

**Adam GAŁĘZIA**FACULTY OF AUTOMOTIVE AND CONSTRUCTION MACHINE ENGINEERING, WARSAW UNIVERSITY OF TECHNOLOGY  
Narbutta 84, 02-524 Warsaw**Averaged signal measures of TKEO energy waveform in detection of tooth break in gearbox**

M.Sc. Eng. Adam GAŁĘZIA

Author is a Ph.D. student at Faculty of Automotive and Construction Machine Engineering. His scientific interests are related with condition monitoring of gearboxes and bearings, application and development of condition monitoring systems. His research area also covers acoustics of vehicles and machines.



e-mail: agalezia@simr.pw.edu.pl

**Abstract**

Fatigue break of tooth is one of two main mechanisms of damaging the toothed wheels. The Teager-Kaiser energy operator (TKEO) can be used for detection of transient events such as impulses resulting from disturbances of mating teeth in a gearbox related with decrease in the stiffness of a given tooth. The paper presents an application of averaged signal measures, such as RMS, kurtosis or spectral moments, of TKEO waveform in the task of detecting the progressing fatigue break of a tooth in a gearbox.

**Keywords:** Teager-Kaiser energy operator, fatigue break of tooth, gearbox

**Uśrednione miary sygnału przebiegu energii Teagera-Kaisera w wykrywaniu wylamania zęba w przekładni zębatej****Streszczenie**

Zmęczeniowe wylamanie zęba jest jednym z dwóch podstawowych mechanizmów zniszczenia przekładni zębatej. Z reguły wylamanie rozwija się szybko i jest trudne do wykrycia. Innym głównym mechanizmem degradacji przekładni jest zniszczenie powierzchni kontaktu zębów. Operator energetyczny Teagera-Kaisera może być zastosowany do wykrywania chwilowych zaburzeń w sygnałach takich jak impulsy spowodowane zakłuceniem współpracy zębów na skutek obniżenia sztywności określonego zęba. Wejście w przypór następną parę zębów jest związane z pojawieniem się impulsu [1]. Czułość operatora energetycznego Teagera-Kaisera (TKEO) umożliwia wykrywanie chwilowych zaburzeń wcześniej niż przy wykorzystaniu metod bazujących na demodulacji sygnału metodą transformaty Hilberta. Artykuł przedstawia zastosowanie uśrednionych miar sygnałów, takich jak RMS, kurtoza czy momenty widmowe, wyliczanych dla przebiegu energii Teagera-Kaisera na potrzeby wykrywania rozwijającego się zmęczeniowego wylamania zęba. Energia Teagera-Kaisera była liczona z przebiegów czasowych sygnałów drganiowych zarejestrowanych w trakcie przyspieszonych badań zmęczeniowego wylamania zęba. Badania zostały przeprowadzone na specjalnym stanowisku do badań wytrzymałościowych zębów. W trakcie badań nie inicjowano sztucznie pęknięcia w stopie zęba.

**Słowa kluczowe:** operator energetyczny Teagera-Kaisera, zmęczeniowe wylamanie zęba, przekładnia zębata.

**1. Introduction**

Use of new, lighter materials, care for the environment and socio-economical costs of machine failures are the main reasons for constant development of machines and reliable techniques of determination of technical condition. One can list use of different physical phenomena for detection of early stages of failures [2, 3, 4], new techniques of signal processing [5, 6] or application of sensors embedded in a structure [7]. One of interesting signal processing techniques is the Teager-Kaiser energy operator (TKEO).

Formation and development of a failure in a machine or its component results in change of the machine technical condition.

This influences the energetic structure, in particular the amplitude and frequency structure, of a vibration signal generated by a working machine [8, 9]. Early detection of change in energy can allow determining the early phase of a failure and monitoring its further development. As a result, it is possible to choose optimal maintenance strategy and obtain measurable savings.

