

A unified model of a spinning disc centrifugal spreader

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Abstract. A mathematical model was built of the motion of fertilizer particles on the spinning disc and in the air. The Langevin's stochastic differential equation was applied to describe the random disturbances affecting fertilizer particles moving on the disc spreader. The verification of experimental results showed good agreement with the simulation results obtained from the stochastic model. Our results bring new cognitive value in the field of process modeling of mineral fertilizers spreading with disc spreaders and can be used to improve the existing spreaders as well as to design new ones.

Key words: spinning disc, centrifugal fertilizer spreader, Langevin equation

INTRODUCTION

An analysis of the literature on the subject shows that currently only a few issues associated with the process of spreading granular fertilizers with disc spreaders have been explained. Experimental studies have focused on analyzing the motion of fertilizer particles along the spreading disc, and their purpose has been to specify the velocity and angle of fertilizer particles leaving the disc [26,27,28]. Few publications refer to operation and testing investigations and are of a fragmentary nature [9, 11, 29, 30]. Theoretical studies focus mainly on an analysis of the motion of fertilizer particles on the rotating disc [4, 10, 25]. Differences between computer simulation results and the corresponding results from experimental studies are a consequence of deficiencies in the applied mathematical models describing the behaviour of fertilizer particles during spreading. This is due to oversimplification at the stage of formulating mathematical dependencies describing the process of spreading fertilizers with disc spreaders. Among the most frequent assumptions found in the literature on the subject [3, 6, 8, 14] simplifying the mathematical description of the motion of fertilizer particles along the spreading disc is the ignoring of the interactions between the fertilizer particles and the deflections of particles from the rotating disc and blades. The simulation of the motion of the fertilizer mass along the spreader disc is limited to the description of the

motion of individual particles and adding together their trajectories using the superposition method. The discrete elements method (DEM) is used to describe the motion of fertilizer particles on the spreading disc, with attempts to include the interactions between them, but with no reference to the structural and operational parameters of the spreading systems [21, 22, 23, 24]. In the case of modelling of the motion of fertilizer particles in the air, the particles are assumed to have a spherical shape, while not taking into account their rotational motion around their own axis and collisions in the air. Literature analysis [4, 5, 6, 14] shows that most authors claim that the most differences between the actual behaviour of fertilizer particles and the assumed model of the process occur during their motion on the rotating disc. Examples of differences between the fertilizer mass distribution obtained during computer simulations and experimental results can be found in investigation results presented in the discussed study [14]. According to the authors of the said study, this is also due to the exclusion of interactions between fertilizer particles moving along the disc. Without an appropriate adjustment of input data it is not possible to obtain simulation results consistent with the experimental data.

The objective of the study was to develop a mathematical model of spreading mineral fertilizers with centrifugal spreaders that includes random shocks to which fertilizer particles moving along the disc are subjected. During the creation of a stochastic model it was assumed that the shape of fertilizer particles is spherical, and that they are uniform in shape, size and density, and also that the interactions between fertilizer particles moving along the spreader disc can be described with the use of stochastic forces [18,19].

THE EQUATION FOR THE MOTION OF FERTILISER PARTICLE ON THE DISC

Figure 1 presents the distribution of forces acting on a fertilizer particle moving along the rotating disc blade of the spreader moving with angle velocity ω .

