*model, dynamic system, shaft, elastic deformable condition, equation, operational transmittance, regulator*

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# MATHEMATICAL MODEL OF IDENTIFICATION AND AUTOMATION THE PROCESS OF SHAFT MACHINING IN ELASTIC-DEFORMABLE CONDITION

**Abstract**

*The specification of the low rigidity parts machining process is consider by introducing suitable equations of constrains, which describe additional elastic deformations in one of equations describing the force controlling influence. This paper introduces general and detailed mathematical models of the DS of turning the longitudinal, low rigidity shafts. Research results and examples of their approximation were introduced. Some method of synthesis and examples structure of regulator P were shown for one detailed model of DS under received approximated operational transmittance of DS. The way of controlling the accuracy of shafts turning in the elastic deformable condition and controlling system was described.*

## 1. INTRODUCTION

Continued efforts aimed at obtaining high-quality machining on machine tools under conditions of various interferences affecting the technological system (TS) have led to the application of adaptive control (AC) systems in the machine-building industry [1, 2]. The problem of improvement of such systems is particularly relevant under ESP conditions, in the realization of so-called "no-man" technology. Development of a mathematical model (MM) of control object (CO) in the dynamics, adequate to the original object, is a prerequisite for substantiated approach to the solution of the problem of analysis of stability

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of automatic control systems (ACS) or AC and synthesis of correcting elements, in accordance with required quality indices of transition process control. Whereas, in similar systems, indexes of quality of control of the input variable – elastic deformation in the dynamic TS - characterize directly the errors of shape of the machined parts, determined by the effect of rapidly changing interferences of the type of change in material allowance for machining or variability of the physicochemical properties of the machined material.

The dynamic system (DS) of the process of machining is a technological system –i.e. a machine tool together with the realized technological process (TP) of machining (turning, grinding, drilling, milling) [3, 5].

In the identification of DS the systemic approach includes the following fundamental stages [5]:

- analysis of input data for the identification;
- formulation of control strategy oriented at a specific subsystem of basic machine tools, in accordance with input data in designing ACS, AC;
- exclusion of invariant, relative to their spectrum, input effects of subsystems and components within the limits of technical capability of ACS, AC and the machine tools;
- analysis of possible structures of *MM* of control system with respect to their function, types of components and connections between them, number of levels of hierarchy, principles of connection, and permanence of the connections.

With a lack of sufficiently complete and detailed information on the object of control, calculated characteristics may significantly differ from the true ones. The parameters (settings) of regulators adopted in designing do not guarantee the required quality of control, or even stability of the system. Apart from this, the analysed systems are characterized by extensive variability of parameters of the CO. Those determinations indicate the complexity of the problem of ensuring stability of the ACS and the necessity of taking special care in the approach to the problem of defining its structure and synthesis of the corrective devices.

## 2. IDENTIFICATION OF DYNAMIC SYSTEMS OF SHAFT TURNING

In the case when there is complete information on the object of control it is possible to design a model using the analytical method. Such a procedure, leading to the identification of the structure and parameters of a model, is referred to as the analytical identification. For complex systems, development of MM with the analytical method frequently requires additional experimental tests aimed at the verification of theoretical results and at determination of some of the model parameters.

The presented schematic of the structure of MM shows that the basic scope of work in the design of MM is based on in-depth theoretical analysis of connections between the variable parameters and on revealing the relationships describing the processes taking place within the object.

The possibility of linearization of equations of motion of the particular components of the DS follows also from the commonly accepted view that assurance of high requirements with respect to precision of adjustment is reduced to realization of adjustment systems operating at “small” deviations of variables. Therefore, the dynamic system of the process of drilling can be considered as multi-dimensional CO with subsystems in the form of the technological process and an elastic system. The structure of the CO includes circuits of feedbacks from the elastic system caused by force effects that appear in the course of realization of the technological process.

References [4, 5, 6] present a system of equations and a generalized structural schematic of MM of the dynamic system of shaft turning. The developed system of equations and the structural schematic of MM take into account the geometry of the machined layer and of the machining force in turning, elastic properties of the TS, process of forming of cross-section of the machined layer (ML). The process of forming of cross-section of the ML takes into account the phenomenon of machining “following the feed ridge” which consists in that the components of the machined layer of the material at the current moment are defined by the temporary position of the cutting edge and by its coordinates at the moment of the preceding revolution of the semi-finished product, i.e. at a time-lag of a single revolution. At the same time the effect of elastic deformation for coordinate Z on the depth of turning is taken into account.

The process of forming of the cross-section of ML is under strong effect of the phenomenon of machining “following the feed ridge” and by elastic deformations in the DS. The process of forming of ML cross-section can be described with a system of integral-differential equations with delayed argument. Variables characterizing the ML cross-section depend on the input variables and on the elastic deformation in the DS. In the vector of the technological variables, formed by the dynamic system, two components can be distinguished – one defined by the vector of input effects and the other by the vector of elastic deformations.

Elements of the vector of input values are the control values in the form the straight feed rate, rotational speed of the machined part, and also interference in the form of changes in the hardness of the machined material and in the machining allowance relative to the length and diameter of the machined part.

