHD* with Narrow-Lane Hardware Delays are formed Uncalibrated Hardware Delays. Only few organizations provide *UHD*, e. g. UMB, CNES or UofC, using another methods or techniques for solved this biases. In submitted paper, new strategy was showed for *WHD* estimated. Computations were executed in SciTEC v.1.0.0 software. Reference data was taken from RYKI station. Melbourne-Wübbena linear combination was utilized in computations as a basic mathematical model. Firstly results of *WHD* are optimistic. In this paper 4 experiments were presented. In primary experiment, example results of *WHD* were characterized. Maximum value of *SWHD* has got G18 satellite (e. g. 2.232 ns) and minimum value G07 satellite (e.g −1.864 ns), respectively. Accuracy of *SWHD* is less than 2 ns. In second experiment, *SWHD* are presented for few days. Magnitude order of *SWHD* are less than ± 2.5 ns (nearly to ± 0.75 m). More than 90% available satellites over 6 days have got value between ± 1.5 ns. In third studies, mean values of *SWHD* over few days are showed. Magnitude order of mean *SWHD* is ± 1 ns, what corresponds to 0.3 cycle of wavelength in L6 combination. Mean value of standard deviation equals to 0.628 ns after 6 days of observations. Daily repeatability of *RWHD* was determinated in the last experiment. *RWHD* is very stable over few days and mean value of *RWHD* is about −0.154 ns, with mean error ± 0.770. Standard deviation of daily repeatability *RWHD* parameter is less than 0.07 ns, what corresponds to 2% of wavelength in L6 combination.

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