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Method of identifying a type 2 membership function and application to decision-making problems*

by

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Abstract: Tanaka (1991) suggested that the parameters of a linear regression model should be made fuzzy in order to better reflect the nature of the system, involving a definite degree of variability, and created a fuzzy linear regression model. This model can be formulated in the form of a linear programming problem that minimizes the span between the upper and lower limits under the constraints that include all data. In recent years, all the attention in this context has been focused on a fuzzy number that has an indifferent zone. A fuzzy number that we consider here is defined by using a type 2 membership function. This paper addresses the fact that a type 2 membership function has the upper and lower limits and shows that a type 2 membership function can be identified by expanding a fuzzy linear regression model into a fuzzy linear polynomial regression model. Finally, after a proposed fuzzy polynomial model is identified, a mathematical model is developed for a fuzzy decision-making method that accounts for an indifferent zone.

Keywords: type 2 membership function, fuzzy linear regression model, fuzzy log-linear regression model, fuzzy linear polynomial regression model, indifferent zone, decision rule on a fuzzy event

1. Introduction

Tanaka (1991) presented an example of the way in which what we can call the swing of a system can be identified by endowing the parameters of a linear

*Dr Yoshiki Uemura published a number of articles in *Control & Cybernetics* during the 1990s and in the early 2000s. A number of later submissions that have not been published until now, have been under consideration for a long time, owing to various reasons, which include, in particular, problems with the clarity of presentation and language, as well as prolonged sickness of Dr Uemura. At present time, the Editorial Office disposes of several texts from Dr Uemura and it is planned to release at least some of them, after careful editing, with appropriate comments, concerning the status of these texts regarding the refereeing process. Thus, the present paper, submitted in the year 2009, has passed the proper refereeing process with success, but its publishing was delayed due to language and presentation problems. The Editorial Office made, therefore a special effort in order to bring the text as close as possible to the adequate quality.

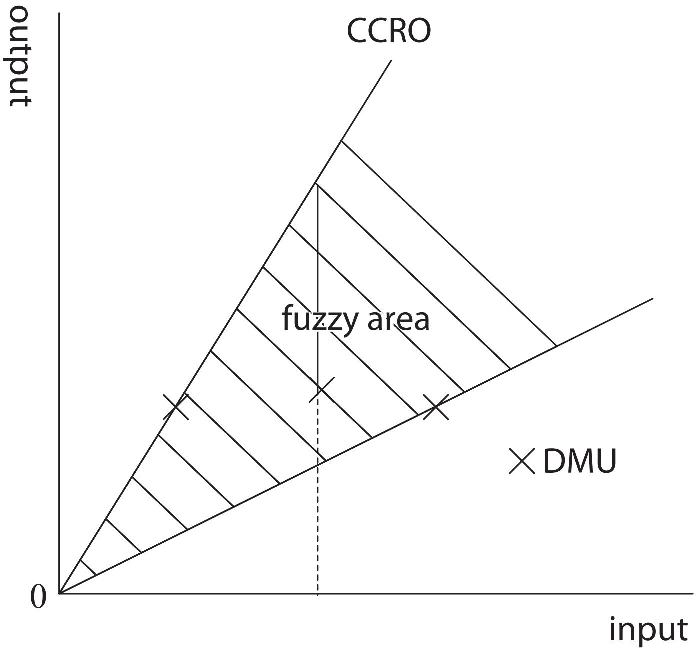

Figure 1. Fuzzy regression model and the DEA CCRO

regression model with some degree of vagueness. The resulting fuzzy linear regression model has the upper and lower limits so as to include all data. On the other hand, Uemura (1996) introduced a fuzzy log-linear regression model that leads to fuzzyfication of the parameters of the Cobb-Douglas model, and applied that model to the problem related to evaluation of efficiency of a business entity. It should be noted that applying a fuzzy regression analysis to the problem related to evaluation of efficiency of a business entity would involve a DEA CCRO model, see Fig. 1.

In recent years, considerable research has been carried out on the application of type 2 membership functions to the decision-making problems, especially in the field of fuzzy inference, considering the assumption that a fuzzy number itself has an indifferent zone. However, no reports as yet have been published on how to identify a type 2 membership function. In this paper, the parameters of a linear polynomial model will be rendered fuzzy to formulate a fuzzy linear polynomial regression model, and the method of identifying a type 2 membership function will be formulated. In another study, Uemura (1991) introduced the decision-making method, related to a fuzzy event by assuming that a decision maker defines a fuzzy event as being a reflection of the natural conditions in a statistical decision-making method. As a case study, application was then presented to the state of silence, which is associated with the field of human sentiment. In the present paper, after type 2 membership function is identified by using the proposed fuzzy linear polynomial regression model, the rules of decision making for the fuzzy events that are characterized by the state of non-discrimination will be addressed as a case study.

2. Method of identifying a type 2 membership function

This section describes the method of identifying two type 2 membership functions. First of all, suppose all data are already normalized.