The vector of elastic deformations is determined by the vectors of machining forces and of control values entering the system of vibrational stability assurance. Dynamic properties of the equivalent elastic system can be approximated with quadratic equations [7]. The choice of the vector of technological variables is significantly affected by the phenomenon of machining “following the feed ridge”, manifest in that the momentary values of the components of the said vector are determined by the values of elements of the input vector and of the vector of elastic deformations not only at the current moment but also at the time of the preceding revolution of the machined part. Due to this the dynamic system is described with a system of integral-differential equations with variable delayed argument.

As a result of analysis of the processes occurring in the dynamic system of machining a system of equations and functions of transition were obtained, as well as the generalized structure of the control object.

### 2.1. Identification of turning of low-rigidity shafts

To improve the precision of machining of shafts with low rigidity, technological methods were developed for the control of machining precision, based on change in the elastic-deformable condition [8, 9]. As control effects, in accordance with the developed classification [9], particular force control effects are employed, or their combinations – axial and eccentric tension, control by means of additional force effects aimed at compensation of force factors from the machining process, bending moments at supports, control of force-induced bending-torsional strain.

MM of various technological systems of machining with control of the elastic-deformable status for stabilised parameters, presented in the form of deflection functions, were obtained with the assumption that a banding force acting on the machined part is an external variable that is independent of the elastic deformations in the DS. This approach is based on not including the closing of the elastic system through the process of machining and does not introduce new errors into results of analyses of static characteristics of the CO. Analysis of the structure of a suitable MM of a control object for transition parameters is not possible without taking into consideration the specifics of processes within the machining zone and the closing of the DS through the process of machining.

MM of the considered control object – DS with control of the elastic-deformable status of parts with low rigidity was constructed on the basis of general principles of creating MM of DS [4, 5, 6] of machining, with the specifics of the process of machining of parts with low rigidity being accounted for by the introduction of suitable equations of constraints [5, 11, 12], reflecting mutual relationships between additional elastic deformations $\Delta g_{\xi}$, into one of the equations representing the force control effects of the system of equations.

Equivalent elastic deformations of the *TS* in the machining of parts with low rigidity can be represented in the form of two components:

$$g_\zeta = g_{\zeta obr.} + g_{\zeta cz.} \ , \tag{1}$$

where: $g_{\zeta obr.}$ and $g_{\zeta cz.}$ – elastic deformations of the machine tool, fixture, tool and part for each coordinate, respectively; $\zeta \in \{x, y, z\}$.

The first component in this expression for the TS under consideration is, in principle, lower by one order of magnitude and can be neglected.

Elastic deformations of the TS in the radial direction $g_y$ in accordance with the deflection equations [8], at set parameters without the inclusion of closed status of the CO, may be considered as a deterministic non-linear function of the part parameters $L, d, EI$; components of the machining force $F_c, F_p, F_f$; coordinates $x$ of machining force application on the length of the semi-finished product and various regulatory effects in the form of: tensile force $F_{x1}$; eccentric tensile force creating two regulatory effects $F_{x1}$ and moment $M = F_{x1} \cdot e$, where $e$ - eccentric of the tensile forces; one or more additional forces $F_{dod.i}$; bending moments $M_i$; torsional moment $M_{skr}$ or their combinations:

$$g = f(L, d, EI, F_c, F_p, F_f, F_{x1}, e, F_{dod.i}, M_i, M_{skr}, x) \ . \tag{2}$$

Assuming that the true feed rate and the rate of change of coordinate $x$ are relatively small, in the analysis of transition processes the change in coordinate $x$ in the function of time can be left out. Therefore, relation (2) in the operator form can be written as:

$$\begin{aligned} g_y(s) &= K_{xy} \cdot F_f(s) + K_{yy} \cdot F_p(s) + K_{zy} \cdot F_c(s) + K_{F_{x1}} \cdot F_{x1}(s) + \\ &+ K_e \cdot e(s) + K_{F_{dod.i}} \cdot F_{dod.i}(s) + K_{M_i} \cdot M_i(s) + K_{M_{skr}} \cdot M_{skr}(s) \end{aligned} \ , \tag{3}$$

where: dual indexes at coefficients $K$ mean that coefficients $K_{xy}, K_{zy}$ indicate the effect of increase in the values of components $F_f, F_c$ on increase in the level of elastic strain on coordinate $y$; $K_e = K_e^{'} \cdot F_{x1_0}$.

The gain coefficients of linear equations are defined as fragmentary derivatives of the strain function along the respective coordinate. For example, for the TS of machining with the effect of axial tensile force $F_{x1}$, causing the elastic-deformable condition, from the system of elastic deformations we obtain [5, 11]:

$$K_{yy} = \left( \frac{\partial g_y}{\partial F_p} \right)_0 = \frac{L^3 \cdot [1 - \cos(2\pi x_0 / L)]^2}{2\pi^2 \cdot (4\pi^2 \cdot EI + F_{x1_0} \cdot L^2)} \ , \tag{4}$$

$$K_{F_{x1}} = \left(\frac{\partial g_y}{\partial F_{x1}}\right)_0 = -\frac{F_p \cdot L^5 [1-\cos(2\pi x_0 / L)]^2}{2\pi^2 \cdot (4\pi^2 \cdot EI + F_{x1} \cdot L^2)} = -\frac{g_{y_0} \cdot L^2}{4\pi^2 \cdot EI + F_{x1} \cdot L^2} \ , \qquad (5)$$

where: $F_{x1_0}, g_{y_0}$ – values of tensile force and elastic strain of the part along coordinate $y$ at the point of linearization (values of variables relative to which increases of variables are given).