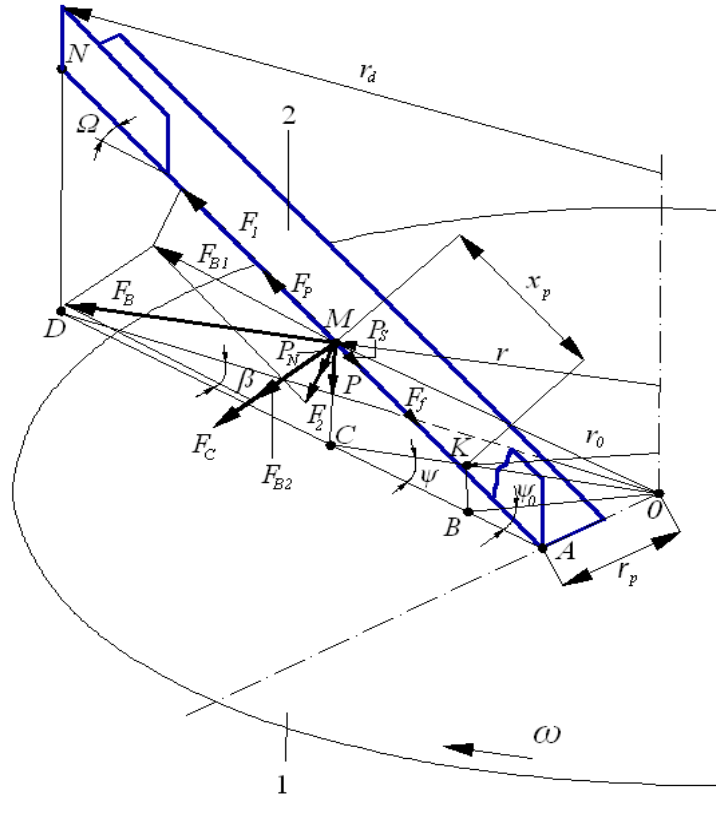


Fig. 1. Distribution of forces acting on the fertilizer particle motion along the vane of spreader disc: 1 – disc, 2 – vane, r – radial coordinate of particle fertilizer, r_d – disc radius, r_p – distance between the vane and axis of disc rotation, r_0 – the feed radius, F_p – inertial force, F_f – friction force, F_C – Coriolis force, F_B – centrifugal force, F_{B1} – component of the force F_B , F_{B2} – normal component of the force F_B , F_1 – tangential component of the force F_{B1} , F_2 – normal component of the force F_{B1} , P – gravity force, P_N – normal component of gravity force, P_S – tangential component of gravity force, β – angle between the vane and disc radius, Ψ – angle between the vane and radius r , Ψ_0 – angle between the vane and radius r_0 , ω – angular velocity of the disc, Ω – angle between the vane and horizontal plane

In the analysed case the blade is inclined towards the horizontal plane at angle Ω and deviates from the disc's radius at angle β . The following forces act on the fertilizer particle located on the disc at distance r from the rotation axis (point M) [1, 2, 7]:

– centrifugal force:

$$F_B = m\omega^2 r, \quad (1)$$

– Coriolis force:

$$F_C = 2m\omega\cos\Omega \frac{dx_p}{dt}, \quad (2)$$

– gravity force:

$$P = mg, \quad (3)$$

– friction force:

$$F_f = \mu_d(P_N + F_2) + \mu_v(F_C + F_{B2}), \quad (4)$$

where:

g – gravitational acceleration [$\text{m}\cdot\text{s}^{-2}$],

μ_d – friction coefficient for particle-disc interaction [-],

μ_v – friction coefficient for particle-vane interaction [-].

From the equilibrium condition of the forces acting on the fertilizer particle at point M (Fig. 1) it is possible to obtain the following equation for the resultant force F_p acting on the fertilizer particle:

$$F_p = F_1 - P_S - F_f, \quad (5)$$

where:

$$F_p = m \frac{d^2 x_p}{dt^2}, \quad (6)$$

$$F_1 = F_{B1} \cos\Omega, \quad (7)$$

$$F_2 = F_{B1} \sin\Omega, \quad (8)$$

$$P_S = P \sin\Omega, \quad (9)$$

$$P_N = P \cos\Omega, \quad (10)$$

$$F_{B1} = F_B \cos\Psi, \quad (11)$$

$$F_{B2} = F_B \sin\Psi. \quad (12)$$

Substituting eqs (1-4) and (6-12) in equ (5), the equation of motion of the particle on the disc for straight vane (equ (13)) becomes:

$$\begin{aligned} \frac{d^2 x_p}{dt^2} + 2\mu_v \omega \cos \Omega \frac{dx_p}{dt} - r \omega^2 \cos \psi (\cos \Omega - \mu_d \sin \Omega) \\ + \mu_v r \omega^2 \sin \psi + g (\sin \Omega + \mu_d \cos \Omega) = 0. \end{aligned} \quad (13)$$