At present a number of techniques are used for detection of symptoms of change in the technical state, e.g.: empirical mode decomposition [10, 11], wavelet analysis [6], time-frequency methods [12, 13]. However, new techniques are developed and applied in the task of condition monitoring. One of them is the Teager-Kaiser energy operator applied with success to bearing fault diagnosis [14, 15, 16, 17] or detection faults of gears [18, 19].

The problem of fatigue break of a tooth is one of the basic mechanisms of damaging the toothed wheels [9, 13]. The paper presents an application of averaged signal measures calculated from the energy waveform in detection of the progressing break of a tooth in a gearbox. The waveform of energy was calculated from the raw vibration signal, recorded during accelerated fatigue brake of tooth, using the Teager-Kaiser energy operator.

This paper is based on poster presentation of research made by the author and professor Stanisław Radkowski, presented last year during The International Conference on Diagnostics of Processes and Systems in Łagów Lubuski.

**2. The Teager-Kaiser energy operator – brief information**

The Teager-Kaiser energy operator is a non-linear operator which applied to a time signal calculates the waveform of measure interpreted as the waveform of energy of this signal [20]. The operator was described and analyzed in many publications [21, 22, 23, 24]. Its version for continuous signals is defined as:

$$\Psi(x(t)) = \dot{x}^2(t) - x(t)\ddot{x}(t) \quad (1)$$

while for discrete signals it is:

$$\Psi_d(x_n) = x_n^2 - x_{n-1}x_{n+1} \quad (2)$$

In this publication the authors will constantly refer to discrete signals and TKEO of discrete signals. For simplicity  $\Psi(x)$  will be used as denotation of the result of energy operator acting on the analyzed signal and will be called the measure of energy of a signal.

The Teager-Kaiser energy operator acting on a time signal calculates the waveform of the measure of energy of the analyzed signal. As a result, new time signal  $\Psi(x)$  is obtained. As pointed out in [20, 22, 23, 24],  $\Psi(x)$  can have negative values and its result can differ depending on the parameters of the analyzed signal, such as the depth of modulations, the sampling frequency, the influence of noise.

An interesting property of energy operator (2) is its sensitivity to sudden changes in the analyzed signal, for example in the form of a transient disturbance of the waveform.

**3. Description of experiment**

The fatigue break of a tooth is an important research problem in condition monitoring of gearboxes. To investigate this problem, tests of accelerated fatigue break of tooth were performed on the

back-to-back tester (Fig. 1). On this test stand, electric engine (1), via clutch (2), puts in motion shaft (4) coupling pinions of gearboxes. Shaft (8) was made as one part. Shaft (4) consists of two parts allowing introducing load to tested gearbox (7) using tightening clutch (5). In tested gearbox (7) spur gears were used (6). The gears were made from carburized steel 20H2N4A hardened up to 60 HRC. In closing gearbox (3) helical gears were used. During tests, the tested gearbox was damaged.

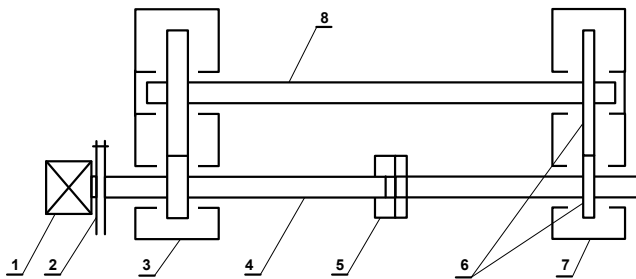


Fig. 1. Schematic diagram of the test stand [25]  
Rys. 1. Schemat stanowiska badawczego [25]

In the examined samples no artificial crack was introduced. The accelerated fatigue break of tooth was performed until complete break of tooth (Fig. 2).

The back-to-back tester has the following parameters [26]:

- maximum tightening moment 1500 Nm;
- engine rotation speed 1500 rpm;
- gear ratio of in both gearboxes 1,296;
- module pitch of gears in tested gearbox 4mm;
- number of teeth of pinion in tested gearbox  $z_1 = 27$ ;
- number of teeth of wheel in tested gearbox  $z_2 = 35$ ;
- distance of wheels axes 125 mm.