In the special case under consideration the remaining coefficients in relation (3) are equal to zero. Coefficients of gain, corresponding to different DS at various methods of loading (i.e. with axial-radial bending and various methods of fixing) in machining of elastic-deformable parts, obtained in an analogous manner, are presented in [5, 11] – $x_0$ - coordinate of cutting edge position on machining length at the point of linearization [5, 11]. The additional elastic strains $g_x, g_z$ with respect to coordinates $x$ and $z$, as a result of the action of the control force effects under consideration, basically do not have any significant effect on the dynamic properties of the CO and can be treated as negligible.

In accordance with the result of studies in ref. [13], the components of machining force without inclusion of the contact strain at the surface of application are written as:

$$F_c = Q_{pw} \cdot a \cdot b \ , \quad F_p = Q_{pw} \cdot a \cdot b \cdot K'_y \ , \quad F_f = Q_{pw} \cdot a \cdot b \cdot K'_x \ ,$$

where: $Q_{pw}$ – relative work of formation of shaving,

$K'_y, K'_x$ – constant coefficients for given conditions of machining.

Hence

$$m_z = \left(\frac{\partial F_c}{\partial a}\right)_0 = Q_{pw_0} \cdot b_0 \cdot K_z \ , \ m_y = \left(\frac{\partial F_p}{\partial a}\right)_0 = Q_{pw_0} \cdot b_0 \cdot K_y \ ,$$

$$m_x = \left(\frac{\partial F_f}{\partial a}\right)_0 = Q_{pw_0} \cdot b_0 \cdot K_x \ \ n_z = \left(\frac{\partial F_c}{\partial b}\right)_0 = Q_{pw_0} \cdot a_0 \cdot K_z \ ,$$

$$n_y = \left(\frac{\partial F_p}{\partial b}\right)_0 = Q_{pw_0} \cdot a_0 \cdot K_y \ , \ n_x = \left(\frac{\partial F_f}{\partial b}\right)_0 = Q_{pw_0} \cdot a_0 \cdot K_x$$

and

$$n_y m_x = Q_{pw_0} a_0 K_y Q_{pw_0} b_0 K_x \ , \quad m_z n_x = Q_{pw_0} b_0 K_z Q_{pw_0} a_0 K_x \ ,$$

$$m_y n_x = Q_{pw_0} b_0 K_y Q_{pw_0} a_0 K_x \ , \quad n_z m_x = Q_{pw_0} a_0 K_z Q_{pw_0} b_0 K_x \ ,$$

$$n_y m_x = m_y n_x \ , \quad m_z n_x = n_z m_x \ .$$

The relations given above permit simple transformations of coefficients $A$ and $B$ included in corresponding operator transmittances (OT) of the CO with relation to various control and interfering effects.

In referenced works [5, 11, 12] the authors analysed the possibility of replacing the obtained relations of OT with approximated ones, application of which significantly simplifies calculation of characteristics of DS MM. The analysis was made according to the criterion of recreation of true characteristics of MM with approximated relations in the time and frequency planes; it was demonstrated that the form of approximating relations should be chosen taking into account the numerical value of coefficient $B$. It was also determined that the value of $B = 0{,}1$ is the "limit" at which the switch from one form of approximating relation to another is justified. The value of coefficient $B$ is defined as the ratio of rigidity of equivalent elastic system to gain coefficients of the process of machining and can be adopted as an index of relative rigidity of DS. Broad ranges of variability of machining parameters on machine tools, e.g. of change in the hardness of the machined material, machining allowance, cutting edge geometry, determine broad ranges of variability of coefficients $m_x, m_y, K_{\kappa_r}, K_x, K_{yy}$ and $B$, respectively.

Calculations show that in machining of low-rigidity shafts and in roughing and profiling of parts with normal rigidity the values of coefficient $B$ are notably greater than the limit value of $B = 0{,}1$; in this case also the approximating relations for OT according to (11), (14), (15) should be built by splitting the exponential function $e^{-s\tau}$ into a Pade series which, keeping the first two components, may be written as:

$$e^{-s\tau} = \left(1 - \frac{1}{2} s \cdot \tau + \frac{1}{12} s^2 \cdot \tau^2\right) / \left(1 + \frac{1}{2} s \cdot \tau + \frac{1}{12} s^2 \cdot \tau^2\right) \ . \tag{6}$$

In the case of control of the elastic-deformable condition of parts with low rigidity through the application of tensile force $F_{x1}$ the structure of CO has been developed in [5, 11].