As equation (13) contains two variables, r and x_p , determining the location of the fertilizer particle on the disc, the equation was converted as follows:

$$\frac{d^2 r}{dt^2} + C_a \left(\frac{dr}{dt} \right)^2 + C_b \frac{dr}{dt} + C_c r + C_d = 0, \quad (14)$$

where the coefficients C_a , C_b , C_c and C_d are defined as follows:

$$C_a = -\frac{r_p^2}{r(r^2 - r_p^2)}, \quad (15)$$

$$C_b = 2\mu_v \omega \cos \Omega, \quad (16)$$

$$C_c = (\mu_d \sin \Omega \cos \psi - \cos \Omega \cos \psi + \mu_v \sin \psi) \cos \Omega \omega^2 \sqrt{1 - \left(\frac{r_p}{r} \right)^2}, \quad (17)$$

$$C_d = (\sin \Omega + \mu_d \cos \Omega) \cos \Omega g \sqrt{1 - \left(\frac{r_p}{r} \right)^2}. \quad (18)$$

The model of the motion of the fertilizer particle on the spreader disc described in equation (14) is a deterministic model. Applying this model always leads to identical solutions with fixed initial conditions. In real conditions fertilizer particles moving along the spreader disc are subjected to random shocks, which the model does not take into consideration. Fertilizer particles moving in a stream interact with one another, which, at the stage of the particles' motion on the disc, causes changes to their velocity and directions of leaving the disc.

In mechanics, stochastic differential equations are used for the description of random shocks to which physical systems are subjected. An example of a stochastic differential equation very frequently used to describe physical systems subjected to random shocks is the Langevin equation.

The Langevin differential equation describing the motion of fertilizer particles on the rotating spreader disc can be expressed as follows:

$$\frac{d^2 r}{dt^2} + C_a \left(\frac{dr}{dt} \right)^2 + C_b \frac{dr}{dt} + C_c r + C_d = \sigma_F \zeta(t), \quad (19)$$

where:

$\zeta(t)$ – the stochastic Langevin force,
 σ_F – the intensity of stochastic forces.

The Langevin force occurring in equation (19) has the following properties:

$$\zeta(t) = 0, \quad (20)$$

$$\zeta(t)\zeta(t_0) = 2\delta(t-t_0), \quad (21)$$

where: $\delta(t-t_0)$ is the Dirac function defined in the following way:

$$\delta(t-t_0) = \begin{cases} 0 & \text{for } t \neq t_0. \\ +\infty & \text{for } t = t_0 \end{cases} \quad (22)$$

This study contains a numerical solution of the non-linear equation (14) via the Runge-Kutta method of the fourth order [17], and the stochastic equation (19) via the Runge-Kutta method of the fourth order for stochastic equations [12] with the following initial and boundary conditions:

$$\begin{aligned} t = 0: \quad r = r_0, \quad \varphi = \varphi_i, \quad \frac{dr}{dt} = 0, \\ t = t_k: \quad r = r_d, \quad \varphi = \varphi_i + \varphi_u, \quad \frac{dr}{dt} = V_R. \end{aligned} \quad (23)$$

Figure 2 presents the trajectory of a fertilizer particle moving on the rotating disc of the spreader at angle velocity ω . In the simulation model, the coordinates of point K of feeding the fertilizer particle to the disc (φ_i , r_0) are generated on a random basis.

The solution of the deterministic (14) or stochastic (19) equation makes it possible to determine the horizontal component V_R of the fertilizer particle's radial velocity at time $t = t_k$ of the particle's leaving the spreader

disc, where t_k is the time at which the particle moved from r_0 to r_d .

Determining the trajectory of the fertilizer particle's motion in the air requires calculating initial velocity V_0 in the air, angle α_0 between the horizontal plane and the direction of initial velocity V_0 , and angle θ of the fertilizer particle's leaving the disc. Figures 2 and 3 present the distribution of initial velocity V_0 of the fertilizer particle at the end of the blade.