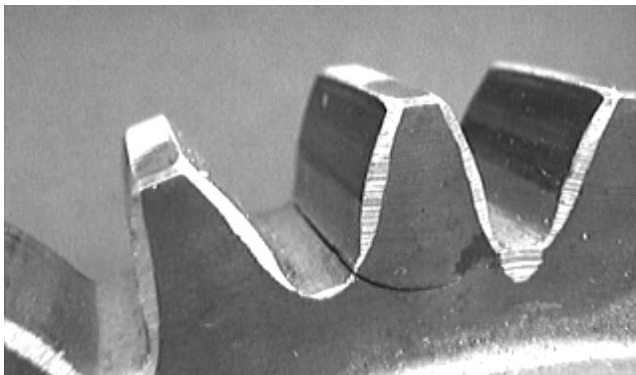


Fig. 2. Broken tooth – effect of tests [27]  
Rys. 2. Wylamany ząb – efekt badań [27]

The vibration signals were recorded using a Bruel&Kjaer 4504A vibration sensor mounted on the housing of the gearbox with sampling frequency 20000 Hz. The trigger signal was recorded for information regarding the rotation speed. During the presented measurement, the waveforms were recorded continuously. The total time of the experiment was 71 minutes and 18 seconds. For the purpose of analysis the complete recording was divided into 6-second segments (files) of the signal. Each segment was immediate continuation of the preceding segment with no loss of data.

The vibration signal measured in the vertical direction was chosen for further analysis, as more informative, because this direction was more parallel to the line of contact.

#### 4. Analysis of vibration signal from fatigue tooth break test

The TKEO measure of energy of the vibration signal from a single segment was calculated. Next, the averaged signal measures, such as RMS, kurtosis or spectral moments were calculated from the waveform of the measure of energy. The change in their values was analyzed.

Unfortunately, the averaged signal measures, calculated from the waveforms of the TKEO measure of energy obtained for raw vibration signals, revealed no unique changes or their changes appeared just before the brake of the tooth. This was related with the wide band of the analyzed signal. Also the vibration difference signal [28] appeared to be insufficient for early detection of the fatigue break of tooth. As a result, attention was focused on the measure of energy calculated from the band-pass filtered signals.

Next, the analysis of the vibration signal from the fatigue tooth break test were performed in 2 stages: first the analysis was performed for narrow-band signals filtered around the meshing frequencies (e.g.: 640-700, 1310-1370, 1980-2040 etc.), next the narrow-band signals filtered between the meshing frequencies were analyzed (e.g.: 0-640, 700-1310, 1370-1980 etc.).

The raw vibration signal, from each 6-second segment, was band-pass filtered in aim to extract different bands of signals. Next, from each such filtered signal, the TKEO measure of energy was calculated giving the waveform of  $\Psi(x)$ . The waveforms of  $\Psi(x)$  were analyzed using time-domain and spectrum-domain approaches. As a result, from each segment, the number of 6-seconds  $\Psi(x)$  waveforms was achieved, each for different band of filtration of the vibration signal. Single values of parameters, such as RMS, kurtosis, spectral moment, were calculated from each  $\Psi(x)$ . Figure 3 presents the flow diagram of the performed analysis.

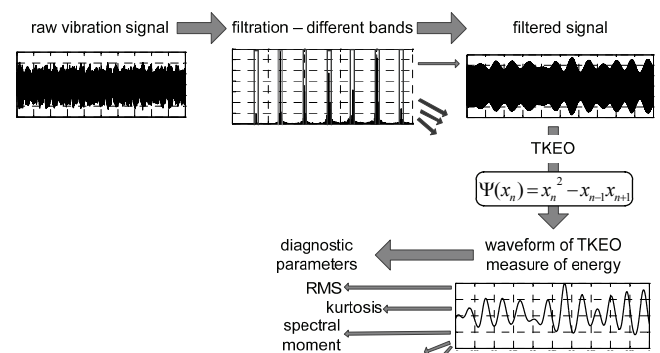


Fig. 3. Flow diagram of the performed analysis  
Rys. 3. Schemat wykonanych analiz

It shows the filtration of the raw vibration signal around the meshing frequencies, which was the first stage of the performed analysis. Later, the filtration was made between the meshing frequencies.