On the basis of the schematic given in [5, 11], after transformation, the relation for OT of the dynamic system when increase in elastic deformations $g_y$ in the radial direction is adopted as the initial variable is reduced to the form of:

$$G_{F_{x1}}(s) = \frac{g_y(s)}{F_{x1}(s)} = K_0 \cdot \frac{1 + A' \cdot (1 - e^{-s\tau})}{1 + B' \cdot (1 - e^{-s\tau})} \;, \tag{7}$$

where: $$K_0 = K_{F_{x1}} \cdot \frac{1}{1 + K_{yy} \cdot n_y + K_{xy} \cdot n_x + K_{bz} \cdot K_z \cdot n_z} \;, \tag{8}$$

$$A' = m_x \cdot K_x + K_{\kappa_r} \cdot m_y \cdot K_y \;, \tag{9}$$

$$B' = \frac{m_x \cdot K_x + K_{\kappa_r} \cdot m_y \cdot K_{yy} \left[ 2 + K_{yy} \cdot n_y + K_{bz} \cdot n_z + K_{xy} \cdot m_x / (K_{yy} \cdot m_y) + K_{bz} \cdot K_z \cdot m_z / (K_{yy} \cdot m_y) \right]}{1 + K_{yy} \cdot n_y + K_{xy} \cdot n_x + K_{bz} \cdot K_z \cdot n_z} . \tag{10}$$

For known values of coefficients included in relations (7) – (10), the relations can be notably simplified. Calculations show that in machining of parts with low rigidity with application of force effects components containing $K_{bz}$ and $K_{xy}$ can be basically left out. In such a situation, the relation for $B'$ gets considerably simplified, and the expression for coefficients $K_0$ is notably reduced. Denominator of OT of operator transmittance for DS determined from the relation in control of straight feed [4, 6] is reduced, as shown above, to the form of denominator of aperiodic component of the first or second order. To transform the numerator of OT to a typical form one can also employ splitting the function $e^{-s\tau}$ into a Pade series, and then the analysed OT will assume the form of:

$$G_{F_{x1}}(s) = K_0 \cdot \frac{T_3^2 \cdot s^2 + T_3' \cdot s + 1}{(T_1 \cdot s + 1) \cdot (T_2 \cdot s + 1)} \;, \tag{11}$$

The time constants $T_1$ and $T_2$ are determined from the relation:

$$T_{1,2} = 0{,}5\tau \cdot \left[ 0{,}5 + B \pm \sqrt{(0{,}5 + B)^2 - 1/3} \right] \tag{12}$$

by substituting in it $B'$ to replace $B$, and the time constants in the numerator are then equal to:

$$T_3 = 0{,}289\tau; T_3' = (0{,}5 + A') \cdot \tau \quad . \tag{13}$$

**2.2. Simplification of mm of dynamic system of shaft turning in the elastic-deformable condition**

Further transformations of the numerator of OT (7) should be made with the inclusion of time constants $T_3$ and $T_3'$ which depend on $A'$. If $A' < 0{,}077$, then the OT of UD can be written in the following typical form:

$$G_{F_{x1}}(s) = \frac{g_y(s)}{F_{x1}(s)} = K_0 \cdot \frac{T_3^2 \cdot s^2 + 2\varepsilon \cdot T_3 \cdot s + 1}{(T_1 \cdot s + 1) \cdot (T_2 \cdot s + 1)} \quad , \tag{14}$$

where: $\varepsilon$ – coefficient of attenuation

$$\varepsilon = \frac{0{,}5 + A'}{0{,}577} \quad . \tag{15}$$

In the case when $A' \geq 0{,}078$, the approximating relation for the analysed OT assumes the form of:

$$G_{F_{x1}} = \frac{g_y(s)}{F_{x1}(s)} = K_0 \cdot \frac{(T_4 \cdot s + 1) \cdot (T_5 \cdot s + 1)}{(T_1 \cdot s + 1) \cdot (T_2 \cdot s + 1)} \quad , \tag{16}$$

where: $T_{4,5} = 0{,}5\tau \cdot \left[ 0{,}5 + A' \pm \sqrt{(0{,}5 + A')^2 - 1/3} \right]$.

In an analogous way, on the basis of the generalized structural schematic and system of equations [5, 11] models of DS were obtained for other control effects. The approximating relations of dynamic system OT for various control effects differ from those presented here only in the value of the gain coefficient $K_0$ of the CO. Instead of coefficient $K_{F_{x1}}$ in relation (8) for $K_0$, in such a case coefficients of gain for the respective effects $K_e, K_{F_{dod.i}}, K_{M_i}, K_{M_{skr.}}$ are inserted. The values of those coefficients can be calculated according to the relations given in [5, 11].

In many cases, with accuracy sufficient for practical engineering calculations, approximating relations for OT (7) should be built with the use of the first component of the splitting of function $e^{-s\tau}$ into a Pade series:

$$e^{-s\tau} = (1 - \frac{1}{2} s \cdot \tau)/(1 + \frac{1}{2} s \cdot \tau) \quad . \tag{17}$$

Table 1 presents operator transmittances, coefficients of gain and time constants for the generalized and the detailed MM of dynamic system for the turning of low-rigidity shafts in the elastic-deformable condition [5, 11].