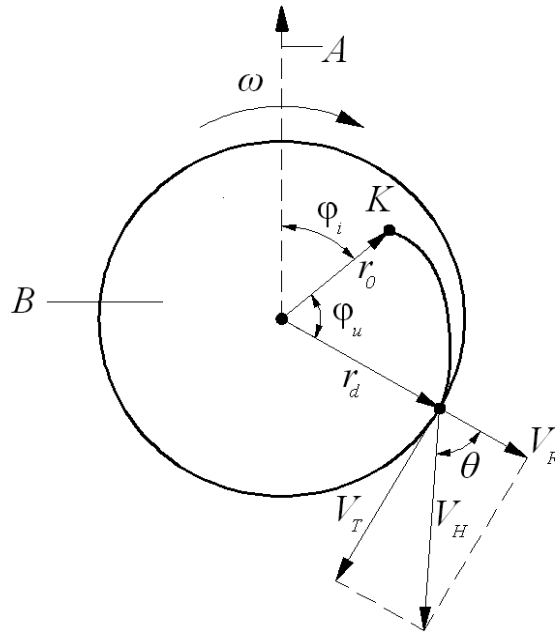


Fig. 2. Trajectory of the fertilizer particle on disc: A – travel direction, B – disc, V_H – horizontal component of the outlet angle of fertilizer particle from disc, V_T – tangential component of V_H velocity, V_R – radial component of V_H velocity, θ – outlet angle of fertilizer particle from disc, ω – angular velocity of the disc, r_d – disc radius, r_0 – the feed radius, φ_i – angle of the initial particle position on disc, φ_u – angle of the particle position on disc

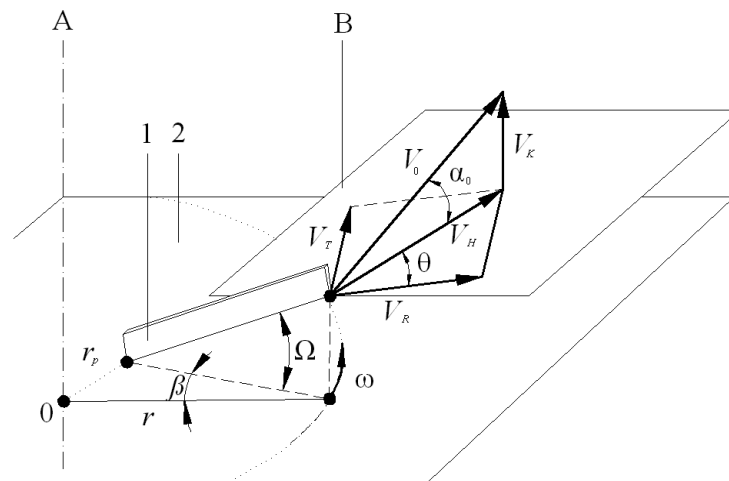


Fig. 3. Distribution of the velocity of fertilizer particle at the edge of vane: 1 – disc, 2 – vane, A – axis of disc rotation, B – horizontal plane, Ω – angle between the vane and horizontal plane, r – radial coordinate of particle fertilizer, ω – angular velocity of the disc, r_p – distance between the vane and axis of disc rotation, θ – outlet angle of fertilizer particle from vane, V_H – horizontal component of outlet velocity of fertilizer particle from disc, V_T – tangential component of V_H velocity, V_R – radial component of V_H velocity, V_K – vertical component of outlet velocity of fertilizer particle from disc, V_0 – outlet velocity of fertilizer particle from disc, α_0 – the initial vertical angle of the particle in the air

The initial velocity V_0 of the fertilizer particle in the air is (Fig. 3):

$$V_0 = \sqrt{V_H^2 + V_K^2} \tag{24}$$

In order to determine the value of angle θ , the value of the component of tangent V_T of the velocity of the fertilizer particle on the horizontal plane must be calculated first (Fig. 3).

$$V_T = \omega r_d - V_R \frac{r_p}{\sqrt{r_d^2 - r_p^2}} \tag{25}$$

Knowing the values of velocity components V_R and V_T from expression (26), it is possible to calculate angle θ of the fertilizer particle's leaving the disc ($r = r_d$):

$$\operatorname{tg} \theta = \frac{V_T}{V_R}. \quad (26)$$

Inserting expression (25) into equation (26) yields the value of angle θ of the fertilizer particle's leaving the disc:

$$\theta = \operatorname{arctg} \left[\frac{\omega r_d \sqrt{r_d^2 - r_p^2} - V_R r_p}{V_R \sqrt{r_d^2 - r_p^2}} \right]. \quad (27)$$

Knowing the value of velocity V_R and angle θ of the fertilizer particle's leaving the disc, it is possible to calculate the velocity of the particle relative to blade V_{XP} at the time of leaving the disc:

$$V_{XP} = \frac{V_R}{\cos \Omega \cos \beta}. \quad (28)$$

and the horizontal component V_H and the vertical component V_K of initial velocity V_0 of the fertilizer particle in the air:

$$V_H = \frac{V_R}{\cos \theta}, \quad (29)$$

$$V_K = V_{XP} \sin \Omega = \frac{V_R}{\cos \beta} \operatorname{tg} \Omega. \quad (30)$$