During the first stage of analysis, the vibration signal was band-pass filtered around first 6 harmonics of meshing frequency. The bands of filtration were 60 Hz wide and symmetric around the nominal basic value of the harmonic of meshing frequency, equal to 670 Hz.

Figure 4 presents a part of the vibration signal filtered in the band 1310 – 1370 Hz, i.e. around the second harmonic of meshing frequency (top figure) and the corresponding waveform of the measure of energy (bottom figure).

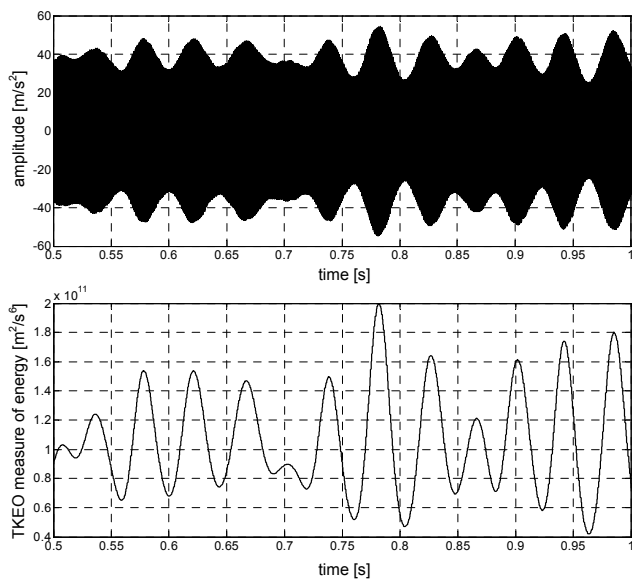


Fig. 4. Filtered vibration signal and corresponding waveform of TKEO measure of energy

Rys. 4. Przefiltrowany sygnał i odpowiadający mu przebieg miary energii Teagera-Kaisera

Only last 213 segments were chosen for the detailed analysis aiming at detecting the development of the fatigue break of tooth. The vibration signal, from each of these segments, was band-pass filtered in the defined bands. As a result, a set of waveforms was obtained. Next, from each filtered signal, the waveform of  $\Psi(x)$  was calculated. From each  $\Psi(x)$  the time-domain and spectrum-domain parameters were calculated, such as: mean value, RMS, variance, kurtosis, skewness, crest factor, spectral moments of order 0 and 1, coefficient of width of band [9].

Figure 5 shows the change in the mean value of the waveform of  $\Psi(x)$ , calculated for different bands of filtration of the vibration signal. The values of the parameter change in a wide range. For most bands there is no significant change that could testify the development of crack in the base of tooth.

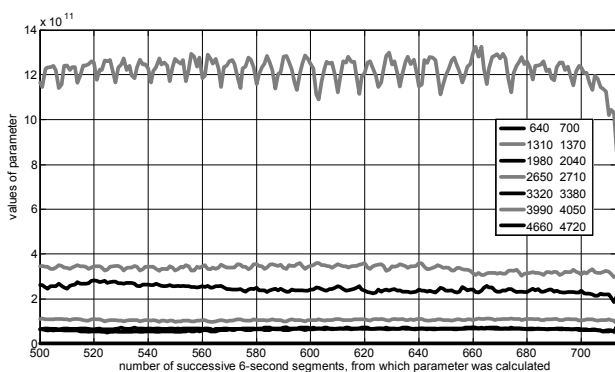


Fig. 5. Change of the mean value of  $\Psi(x)$ , from vibration signals filtered in different bands

Rys. 5. Zmiany wartości średniej  $\Psi(x)$ , dla przebiegów z różnych pasm filtracji

The analysis of other averaged signal measures of measure of energy reveal that information about the developing break is hidden only in some bands and does not manifest itself in a clear way. Also the results of the analysis of the spectral parameters of spectrum of measure of energy, calculated based on band-passed around the harmonics of mesh frequency of the filtered vibration signals, were unsatisfactory. None of parameters reveal unique tendency of change resulting from the growth of fatigue crack.