2.1. Adaptation of a fuzzy linear regression model

All regions are segmented by points at the height of 1 along the y axis, and the medial point $((a+b)/2, 1)$ between the maximum point $(a, 1)$ and the minimum point $(b, 1)$ along the x axis. For the region below the medial point the problem is formulated as the positive regression. For the region above the medial point the problem is formulated as the negative regression, see Fig. 2:

$$\begin{aligned}
&x_i \leq \frac{(a+b)}{2} \\
&min \sum_{i=1}^{n} \left[a_H x_i + b_H - (a_L x_i + b_L)\right] \\
&\text{s.t.} \\
&a_H x_i + b_H \leq y_i \ (i = 1l \ldots, n) \\
&a_H x_i + b_L \geq y_i \ (i = 1l \ldots, n) \\
&a_H, \alpha_L, b_H, b_l \leq 0
\end{aligned} \tag{1}$$

$$\begin{aligned}
&x_i \geq \frac{(a+b)}{2} \\
&min \sum_{i=1}^{n} \left[-a_H x_i + b_H - (-a_L x_i + b_L)\right] \\
&\text{s.t.} \\
&-a_H x_i + b_H \leq y_i \ (i = 1l \ldots, n) \\
&-a_L x_i + b_L \geq y_i \ (i = 1l \ldots, n) \\
&a_H, a_L, b_H, b_L \geq 0
\end{aligned} \tag{2}$$

where in both (1) and (2), (x_i, y_i) are data items.

2.2. Adaptation of a fuzzy linear regression polynomial model

A linear polynomial of second degree can be identified without dividing regions by a medial point. Although a membership function that is higher than the

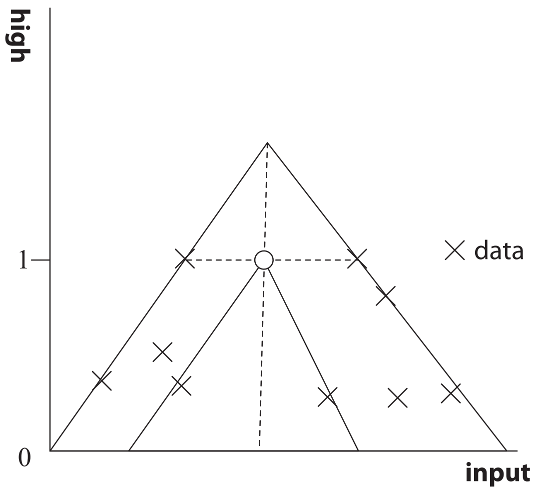

Figure 2. Identification for the type 2 membership function by the fuzzy regression model

height of 1 is derived, it can be identified as being like a trapezoid by cutting off the upper portion of the respective curve at the height of 1. If a membership function, reaching the height higher than 1, is usable, it is better to use a type 2 membership function, identified as this is considered here, see Fig. 3.

$$min \sum_{i}^{n} = 1\ [a_{2H}x_i^2 + a_{1H}x_i + b_H - (a_{2L}x_i^2 + a_{1L}x_i + b_L)]$$

s.t.

$$a_{2H}x_i^2 + a_{1H}x_i + b_H \leq y_i\ (i = 1l \ldots, n)$$

$$a_{2L}x_i^2 + a_{1L}x_i + b_L \geq y_i\ (i = 1l \ldots, n)$$

$$a_{2H}, a_{1H}, b_H, a2L, a1L, b_l \geq 0$$

where, again, (x_i, y_i) are data items.

2.3. Discussion

If a type 2 membership function is interpreted as defining an indifferent zone, a dome shape is suitable for showing an implication that the possibility of presenting non-discrimination is gradually decreased. Moreover, even if the interpolation of data is taken into consideration, the application of a fuzzy linear polynomial regression model is also appropriate. This paper addresses the issue of representing a fuzzy number. In the case of a general fuzzy set, a fuzzy number is formulated by using a polynomial of $2c$ degree for the number of crests c

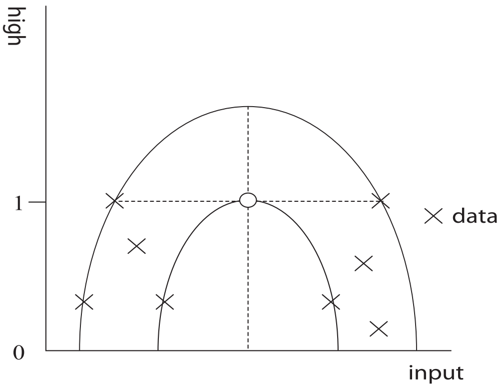

Figure 3. Identification for the type 2 membership function by a fuzzy polynomial

as follows:

$$min \sum_{i=1}^{n} \; [a_{2CH}x_i^{2c} + \ldots + a_{1H}x_i + b_H - (a_{2CL}x_i^{2c} + \ldots + a_{1L}x_i + b_L)]$$

s.t.

$$a_{2CH}x_i^{2c} + \ldots + a_{1H}x_i + b_H \leq y_i \; (i = 1l \ldots, n)$$
$$a_{2CL}x_i^{2c} + \ldots + a_{1L}x_i + b_L \geq y_i \; (i = 1l \ldots, n)$$
$$a_{2CH}, \ldots a_{1H}, b_H, a2CL, a1L, b_l \geq 0$$

where, as before, (x_i, y_i) $(i = l \ldots, n)$ are data items.