The initial value of the angle of inclination α_0 of the initial velocity vector V_0 of the fertilizer particle in the air relative to the horizontal plane can be determined through the following expression (Fig. 3):

$$\frac{d^2 x_{pp}}{dt^2} = -\frac{1}{2} C_D \frac{\rho_a}{\rho_p} \frac{S_p}{V_p} \frac{dx_{pp}}{dt} \sqrt{\left(\frac{dx_{pp}}{dt} \right)^2 + \left(\frac{dz_{pp}}{dt} \right)^2}, \quad (33)$$

$$\frac{d^2 z_{pp}}{dt^2} = -g - \frac{1}{2} C_D \frac{\rho_a}{\rho_p} \frac{S_p}{V_p} \frac{dz_{pp}}{dt} \sqrt{\left(\frac{dx_{pp}}{dt} \right)^2 + \left(\frac{dz_{pp}}{dt} \right)^2}, \quad (34)$$

where:

- C_D – air resistance coefficient [-],
- S_p – resistance surface of the particle [m^2],
- V_p – volume of the particle [m^3],
- ρ_a – density of the air [$\text{kg} \cdot \text{m}^{-3}$],
- ρ_p – density of the particle [$\text{kg} \cdot \text{m}^{-3}$].

Solving the system of differential equations (33) and (34) with initial and boundary conditions (35), it is possible to determine distance x_k between the point of the

$$t = 0: \quad x_{pp} = 0 \quad z_{pp} = h; \quad \frac{dx_{pp}}{dt} = V_H; \quad \frac{dz_{pp}}{dt} = V_K, \quad (35)$$

$$t = t_k: \quad x_{pp} = x_k; \quad z_{pp} = 0,$$

where: horizontal component V_H and vertical component V_K of the resultant velocity V_0 are as follows:

$$V_H = V_0 \cos \alpha_0, \quad (36)$$

$$V_K = V_0 \sin \alpha_0. \quad (37)$$

$$\operatorname{tg} \alpha_0 = \frac{V_K}{V_H}. \quad (31)$$

Inserting dependencies (29) and (30) into equation (31) yields the following expression for the value of angle α_0 :

$$\alpha_0 = \operatorname{arctg} \left[\frac{\cos \theta}{\cos \beta} \operatorname{tg} \Omega \right]. \quad (32)$$

In the simulation model, angles α_0 and θ and initial velocity V_0 of the fertilizer particle in the air are determined through equations (32), (27) and (24). Knowing these values, the trajectory of motion of the fertilizer particle in the air and the place of its falling on the ground can be determined.

THE EQUATION FOR THE MOTION OF A FERTILISER PARTICLE IN THE AIR

At the moment of leaving the blade, the velocity value of the fertilizer particle velocity is V_0 . In the case of a conical disc ($\Omega \neq 0$), the velocity vector V_0 forms angle α_0 with the horizontal plane. A fertilizer particle moving in the air is influenced by the gravitational force, air-resistance force and lift force. Because the density of fertilizer particles is considerably higher than air density, in the equation for the particle's motion in the air, the air resistance force was ignored. General equations of the motion of fertilizer particles in the air are as follows [14, 16]:

fertilizer particle's leaving the spreading disc and the place of its falling on the ground (point M, Fig. 4).

Another important step in the presented simulation model is determining the coordinates (x_{toi} , y_{toi}) of the place of the fertilizer particle's falling on the ground in the coordinate system related to the spreader discs. Let us assume that at initial time $t = 0$ the fertilizer particle is on the spreader disc at point K at distance r_0 from the centre of the disc (Fig. 4).