Searching for the diagnostic information, attention was paid to bands between the harmonics of mesh frequency of the raw vibration signal (2<sup>nd</sup> stage of analysis). Filtration bands of the vibration signal were chosen in the following way: the first band covered the range from the lowest frequency (constant component was not analyzed) to the frequency equal to 30 Hz lower than the nominal mesh frequency. Next bands covered the frequency range from the frequency higher by 30 Hz than the harmonic of mesh frequency to the frequency lower by 30 Hz than the next harmonic of mesh frequency. From so filtered signals the TKEO measure of energy was calculated and analyzed. The mentioned above, time-domain and spectral-domain parameters were calculated from the successive waveforms of measure of energy.

Figure 6 presents the changes in the RMS value calculated from  $\Psi(x)$  for different bands of filtration of the vibration signal. For some bands, such as 2710 – 3320 Hz or 3380 – 3990 Hz, the values of the parameter have clear increasing tendency testifying the development of fatigue breaking of tooth. For those bands behavior of other parameters such as kurtosis or skewness reveal similar character of changes.

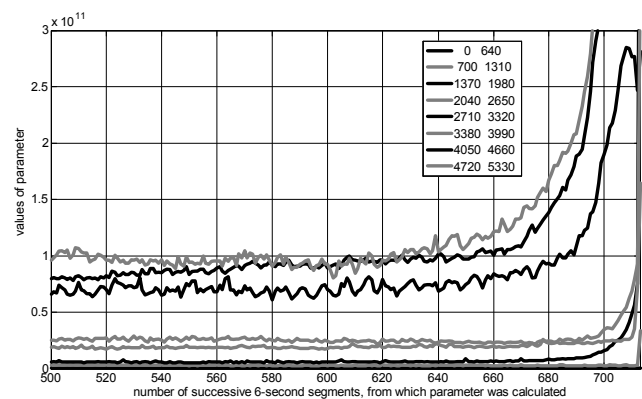


Fig. 6. RMS value of waveform of  $\Psi(x)$ , from vibration signals filtered in bands between harmonics of mesh frequency

Rys. 6. Wartości RMS dla przebiegów  $\Psi(x)$  wyliczonych z sygnałów przefiltrowanych w pasmach między harmonicznymi zazębienia

Also the spectral parameters calculated from  $\Psi(x)$  waveforms, of mentioned above bands of the vibration signal, can be used in inference on developing break of tooth. For the bands of filtration of the vibration signal: 2710 – 3320 Hz and 3380 – 3990 Hz, majority of parameters change their values in a unique way (increase or decrease in values) with development of crack. Figure 7 shows the change in the value of the coefficient of band width.

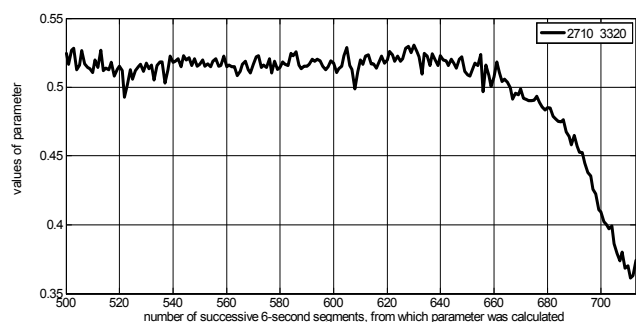


Fig. 7. Coefficient of the band width of  $\Psi(x)$  spectrum calculated for a given filtration band

Rys. 7. Współczynnik szerokości pasma widma  $\Psi(x)$  wyliczony dla danego pasma filtracji

A unique change in the parameters can be observed at least from 660<sup>th</sup> segment, i.e. for more than 5 minutes before the complete break off of tooth. Similar changes can be observed for other spectral parameters.