**Tab. 1. Operator transmittances, coefficients of gain and time constants of generalized and simplified MM of dynamic system of turning of shafts with low rigidity in elastic-deformable condition**

| No | $K_{\kappa_r} \neq 0, \quad \kappa_r \neq 90^\circ$ | | | $K_{\kappa_r} = 0, \quad \kappa_r = 90^\circ$ | | |
|---|---|---|---|---|---|---|
| | **Dynamical System Operator Transmittance** | **Coefficient of Gain** | **Time Constants** | **Dynamical System Operator Transmittance** | **Coefficient of Gain** | **Time Constants** |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | **Using first two elements of Padé Approximation for $e^{-s\tau}$:**<br>$G_{T1}(s) = K_0 \frac{T_3^2 s^2 + T_3' s + 1}{(T_1 s + 1)(T_2 s + 1)}$ | $K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{bz} n_z K_z + K_{yy} n_y}$<br>$A_1 = m_x K_x + m_y K_y K_{\kappa_r}$<br>$B_1 = [m_x K_x + K_{\kappa_r}(m_y K_y + m_x K_{xy} +$<br>$+ K_{xy} n_x K_y m_y) + K_{bz} K_z K_{\kappa_r}(m_z +$<br>$+ n_z m_y K_y + K_{yy} m_y K_{\kappa_r}(1 + n_y K_y)\ ]/$<br>$/(1 + K_{xy} n_x + K_{bz} K_z n_z + K_{yy} n_y)$ | $T_{1,2} = 0{,}5\tau[0{,}5 + B_1 \pm$<br>$\pm \sqrt{(0{,}5 + B_1)^2 - 1/3}\ ]$<br>$T_3 = 0{,}289\tau$<br>$T_3' = (0{,}5 + A_1)\tau$ | **Using first two elements of Padé Approximation for $e^{-s\tau}$:**<br>$G'_{T1}(s) = K_0 \frac{T_3^2 s^2 + T_3' s + 1}{(T_1 s + 1)(T_2 s + 1)}$ | $K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{bz} n_z K_z + K_{yy} n_y}$<br>$A_1' = m_x K_x$<br>$B_1' = \frac{m_x K_x}{1 + K_{xy} n_x + K_{bz} n_z K_z + K_{yy} n_y}$ | $T_{1,2} = 0{,}5\tau\left[0{,}5 + B_1' \pm\right.$<br>$\left.\pm \sqrt{(0{,}5 + B_1')^2 - 1/3}\ \right]$<br>$T_3 = 0{,}289\tau$<br>$T_3' = (0{,}5 + A_1')\tau$ |
| 1 | $m_x K_x << 1$ | $K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{bz} n_z K_z + K_{yy} n_y}$<br>$A_2 = m_y K_y K_{\kappa_r}$<br>$B_2 = \{K_{\kappa_r}(m_y K_y + K_{xy} n_x m_y K_y) +$<br>$+ K_{xy} m_x (K_{\kappa_r} - n_x K_x) + K_{bz} n_z K_z \times$<br>$\times [K_{\kappa_r} n_z m_y K_y + m_z (K_{\kappa_r} - n_x K_x)\ ] +$<br>$\times [K_{\kappa_r} n_z m_y K_y + m_z (K_{\kappa_r} - n_x K_x)\ ] +$<br>$/(1 + K_{xy} n_x + K_{bz} K_z n_z + K_{yy} n_y)$ | $T_{1,2} = 0{,}5\tau[0{,}5 + B_2 \pm$<br>$\pm \sqrt{(0{,}5 + B_2)^2 - 1/3}\ ]$<br>$T_3 = 0{,}289\tau$<br>$T_3' = (0{,}5 + A_2)\tau$ | $G'_{T1}(s) = K_0 \frac{T_3^2 s^2 + 2\varepsilon T_3 s + 1}{(T_1 s + 1)(T_2 s + 1)}$ | $K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{bz} n_z K_z + K_{yy} n_y}$<br>$A_2' = 0$<br>$B_2' = n_x K_x (m_z K_{bz} K_z + m_x K_{xy} +$<br>$+ m_y K_{yy})/$<br>$/(1 + K_{xy} n_x + K_{bz} n_z K_z + K_{yy} n_y)$ | $T_{1,2} = 0{,}5\tau\left[0{,}5 + B_2' \pm\right.$<br>$\left.\pm \sqrt{(0{,}5 + B_2')^2 - 1/3}\ \right]$<br>$T_3 = 0{,}289\tau$<br>$T_3' = 0{,}5\tau$<br>$\varepsilon = \frac{T_3'}{2T_3} = 0{,}866$ |
| 1 | $m_x K_x << 1, \quad K_{bz} n_z K_z << 1$ | $K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{yy} n_y}$<br>$A_2 = m_y K_y K_{\kappa_r}$<br>$B_3 = \{K_{\kappa_r}(m_y K_y + K_{xy} n_x m_y K_y) +$<br>$+ K_{xy} m_x (K_{\kappa_r} - n_x K_x) +$<br>$+ K_{yy} m_y (K_{\kappa_r} - n_x K_x)\}/$<br>$/(1 + K_{xy} n_x + K_{yy} n_y)$ | $T_{1,2} = 0{,}5\tau[0{,}5 + B_3 \pm$<br>$\pm \sqrt{(0{,}5 + B_3)^2 - 1/3}\ ]$<br>$T_3 = 0{,}289\tau$<br>$T_3' = (0{,}5 + A_2)\tau$ | | $K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{yy} n_y}$<br>$A_2' = 0$<br>$B_3' = n_x K_x (m_x K_{xy} + m_y K_{yy})/$<br>$/(1 + K_{xy} n_x + K_{bz} n_z K_z + K_{yy} n_y)$ | $T_{1,2} = 0{,}5\tau\left[0{,}5 + B_3' \pm\right.$<br>$\left.\pm \sqrt{(0{,}5 + B_3')^2 - 1/3}\ \right]$<br>$T_3 = 0{,}289\tau$<br>$T_3' = 0{,}5\tau$<br>$\varepsilon = 0{,}866$ |