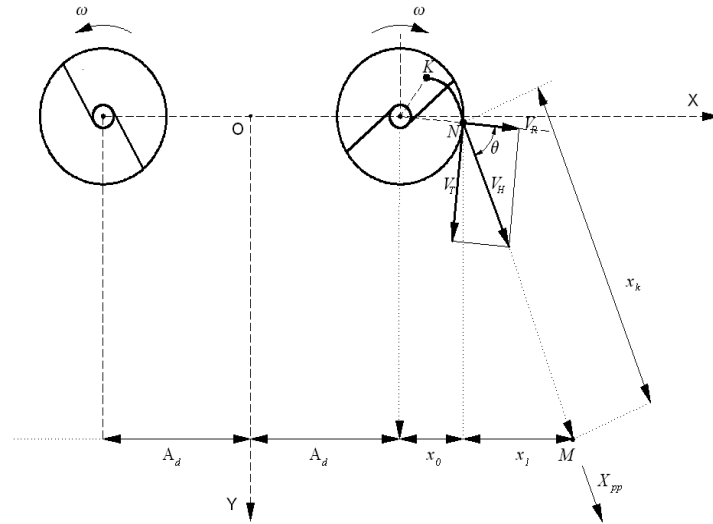


Fig. 4. Position of the fertilizer particle on field: $2A_d$ – distance between two disc centers, V_H – horizontal component of outlet velocity of fertilizer particle from disc, V_R – radial component of V_H velocity, V_T – tangential component of V_H velocity, x_k – distance between the disc edge and the place (M point) of drop, ω – angular velocity of disc, θ – outlet angle of fertilizer particle from disc

At time t_k during the particle's motion from r_0 to r_d the spreader disc will turn by angle $\varphi_u = \omega t_k$ (Fig. 2). Knowing angle θ of the fertilizer particle's leaving the spreader disc and angles φ_i and φ_u , it is possible to determine angles θ_1 and θ_2 :

$$\begin{aligned} \theta_1 &= \varphi_i + \varphi_u + \theta, \\ \theta_2 &= \varphi_i + \varphi_u, \end{aligned} \quad (38)$$

and later the coordinates of the location of the fertilizer particle on the edge of the disc (x_0, y_0) and distances x_1 and y_1 :

$$\begin{aligned} x_0 &= r_d \sin \theta_2, & x_1 &= x_k \sin \theta_1, \\ y_0 &= -r_d \cos \theta_2, & y_1 &= -x_k \cos \theta_1. \end{aligned} \quad (39)$$

The coordinates x_{tot} and y_{tot} of the location of the fertilizer particle on the ground can be obtained via the following equations:

$$\begin{aligned} x_{tot} &= x_0 + x_1 + A_d, \\ y_{tot} &= y_0 + y_1. \end{aligned} \quad (40)$$

RESULTS

In order to verify the mathematical model of spreading mineral fertilizers with centrifugal spreaders, the results of the investigation performed by the author of this study were used, carried out on an experimental site based on the author's own design. Urea from the current batch manufactured by the Zakłady Azotowe Puławy SA was used for the investigation.

The developed mathematical model of the description of spreading mineral fertilizers with centrifugal spreaders, which includes the impact of random shocks occurring as a result of interactions between fertilizer pellets in a stream, were subjected to validation consisting of the evaluation of the degree of consistency between the values specified on the basis of the model with the actual values obtained on site. This procedure was applied for significant reasons, as the results presented by other

authors [11, 15, 29, 30] are not accompanied by documented conditions of measurements (physical properties of the fertilizers, structural and operational parameters of the spreading systems, external conditions, etc.). Furthermore, most of the results of the testing investigations of spreaders are presented in graphic form, which makes it impossible to recreate their actual values [11, 29, 30].

In order to carry out a validation of the compared models, the distribution of the fertilizer mass along the width of the spreading strip was used. One of the commonly used and recommended methods for the evaluation of the conformity of the simulation results with the results of the experimental investigations is regression analysis for a linear model [13].

Figures 5-6 present charts of the actual placement of the fertilizer mass in measurement boxes for urea in comparison to the mass calculated via the deterministic and stochastic models. For both models a strong positive correlation was found between the experimental values of fertilizer mass in the measurement boxes and the values calculated with the use of the models. However, it should be mentioned that the regression coefficient (the slope of the straight line) value is significantly different from zero and amounts to 0.9647 for the equation determining the dependence between the actual distribution of the urea mass and that calculated with the stochastic model. The provided value differs slightly from the value of the coefficient in a perfect situation where the simulation results are consistent with the experimental results. However, it should be emphasized that the mentioned coefficient, in comparison with the one based on the data from the deterministic model, is much lower and amounts to 0.9083. In turn, considering constant values in the regression equations, it can be concluded that they are significantly different (9.0257 and 55.034). In the case of the equation based on the simulation results of the deterministic model, the value is quite substantial and demonstrates the lack of concentration of points along line 1:1 on the plane of interest to the researcher.