## 5. Conclusions

The Teager-Kaiser energy operator calculates the measure of energy of signal  $\Psi(x)$  whose changes reflect the changes occurring in the analyzed signal. The analyzed signal should be processed in order to reduce noise. The interesting results are obtained if the vibration signal, from which the TKEO measure of energy is calculated, is band-pass filtered. Due to ease of calculations and sensitivity to transient events, the waveform of  $\Psi(x)$  can be used as diagnostically useful information. The measure of energy of band-pass filtered signals reveals unique information on development of crack caused by loads during the accelerated fatigue test.

## 6. References

- [1] Mączak J.: Local meshing plane analysis as a source of information about the gear quality, *Mech. Syst. Signal Process.* (2012), <http://dx.doi.org/10.1016/j.ymssp.2012.09.012>
- [2] Gałęzia A.: Utilization of components of signals from high frequency range in condition monitoring of bearings, *Diagnostyka*, nr. 3 (55), pp. 35-44, ISSN 1641-6414, 2010.
- [3] Gontarz S., Radkowski S.: Impact of different factors on relationship between stress and eigenmagnetic field in steel specimen. *IEEE Transactions on Magnetics* Vol. 48, no. 3, pp. 1143-1154, (2012), DOI: 10.1109/TMAG.2011.2170845
- [4] Radkowski S., Gumiński R.: Impact of vibroacoustic diagnostics on certainty of reliability assessment, *Engineering Asset Lifecycle Management*, 2010, Part 16, (2010), pp. 574-582, DOI: 10.1007/978-0-85729-320-6\_66.
- [5] Dybała J.: Vibrodiagnostics of gearboxes using NBV-based classifier: A pattern recognition approach, *Mech. Syst. Signal Process.* (2012), <http://dx.doi.org/10.1016/j.ymssp.2012.08.021>
- [6] Delvecchio S.: On the Use of Wavelet Transform for Practical Condition Monitoring Issues, (w: Baleanu D. (Ed.), *Advances in Wavelet Theory and Their Applications in Engineering. Physics and Technology*, InTech), (2012), DOI: 10.5772/35964.
- [7] Orłowska A., Kolakowski P., Holnicki-Szulc J.: Detecting delamination zones in composites by embedded electrical grid and thermographic methods, *Smart Materials and Structures*, vol. 20, no. 10. (2011), DOI: 10.1088/0964-1726/20/10/105009.
- [8] Cempel C., Żółtowski B.: *Engineering of machine diagnostics (Inżynieria Diagnostyki Maszyn)*, publication in Polish, PTDT/ITE, 2004.
- [9] Radkowski S.: Non-linearity and intermodulation phenomena tracking as a method for detecting early stages of gear failures. *INSIGHT*, Vol. 50, Issue 8, pp.: 419-422, DOI: 10.1784/insi.2008.50.8.419, 2008.
- [10] Dybała J., Zimroz R.: Application of Empirical Mode Decomposition for impulsive signal extraction to detect bearing damage – industrial case study, (w: Fakhfakh T. et al. (Eds.) *Condition Monitoring of Machinery in Non-Stationary Operations*, Part 3, Springer, pp. 257–266), 2012, DOI: 10.1007/978-3-642-28768-8\_27.
- [11] Huang N. E., Shen Z., Long S. R., Wu M. C., Shih H. H., Zheng Q., Yen N. C., Tung C. C., Liu H. H.: The Empirical Mode Decomposition and the Hilbert Spectrum for Nonlinear and Nonstationary Time Series Analysis, *Proceedings of the Royal Society of London A* 454, (1998), str.: 903–995, doi:10.1098/rspa.1998.0193.
- [12] Brandt A.: *Noise and Vibration Analysis: signal analysis and experimental procedures*, John Wiley & Sons, (2011).
- [13] Randall B.: *Vibration-based Condition Monitoring: Industrial, Aerospace and Automotive Applications*, John Wiley & Sons, (2011).