| 1 | $K_{xy} << 1$ | $K_0 = \frac{K_{F_{x1}}}{1 + K_{yy} n_y}$<br>$A_2 = m_y K_y K_{\kappa_r}$<br>$B_4 = [K_{yy} m_y K_{\kappa_r}(n_y K_y + 1) +$<br>$+ K_{yy} m_y (K_{\kappa_r} - n_x K_x)/(1 + K_{yy} n_y)$ | $T_{1,2} = 0{,}5\tau[0{,}5 + B_4 \pm$<br>$\pm \sqrt{(0{,}5 + B_4)^2 - 1/3}\ ]$<br>$T_3 = 0{,}289\tau$<br>$T_3' = (0{,}5 + A_2)\tau$ | | $K_0 = \frac{K_{F_{x1}}}{1 + K_{yy} n_y}$<br>$A_2' = 0$<br>$B_4' = n_x K_x K_{yy} m_y /(1 + K_{yy} n_y)$ | $T_{1,2} = 0{,}5\tau\left[0{,}5 + B_3' \pm\right.$<br>$\left.\pm \sqrt{(0{,}5 + B_3')^2 - 1/3}\ \right]$<br>$T_3 = 0{,}289\tau$<br>$T_3' = 0{,}5\tau$<br>$\varepsilon = 0{,}866$ |
| 2 | $A_1 << 0{,}077$<br>$G'_{T1}(s) = K_0 \frac{T_3^2 s^2 + 2\varepsilon T_3 s + 1}{(T_1 s + 1)(T_2 s + 1)}$ | $K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{bz} n_z K_z + K_{yy} n_y}$<br>$A_1 = m_x K_x + m_y K_y K_{\kappa_r}, \quad B_1 = B_1$ | $T_{1,2} = 0{,}5\tau[0{,}5 + B_1 \pm$<br>$\pm \sqrt{(0{,}5 + B_1)^2 - 1/3}\ ]$<br>$T_3 = 0{,}289\tau$<br>$\varepsilon = \frac{0{,}5 + A_1}{0{,}577}$ | $G'_{T1}(s) = K_0 \frac{T_3^2 s^2 + 2\varepsilon T_3 s + 1}{(T_1 s + 1)(T_2 s + 1)}$ | $K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{bz} n_z K_z + K_{yy} n_y}$<br>$A_1' = m_x K_x$<br>$B_1' = \frac{m_x K_x}{1 + K_{xy} n_x + K_{bz} n_z K_z + K_{yy} n_y}$ | $T_{1,2} = 0{,}5\tau\left[0{,}5 + B_1' \pm\right.$<br>$\left.\pm \sqrt{(0{,}5 + B_1')^2 - 1/3}\ \right]$<br>$T_3 = 0{,}289\tau$<br>$\varepsilon = \frac{0{,}5 + A_1}{0{,}577}$ |
| 2 | $m_x K_x << 1, \ A_2 << 0{,}077$ | $K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{bz} n_z K_z + K_{yy} n_y}$<br>$A_2 = m_y K_y K_{\kappa_r}, \quad B_2 = B_2$ | $T_{1,2} = 0{,}5\tau[0{,}5 + B_2 \pm$<br>$\pm \sqrt{(0{,}5 + B_2)^2 - 1/3}\ ]$<br>$T_3 = 0{,}289\tau$<br>$\varepsilon = \frac{0{,}5 + A_2}{0{,}577}$ | $G'_{T1}(s) = K_0 \frac{T_3^2 s^2 + 2\varepsilon T_3 s + 1}{(T_1 s + 1)(T_2 s + 1)}$ | $K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{bz} n_z K_z + K_{yy} n_y}$<br>$A_2' = 0, \quad B_2' = B_2'$ | $T_{1,2} = 0{,}5\tau\left[0{,}5 + B_2' \pm\right.$<br>$\left.\pm \sqrt{(0{,}5 + B_2')^2 - 1/3}\ \right]$<br>$T_3 = 0{,}289\tau$<br>$\varepsilon = 0{,}866$ |
| 2 | $m_x K_x << 1, \ K_{bz} n_z K_z << 1$ | $K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{yy} n_y}$<br>$A_2 = m_y K_y K_{\kappa_r}, \quad B_3 = B_3$ | $T_{1,2} = 0{,}5\tau[0{,}5 + B_3 \pm$<br>$\pm \sqrt{(0{,}5 + B_3)^2 - 1/3}\ ]$<br>$T_3 = 0{,}289\tau$<br>$\varepsilon = \frac{0{,}5 + A_2}{0{,}577}$ | | $K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{yy} n_y}$<br>$A_2' = 0, \quad B_3' = B_3'$ | $T_{1,2} = 0{,}5\tau\left[0{,}5 + B_3' \pm\right.$<br>$\left.\pm \sqrt{(0{,}5 + B_3')^2 - 1/3}\ \right]$<br>$T_3 = 0{,}289\tau$<br>$\varepsilon = 0{,}866$ |
| 2 | $K_{xy} << 1$ | $K_0 = \frac{K_{F_{x1}}}{1 + K_{yy} n_y}$<br>$A_2 = m_y K_y K_{\kappa_r}, \quad B_4 = B_4$ | $T_{1,2} = 0{,}5\tau[0{,}5 + B_4 \pm$<br>$\pm \sqrt{(0{,}5 + B_4)^2 - 1/3}\ ]$<br>$T_3 = 0{,}289\tau$<br>$\varepsilon = \frac{0{,}5 + A_2}{0{,}577}$ | | $K_0 = \frac{K_{F_{x1}}}{1 + K_{yy} n_y}$<br>$A_2' = 0, \quad B_4' = B_4'$ | $T_{1,2} = 0{,}5\tau\left[0{,}5 + B_4' \pm\right.$<br>$\left.\pm \sqrt{(0{,}5 + B_4')^2 - 1/3}\ \right]$<br>$T_3 = 0{,}289\tau$<br>$\varepsilon = 0{,}866$ |