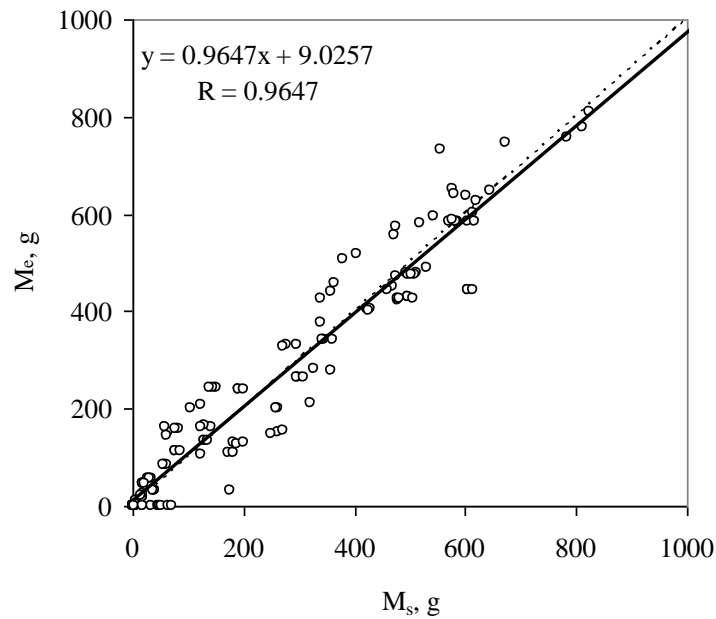


Fig. 5. Regression of real mass of urea in the measuring container (M_e) in relation to mass obtained from stochastic model (M_s); the dashed line represents the ratio of 1:1

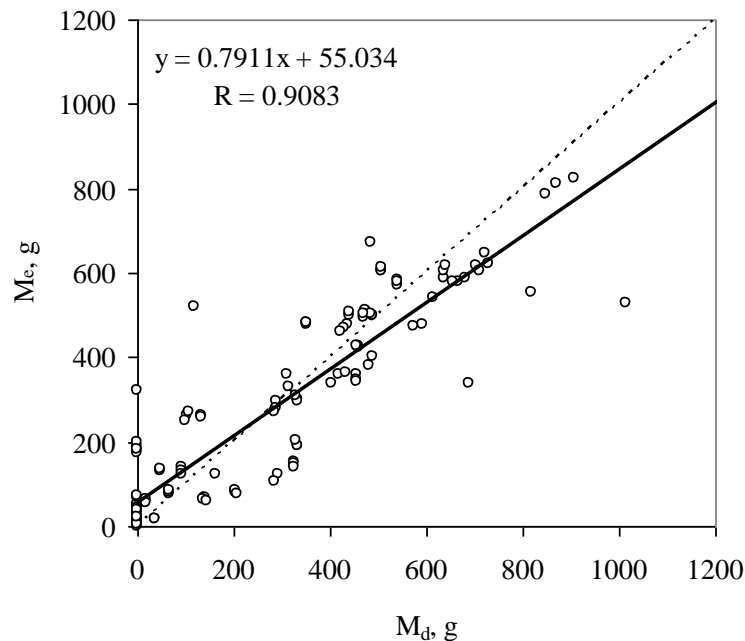


Fig. 6. Regression of real mass of urea in the measuring container (M_e) in relation to mass obtained from deterministic model (M_d); the dashed line represents the ratio of 1:1

CONCLUSIONS

The model for spreading mineral fertilizers developed within the study based on the Langevin stochastic equation describes the mutual interactions between fertilizer particles during their motion along the disc. It also facilitates simulation tests of the transverse distribution of fertilizer mass depending on the physical properties of the fertilizers and the structural and operational parameters of the spreading systems. The developed procedure for simulation calculations, consisting of mutually connected models of the motion of

fertilizer particles along the spreader disc and in the air, can be used for designing centrifugal spreading systems and selecting structural and operational parameters depending on the physical properties of fertilizers. The analysis of the regression dependencies of the fertilizer mass actual distribution and of those calculated on the basis of the stochastic and deterministic models showed that the stochastic model developed in this study more accurately described the process of spreading fertilizers with centrifugal spreading systems. This was confirmed by the R^2 coefficient of determination values for the analysed fertilizer.

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