3. Application to decision-making problems in fuzzy events having indifferent zones

3.1. Some premises

The Bayes decision problem is denoted by $< S, A, U1, \pi, \phi >$, where S is the state of nature, A is a decision, $U1$ is the utility function, and π is the prior probability on ϕ (the parameter space). We denote the fuzzy-Bayes decision by $< F, A, U2, \pi, \phi >$, where F is the set of fuzzy events on S, and $U2$ is the utility function on a fuzzy event on $F \times A$ for the decision in a fuzzy environment. Zadeh (1968) defined the probability of a fuzzy event and the possibility measure of a fuzzy event as follows:

(probability of a fuzzy event)
$P(F_j) = \int_{\phi} \mu_{iFj}(\theta)\pi(\theta)d\theta \; (i = 1, 2)$
(possibility measure of a fuzzy event)
$\Pi(F_j) = \text{supmin} \; \{\mu_{iFj} \;\; (\theta), \Pi \; (\theta)\} \;\; (i = 1, 2)$

where $\Pi_(\theta)$ is an appropriate transformation of $\pi(\theta)$.

We obtain the interval of possibility measure of a fuzzy event as $[m1(F_i, A_i), m2(F_i, A_i)]$. In this paper, we pick up a fuzzy event with the two level membership functions from the indifferent zone $(\mu_{1Fj}(\theta), \mu_{2Fj}(\theta))$, assumed to have the same peak as this was shown in Fig. 4. Assume that the decision maker has the monotonic continuous utility function $U1(A_i, \theta)(i = 1, \ldots, m)$. While this assumption imposes quite strict constraints, we can apply the approach for an appropriately relaxed condition. $U1(A_i, \theta)$ and $\mu_{iFj}(\theta)$ are defined as follows:

$U1(A_i, \theta) : A_i \times \theta \to [0, 1]$
$\mu_{iFj}(\theta) : \theta \to [0, 1]$.

3.2. Decision rule on a fuzzy event

By making full use of the extension principle of a mapping, two utility functions of a fuzzy event are given as

$$\text{(the upper)} \quad U2a(A_i, F_j) = \int \mu_{1Fj}(\theta)/U1(A_i, \theta)$$

$$\sup_{\{\theta|x=U1(Ai,\theta)\}} \mu_{1Fj}(\theta) = \mu_{1Fj}(U1^{-1}(x))$$

$$\text{(the lower)} \quad U2b(A_i, F_j) = \int \mu_{2Fj}(\theta)/U1(A_i, \theta)$$

$$\sup_{\{\theta|x=U1(Ai,\theta)\}} \mu_{2Fj}(\theta) = \mu_{2Fj}(U1^{-1}(x))$$

where $U1^{-1}(U1(A_i, \theta)) = \theta$.

We pick up the representative interval by the level cut as $[U2a(F_j, A_i), U2b(F_j, A_i)]$.

We obtain the fuzzy expected utility as follows:

$[E(A_i), E(A_i)] = [\Sigma m1(F_j, A_i)U2a(F_j, A_i), \Sigma m2(F_j, A_i)U2b(F_j, A_i)]$.

Ordering of the fuzzy expected utilities is accomplished using the index of the inequality relationship of the intervals, proposed by Dubois and Prade (1988): for all $j(i \neq j)$, if $E(A_j) \geq E(A_i)$, the optional decision is A_j.

4. Conclusion

This paper has introduced a fuzzy linear polynomial regression model as a technique to identify a fuzzy number having an indifferent zone. Although the method of identifying is also possible to some extent by using a conventional fuzzy linear regression model, a dome-shape is suitable as the shape of a fuzzy linear polynomial regression model in the condition of non-discrimination. Moreover, this paper has presented a fuzzy event with the strong implication of non-discrimination, and has also described the way to expand the laws of decision-making in a fuzzy event.

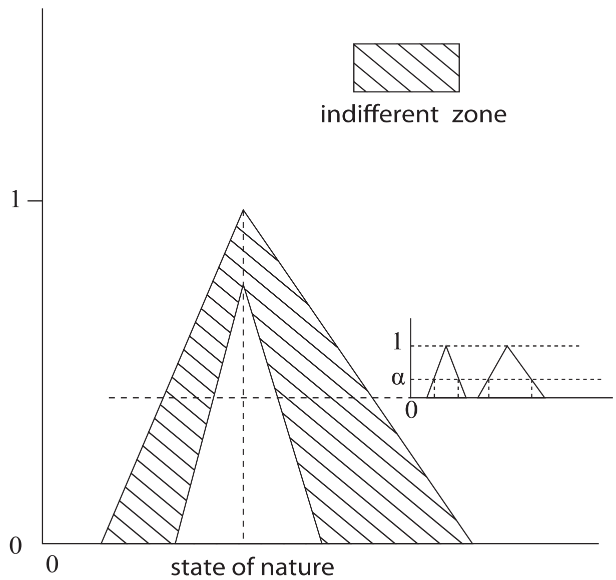

Figure 4. A fuzzy event with two type 2 membership functions

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