**Tab. 1. Operator transmittances, coefficients of gain and time constants of generalized and simplified MM of dynamic system of turning of shafts with low rigidity in elastic-deformable condition (continued)**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 3 | $A_1 \geq 0{,}078$<br>$G_{T2}(s) = \frac{(T_4 s+1)(T_5 s+1)}{T_1 s+1)(T_2 s+1)}$ | $K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$<br>$A_1 = m_x K_x + m_y K_y K_{\kappa_r}$, $B_1 = B_1$ | $T_{4,5} = 0{,}5\tau[0{,}5 + A_1 \pm$<br>$\pm\sqrt{(0{,}5+A_1)^2 - 1/3}\ ]$<br>$T_{1,2} = 0{,}5\tau[0{,}5 + B_1 \pm$<br>$\pm\sqrt{(0{,}5+B_1)^2 - 1/3}\ ]$ | $G'_{T2}(s) = \frac{(T_4 s+1)(T_5 s+1)}{T_1 s+1)(T_2 s+1)}$ | $K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$<br>$A_1{}' = m_x K_x$, $B'_1 = B'_1$ | $T_{4,5} = 0{,}5\tau[0{,}5 + A'_1 \pm$<br>$\pm\sqrt{(0{,}5+A'_1)^2 - 1/3}$<br>$T_{1,2} = 0{,}5\tau[0{,}5 + B'_1 \pm$<br>$\pm\sqrt{(0{,}5+B'_1)^2 - 1/3}$ |
| 3 | $m_x K_x << 1$, $A_2 \geq 0{,}078$ | $K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$<br>$A_2 = m_y K_y K_{\kappa_r}$, $B_2 = B_2$ | $T_{4,5} = 0{,}5\tau[0{,}5 + A_2 \pm$<br>$\pm\sqrt{(0{,}5+A_2)^2 - 1/3}\ ]$<br>$T_{1,2} = 0{,}5\tau[0{,}5 + B_2 \pm$<br>$\pm\sqrt{(0{,}5+B_2)^2 - 1/3}\ ]$ | $G'_{T1}(s) = K_0 \frac{T_3^2 s^2 + 2\varepsilon T_3 s + 1}{(T_1 s+1)(T_2 s+1)}$ | $K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$<br>$A_2{}' = 0$, $B'_2 = B'_2$ | $T_{1,2} = 0{,}5\tau\left[0{,}5 + B_2{}' \pm\right.$<br>$\left.\pm\sqrt{(0{,}5+B_2{}')^2 - 1/3}\ \right]$<br>$T_3 = 0{,}289\tau$<br>$\varepsilon = 0{,}866$ |
| 3 | $m_x K_x << 1$, $K_{bz}n_z K_z << 1$ | $K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{yy}n_y}$<br>$A_2 = m_y K_y K_{\kappa_r}$, $B_3 = B_3$ | $T_{4,5} = 0{,}5\tau[0{,}5 + A_2 \pm$<br>$\pm\sqrt{(0{,}5+A_2)^2 - 1/3}\ ]$<br>$T_{1,2} = 0{,}5\tau[0{,}5 + B_3 \pm$<br>$\pm\sqrt{(0{,}5+B_3)^2 - 1/3}\ ]$ | | $K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{yy}n_y}$<br>$A_2{}' = 0$, $B'_3 = B'_3$ | $T_{1,2} = 0{,}5\tau\left[0{,}5 + B_3{}' \pm\right.$<br>$\left.\pm\sqrt{(0{,}5+B_3{}')^2 - 1/3}\ \right]$<br>$T_3 = 0{,}289\tau$<br>$\varepsilon = 0{,}866$ |
| 3 | $K_{xy} << 1$ | $K_0 = \frac{K_{F_{x1}}}{1+K_{yy}n_y}$<br>$A_2 = m_y K_y K_{\kappa_r}$, $B_4 = B_4$ | $T_{4,5} = 0{,}5\tau[0{,}5 + A_2 \pm$<br>$\pm\sqrt{(0{,}5+A_2)^2 - 1/3}\ ]$<br>$T_{1,2} = 0{,}5\tau[0{,}5 + B_4 \pm$<br>$\pm\sqrt{(0{,}5+B_4)^2 - 1/3}\ ]$ | | $K_0 = \frac{K_{F_{x1}}}{1+K_{yy}n_y}$<br>$A_2{}' = 0$, $B'_4 = B'_4$ | $T_{1,2} = 0{,}5\tau\left[0{,}5 + B_4{}' \pm\right.$<br>$\left.\pm\sqrt{(0{,}5+B_4{}')^2 - 1/3}\ \right]$<br>$T_3 = 0{,}289\tau$<br>$\varepsilon = 0{,}866$ |
| 4 | **Using the first element of Padé Approximation for $e^{-s\tau}$:**<br>$G_{T3}(s) = K_0 \frac{T_2 s+1}{T_3 s+1}$ | $K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$<br>$A_1 = m_x K_x + m_y K_y K_{\kappa_r}$, $B_1 = B_1$ | $T_0 = \tau$, $T_1 = 0{,}5\tau$<br>$T_2 = \tau(0{,}5 + A_1)$<br>$T_3 = \tau(0{,}5 + B_1)$ | **Using the first element of Padé Approximation for $e^{-s\tau}$:**<br>$G_{T3}(s) = K_0 \frac{T_2 s+1}{T_3 s+1}$ | $K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$<br>$A_1{}' = m_x K_x$, $B'_1 = B'_1$ | $T_0 = \tau$, $T_1 = 0{,}5\tau$<br>$T_2 = \tau(0{,}5 + A'_1)$<br>$T_3 = \tau(0{,}5 + B'_1)$ |
| 4 | $m_x K_x << 1$ | $K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$<br>$A_2 = m_y K_y K_{\kappa_r}$, $B_2 = B_2$ | $T_0 = \tau$, $T_1 = 0{,}5\tau$<br>$T_2 = \tau(0{,}5 + A_2)$<br>$T_3 = \tau(0{,}5 + B_2)$ | | $K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$<br>$A'_1 = 0$, $B'_2 = B'_2$ | $T_0 = \tau$<br>$T_1 = T_2 = 0{,}5\tau$<br>$T_3 = \tau(0{,}5 + B'_2)$ |
| | $m_x K_x << 1$, $K_{bz}n_z K_z << 1$ | $K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{yy}n_y}$<br>$A_2 = m_y K_y K_{\kappa_r}$, $B_3 = B_3$ | $T_0 = \tau$, $T_1 = 0{,}5\tau$<br>$T_2 = \tau(0{,}5 + A_2)$<br>$T_3 = \tau(0{,}5 + B_3)$ | | $K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{yy}n_y}$<br>$A_2{}' = 0$, $B'_3 = B'_3$ | $T_0 = \tau$<br>$T_1 = T_2 = 0{,}5\tau$<br>$T_3 = \tau(0{,}5 + B'_3)$ |
| | $K_{xy} << 1$ | $K_0 = \frac{K_{F_{x1}}}{1+K_{yy}n_y}$<br>$A_2 = m_y K_y K_{\kappa_r}$, $B_4 = B_4$ | $T_0 = \tau$, $T_1 = 0{,}5\tau$<br>$T_2 = \tau(0{,}5 + A_2)$<br>$T_3 = \tau(0{,}5 + B_4)$ | | $K_0 = \frac{K_{F_{x1}}}{1+K_{yy}n_y}$<br>$A_2{}' = 0$, $B'_4 = B'_4$ | $T_0 = \tau$<br>$T_1 = T_2 = 0{,}5\tau$<br>$T_3 = \tau(0{,}5 + B'_4)$ |