- [14] Li H., Fu L., Zhang Y.: Bearing Faults Diagnosis Based on Teager Energy Operator Demodulation Technique, In proceeding of: *Measuring Technology and Mechatronics Automation*, 2009. ICMTMA '09. International Conference on, Volume: 1, (2009) DOI:10.1109/ICMTMA.2009.421.
- [15] Liang M., Soltani B. I.: An energy operator approach to joint application of amplitude and frequency-demodulation for bearing fault detection, *Mechanical Systems and Signal Processing*, Vol. 24, (2010), pp. 1473-1494.
- [16] Feng Z., Wang T., Zuo M. J., Chu F., Yan S.: Teager Energy Spectrum for Fault Diagnosis of Rolling Element Bearings, *J. Phys.: Conf. Ser.* 305 012129, (2011), doi:10.1088/1742-6596/305/1/012129.
- [17] Henriquez P., White P. R., Alonso J. B., Ferrer M. A., Travieso C. M.: Application of Teager-Kaiser Energy Operator to the Analysis of Degradation of a Helicopter Input Pinion Bearing, (access: 2012-11-28, available at: [http://jean.fabri.perso.sfr.fr/sfm/papiers\\_surveillance/8\\_Henriquez.pdf](http://jean.fabri.perso.sfr.fr/sfm/papiers_surveillance/8_Henriquez.pdf)).
- [18] Bozchalooi S. I., Liang M.: Teager energy operator for multi-modulation extraction and its application for gearbox fault detection, *Smart Materials and Structures*, Vol. 19, Number 7, 075008, (2010), doi:10.1088/0964-1726/19/7/075008.
- [19] Li H., Zheng H., and Tang L.: Gear Fault Detection Based on Teager-Huang Transform, *International Journal of Rotating Machinery*, vol. 2010, Article ID 502064, 9 pages, 2010. doi:10.1155/2010/502064.
- [20] Kaiser J. F.: On a simple algorithm to calculate the 'energy' of a signal, *Proc IEEE ICASSP-90*, Albuquerque, NM, April 1990, pp. 381-384, 1990.
- [21] Kaiser J. F.: On Teager's Energy Algorithm and its generalization to continuous signals" a simple algorithm to calculate the 'energy' of a signal, *Proc IEEE Digital Signal Processing Workshop*, New Paltz, NY, September 1990, 1990.
- [22] Kvedalen E.: *Signal processing using the Teager Energy Operator and other nonlinear operators*, Master of Science Thesis, University of Oslo, Department of Informatics, 2003.
- [23] Maragos P., Kaiser J. F., Quatieri T. F.: On amplitude and Frequency Demodulation Using Energy Operators, *IEEE Transactions on Signal Processing*, vol. 41, pp. 1532-1550, 1992.
- [24] Maragos P., Potamianos A.: Higher order differential energy operators, *IEEE Signal Processing Letters*, Vol. 2, Issue: 8, pp. 152 – 154, 1995.
- [25] Dybała J.: Detection of early phases of failures with means of artificial intelligence. (Wykrywanie wczesnych faz uszkodzeń metodami sztucznej inteligencji, publication in Polish) *Wydawnictwo Naukowe Instytutu Technologii Eksploatacji, Warszawa–Radom*, (2008).
- [26] Gumiński R.: Use of diagnostic information in analysis of technical risk (Wykorzystanie informacji diagnostycznej w analizie ryzyka technicznego, publication in Polish), Doctor thesis, Warsaw University of Technology, The Faculty of Automotive and Construction Machinery Engineering, Warsaw, (2010).
- [27] Radkowski S.: *Vibroacoustic diagnostics of low-energy failures (Diagnostyka wibroakustyczna uszkodzeń niskoenergetycznych)*, publication in Polish, ITE, Radom, (2002).
- [28] Stewart R.M.: *Some Useful Data Analysis Techniques for Gearbox Diagnostics*, Report MHM/R/10/77, Machine Health Monitoring Group, Institute of Sound and Vibration Research, University of Southampton, 1977.

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