## 3. CONCLUSION

As follows from the performed study, dynamic structures of MM of technological systems for low-rigidity shafts with control of their elastic-deformable condition include, apart from inertial segments characteristic for MM of feed-related control, also overload segments. The occurrence of the overload segments in transmittances of the MM reduces the inertness of the control objects with respect to channels of control of additional force effects. For example, with close values of time constants of the numerator and denominator in relations [12], as happens is numerous cases, the properties of model of CO approach those of the non-inertial segment with transmission coefficient $K_0$.

It should be emphasized that the discussed mathematical description of the *CO* was made with the exclusion of “small” time constants characterizing the dynamic properties of the process of machining and of the equivalent elastic system. Such an approach is justified as the *ACS* or *AC* circuit includes, apart from the object, also an automatic control device and other components with "large" time constants, whose dynamic properties are highly significant in the solution of the problem of stability analysis and synthesis of corrective segments.

Comparison of MM of the object for various control effects permits the statement that with the application of additional force effects the object has a notably lower inertness compared to the case of control focused on the feed channel. Thanks to this in the *ACS* and *AC* of the elastic-deformable condition of parts higher indexes of control quality can be achieved in the dynamics and there is a possibility of effective counteraction of interference caused by changes in material allowance for machining and in the hardness of machined semi-finished products by varying their rigidity on the length of machining.

The results of theoretical research of object's time characteristics by channel of additional force reactions, confirm the above-mentioned conclusion, that DS's properties are, in approximation, equivalent to proportional link when TS's elastic-deformable condition is being controlled. Such simplification is correct only when "low" and "medium" frequencies (dynamical properties of control process and elastic system are not shown) range is being considered. Time-constants of elastic system and cutting process which define limits of the "medium" frequencies range, are between *0,003s* and *0,005s.* Time-constants of executive element, which are applied during constructing SAC by elastic-deformable condition, usually increase the pointed value by an order of magnitude. Hence, the range of important frequencies is defined by the executive element's inertia and is localized more to the left than range of frequencies that are defined by dynamical characteristics of considered object.

In case of interferences in the form of exponential-cosines function, the optimum controller for the model is the typical P controller, which proportionality coefficient is defined by selected level of limitations on the control reaction. The hardware realization of PI controller was presented.

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